

**Supply Chain Analytics**  
**Prof. Dr. Rajat Agrawal**  
**Department of Management Studies**  
**Indian Institute of Technology-Roorkee**

**Lecture-14**  
**Tracking Signal and Seasonality Models**

Welcome back, we are discussing the forecasting methods in a supply chain environment. And so far we have discussed different types of time series analysis methods. We have discussed about the simple methods where we are taking simply average of the past data. And on the basis of that average we are determining the forecast for the coming periods. Then we also discuss about the limitations of those average methods.

And we discussed about a smothering methods, where we consider the demand is fluctuating about a base value. And in those a smothering methods we discussed the some type of very fundamental method where there is no type of characteristic available in our demand data. We started with basic exponential smoothing model in which only there are fluctuations around a base value, a level value.

Then in our last session we discussed a slightly more complicated model where some kind of trend was also available in our demand data. And with the help of trend and base we actually determine the forecasted value for the coming period. Now we are moving to a more complicated model where seasonality is included in our demand data.

**(Refer Slide Time: 01:44)**

Year	Month	Actual Demand	Forecasted Demand
2016	1	100	100
2016	2	110	110
2016	3	120	120
2016	4	130	130
2016	5	140	140
2016	6	150	150
2016	7	160	160
2016	8	150	150
2016	9	140	140
2016	10	130	130
2016	11	120	120
2016	12	110	110
2017	1	100	100
2017	2	110	110
2017	3	120	120
2017	4	130	130
2017	5	140	140
2017	6	150	150
2017	7	160	160
2017	8	150	150
2017	9	140	140
2017	10	130	130
2017	11	120	120
2017	12	110	110

And here we have seen in the last class, that some data is available to us. Where, we are saying that ratio seasonality is present in this demand data. We discussed in the last class, that it is almost impossible from the naked eye to determine the type of data, which is available with us. The second column in this table represents the actual demand of 2016 for various months.

And for 2017 from January to July. Now from this data it is almost impossible that somebody can say there is no component it is a horizontal demand data. Somebody can say, this data represents trend as well as seasonality. And now I am claiming that this data has ratio seasonality. So, for this purpose that whether it has ratio seasonality should be use the ratio seasonality model, should we reuse the trend model, should be use the trend with seasonality model.

We discussed in our last class. That there are different types of forecasting error measures. And, out of that we discussed MAD. That is Mean Absolute Deviation. This is the most important measures which helps us in selecting the appropriate type of forecasting model. We discuss the procedure to calculate the Mean Absolute Deviation. We discuss other type of forecasting error measures also like average error.

We discussed MSE that is Mean Squared Error and then we also discuss about MAPE which is Mean Absolute Percentage Error, but out of various types of forecasting error measures we said that MAD-Mean Absolute Deviation is the most important type of model is the method which

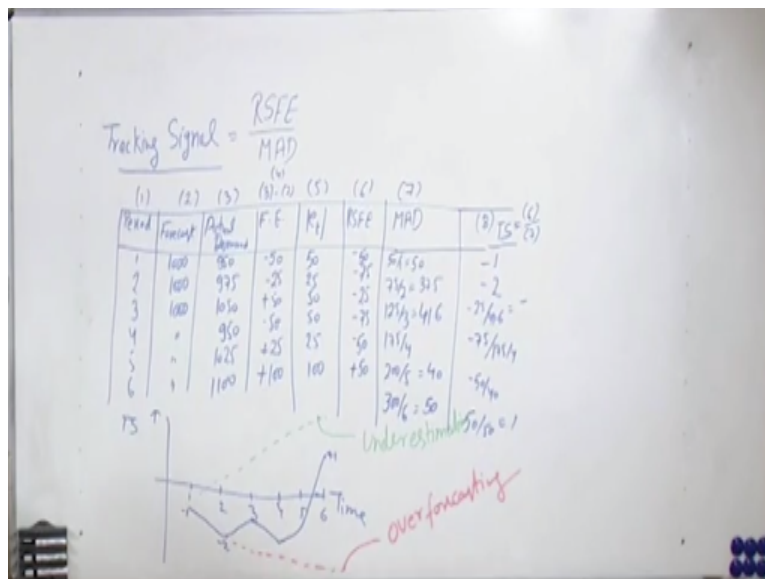
helps us in determining the appropriate type of model for the forecasting purpose. And at the same time since we have talking of smothering methods there are smothering constants alpha, beta, gamma.

What should be the appropriate values of alpha, beta, gamma. That will also be determined using these methods of forecasting error measures. But, along with those methods of forecasting error measures. We also have a more measures which will help us in deciding whether the model which we are using. This model has any kind of biasness or not. Now the meaning of biasness is that this is the actual demand.19, 25, 19, 21 etc., etc.

Now, if continuously if I over forecast my demand is always less than the forecasted value or vice versa. Demand is always more than the forecasted value. So, in both these cases I will say that my forecasting model has some kind of biasness. Either I am regularly over forecasting or I am regularly under forecasting. In a good forecasting model, in a appropriate forecasting model we should have a fluctuating kind of situation.

Sometime, I may over forecast, sometime I may under forecast. So, that the nut nut story should be more like the concept of average error. It should be on a 0 side. So, that is one thing which is determined with the help of one more concept which known as tracking signal.

**(Refer Slide Time: 05:43)**



Now, this concept of tracking signal will give you fair idea. Whether, is our forecasting model is moving in one direction or it is a balance kind of model. Now to understand the calculation of forecast this tracking signal, in case of forecasting. This tracking signal is calculated as RSFE divided by MAD. RSFE this stands for Running Sum of Forecasting Error.

MAD we have already discussed in the last class, that is Mean Absolute Deviation. So we need to calculate Running Sum of Forecasting Error and MAD. And, with this ratio you can calculate the tracking signal. Now I can demonstrate you a simple calculation of tracking signal. And then with the help tracking signal calculation we will also understand the physical significance of the biasness of the forecasting model.

So, now let us have a very sample of tracking signal. You can have the real time calculation also. The reason of discussing this tracking signal in this class is so, that we are discussing the real time decision making. The concept of tracking signal will help us in improving or in adoptive the forecasting model in the real time situation. So this tracking signal calculation, which is I am showing you for a sample data. Let us have six periods.

And for each of these periods let us forecast, let us assume some forecast value , that I am forecasting 1000 units for each of this periods. Now the actual demand for these periods, let us assume some values of the actual demand for these periods. And these demands may be 950, 975, 1050 then again 950, 1025 and 1100. Let us say these are the demand values of over the period. Now with the help of these forecasting values and the actual demand data I can determine my forecasting error which is actual demand.

This is column number 1, this column number 2, this is column number 3. So forecasting error will be column number 3 – column number 2. That is my column number 4. Now, 950-1000, this is -50. 975-1000 this is -25. 1050-1000 this is +50. 950-1000again -50. 1025-1000 +25. 1100-1000 +100. So these are the forecasting errors. The absolute values of these errors, or you can say this values will be without sign.

So these are 50, 25, 50, 50, 25 and 100. Now the next column will be running sum of forecasting error that is my column number 6. The running sum of forecasting error is actually the summation of various elements of column number 4. So this is -50. Then you add -25 in to this, becomes -75. You add 50 in to this so, that is of + sign. So,  $-75+50$  it becomes -25. Then further -20 it becomes  $-75+25$  in remains -50, and +100 so, it finally end with up with +50.

So, these are Running Sum of Forecasting Errors. Then you also calculate with the help of these Dt values your MAD. That is your column number 7. Now what is the MAD.? MAD is actually this et divide by the number of periods. So as you go up you need to add up the values of, absolute values of forecasting errors. So here it is 50 by 1, 50. In the second case  $50+25$  that is 75. And  $75/2$ . So it remains 37.5. Then in third case  $50+25+50$  it is 125 divided by 3.

So it remains like 41.6. Then in the next case  $50 + 50 + 50 + 25$ , 175 divided by 4. For the fifth period it will be  $175 + 25$  that is 200 divided by 5 that is 40. And for the last period  $200 + 100$  that is 300 divided by 6, that is 50. So, you get the values of Mean Absolute Deviation in column number 7. And then in column number 8, we calculate the values of tracking signal. And that will be determined by dividing the values of column number 6 with the values column number 7 respectively.

And -50 divided by 50 you get -1, -75 divided by 37.5 you get -2. Then -25 for third period divided by 41.6. Here the value is again in -, but it is somewhere less than 1, and we can have fine tuned calculations also. For the fourth period, it is -75 divided by 175 by 4. For fifth period it is -50 divided by 40. For the sixth period it is 50 divided by 50, that is 1. So, now you see you started tracking signal from the first period that is -1, it increases to -2.

Then it decreases it remains about only about 0.5. Obviously of with negative sign, here also in the negative sign, here it is slightly more than 1 -1`and it is coming + sign in the sixth period. So, if I plot the values of tracking signal, on a graphical plane, so, this is time one on the x axis, we have the values of the tracking signal. So, if this is the first period, second period, third period, fourth, fifth, sixth like this.

So for these periods we can start with -1, -2 and then in the third period it is coming very close to 0 value. And then in the fourth period it is actually  $75,300$  divided by  $175$ . So it is around 2- less than -2 it is around -1, and in the sixth period it is crossing 1, and it is becoming +1. So you see you have some type of fluctuations though all the fluctuations are in the negative side from period 1 to period 5.

You have all fluctuations which are in the tracking signals but there are fluctuations sometime it is going to -2 sometime coming to 0.5 values also. And, then in the six period it is coming to +1. So here we can say the forecast sometime it is over estimated sometime it is under estimated. So it is a more like a reasonable forecasting model. But if it is like this where you have continuously this type of data.

If you have this type of data which is represented by these red dots, where you are forecasting model is giving you a tracking signal which is represented by this red dots that means you are all the time moving in the negatives and it this means you are all the time doing the over forecasting. This red data is a symbol of over forecasting. Your demand is not that much. But you are doing over forecasting.

On the other head if you start in this session and then you move this way, the dots representing the green line, if this type of tracking signal is there where continuously the value of tracking signal is moving upward. So this type of tracking signal is representing under estimation where I am forecasting less, the demand is more and the physical significance is that I am not able to fulfil the demand of the market. Some of the demand I am not able to fulfil my service level is low.

And as a result of that the customer satisfaction is also low. When I am doing the over forecasting it means I am filing excessive inventory in my supply chain. I am keeping more inventory in my supply chain which is not required because, I am not touching that level of demand which my forecasting is suggesting. As a result of that this excessive inventory in my supply chain has eaten away my profits.

So, both these lines which are represented by green or by red or not desirable. My line ideally should fluctuate above this X axis. Above the time X axis, the still if you see the blue line the line which we got as a result of this data that line is still acceptable. Because, there are certain fluctuations in this line, but if fluctuations are not there we will not be able to accept or we will say that our forecasting model has some type of biasness.

And, that is not desired, so for that reason this tracking signal is a very important tool it is very important you can say outcome which will help us in determining whether the model is having some kind of biasness are not. And therefore this concept of tracking signal must be used in combination with the errors forecasting error measures particularly MAD and tracking signal gives you biasness, and since we plot this tracking signal in real time environment.

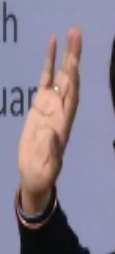
So this tracking signal data will help us for any kind of adaptation of the data. If we require any kind of adaptation this tracking signal will help us that you can modify your model, you can develop your model you can take care whatever happening in your recent data. So, that your model does not exhibit any kind of biasness. And with this now we are coming back to this original problem which we started and now let us see how to go ahead with solution of this model.

So we will talking of we right now just for this sake of understanding have just assumed thus the model or the data is showing the ratio seasonality it is exhibiting the ratio seasonality and with this assumption we are going ahead to understand the process of solving this ratio seasonality model. Now in this particular case this model needs to be de seasonalized first whenever we have is seasonal data.

Whenever we have a seasonal data we need to de seasonalized that and for that purpose as we discussed in the last class we first need to take the average of past demand. So in this case we are taking only 2016 data, for the purpose of initialization of the solution process. We are not taking this 2017 data to start the solution process. I am taking only the first 12 cells for starting the solution process. We will take the average demand of 2016 and the average demand will be this sum of actual demands of 2016 divided by 12.

(Refer Slide Time: 20:43)

- Average Demand of 2016 is 24.07
- To deseasonalize the demand of 2016, we need to divide the demand of each month with average demand. E.g in case of January 2016  $19.36 / 24.07 = 0.804$




And that comes to be 24.07. So this is the average demand for 2016 now we will divide the actual demand of any periods like of January 2016, 19.36 by the average demand and that will give me the seasonal index. So like we are shown the sample calculation here. The actual demand of January 2016 is 19.36 divided by the average demand. I got this seasonal index as 0.804. And so on we do this calculation for all the months of 2016 from 0.804 to 1.128.

(Refer Slide Time: 21:26)

### Data with Seasonal Index for Year 2016

Date	Actual Demand	Index	Seasonal Index	Forecast
Jan-16	19.36		0.804	
Feb-16	23.45		1.057	
Mar-16	19.73		0.821	
Apr-16	21.48		0.892	
May-16	23.77		0.983	
Jun-16	23.42		1.035	
Jul-16	23.79		0.989	
Aug-16	28.35		1.178	
Sep-16	26.8		1.113	
Oct-16	25.22		1.052	
Nov-16	25.22		1.049	
Dec-16	27.34	30	1.138	
Jan-17	32.52	21,048		
Feb-17	31.33	10,968		
Mar-17	25.32	30,055		
Apr-17	27.33	20,053		
May-17	26.38	10,98,79		
Jun-17	25.72	10,10,75		
Jul-17	28.24			



So all these seasonal indices are calculated by dividing the actual demand. These actual demands by the average demand of 2016, that is 24.07 and students can try these calculations to get this seasonal index. Now, these seasonal index of lot of significance, because this seasonal index will



be used for calculating the forecast for the 2017. So we discuss the development of the formula for seasonal index.

And now with the help of the seasonal index and the base values we will determine the forecast for the 2017. You already have the data up to July 2017 in this table. So we will like to determine the forecast for August 2017 and to determine the forecast for August 2017. In our last lecture, we discuss how to develop the formula and now we are directly showing you the developed formula which we saw in the procedure in the last lesson.

**(Refer Slide Time: 22:36)**

• The deseasonalized smoothed average,  $\bar{S}_t$

$$\bar{S}_t = \alpha \left( \frac{D_t}{I_{t-l}} \right) + (1 - \alpha) \bar{S}_{t-1}$$

Updated Trend will be  $I_t = \gamma \left( \frac{D_t}{\bar{S}_t} \right) + (1 - \gamma) I_{t-l}$

Forecast will be  $F_{t+1} = \bar{S}_t I_{t-l+1}$

So this forecast will be the forecast for August 2017 which is represented by  $F_{t+1}$ .  $F_t$  represents the current period so current period is July 2017, so  $t+1$  means 1 period ahead, so that becomes August 2017. So I am forecasting for August 2017 and today I am in July 2017. So I will be using updated base for July 2017 which is represented by  $\bar{S}_t$  that is the updated base value in July 2017 of July 2017.

And this  $I_{t-l+1}$  this 'l' represents the period of seasonality. In this data you have monthly data for 2016. So, you have the demand data available for each month. So, in this case the period of seasonality. Since, monthly data is given is 12 sometime you have quarterly data available to you. So, in that case period of seasonality 'l' will be 4. If you have half yearly data available data to you in that case the period of seasonality is 2.

So 'l' will be 2, so depending upon the type of data available to you 'l' will be in case of half yearly data it will be 2 in case of quarterly data it will be 4 and in case of monthly data it can be 12 and even if you have fortnight data 'l' can be 24 also. So, depending upon the type of data available to you. The values of 'l' will change, so if weekly data is available to you so in case of weekly data 'l' can be 52 also.

So, depending upon since we are considering the monthly data. So, 'l' is 12. Now the formula says  $t-1$  so if I am talking of July 2017. So  $t-1$  will be July 2016 and  $+1$  it means August 2016. So, for forecasting for the month of July 2017 for August 2017. I am going to use the base value of July 2017 and the seasonal index of August 2016 and their multiplication will give me the forecast of August 2017 and for this purpose, I require two things.

One is the updated base value of July 2017 which I am going to calculate from this formula and the second is  $I_{t+1}$  which is the seasonal index of August 2016. So, that seasonal index of August 2016 is already available to me that is 1.178. The updated base value of July 2017 is not available to me. So that I need to calculate and once I multiply this value whichever will come here with the value of August 2016, 1.178.

That will give me the forecast which I will put here for August 2017. So, this value I need to get here multiplied by 1.178 that is the seasonal index of August 2016 will give me the forecast of August 2017. Now for that purpose forgetting the updated base value of July 2017, I will use this formula now in this formula you see what I am taking since this is the base value so here  $D_{t-1}$  this represents again the de seasonalized demand of August of July 2017.

So I will whatever  $D_t$  is the actual demand when I divide this  $D_t$  by seasonal index factor I get the de seasonalized demand or you can see the base value. So whatever is the current base value and then whatever is the previous base value. One period before, so I will take a part of that and then. So that alpha is the smothering constant. A smothering constant multiplied by the de seasonalized demand of July 2017 multiplied.

Then you have the addition of  $S_{t-1}$  the remaining component comes from the base of June 2017. So, if I see from this table I take this value 30.0273 this is  $S_{t-1}$ . This is  $D_t$  29.14 this is  $D_t$ . This  $D_t$  will be divided by 0.988. to get the my de seasonalize demand of July 2017. 29.14 divided by 0.988 multiplied by alpha value+ one component of my updated base + a part of that will come from 30.0273.

And, this will be multiplied by  $1-\alpha$ . So, with this way I will get my updated base value for July 2017. When, I multiplied this by the seasonal index of August 2016. I will get the forecast of August 2017. But at the same time I will like to update my seasonal index also though it will not be used immediately.

It will not be used immediately it will be used for 2018. But, I continuously need to update my seasonal index also and for that purpose this equation will be used of seasonal index where I get the seasonal indexes here. This is  $D_t$  upon updated  $S_t$  which is multiplied by the a smoothing constant of seasonality that is  $\gamma$  and this is the  $t-1$ . This is the seasonality index of the same period one cycle before  $t-1$ .

And this updated trend of July 2017 will be used in year 2018 not now, and we get the updated base value and we already have the old seasonal index multiplication of that will give us the forecast for August 2017. So with this ratio seasonality model where we have some kind of seasonal component and this seasonal component is ratio in nature. So we have got this forecasting of that method.

So we are closing our this session at this end. Now in our next class we will take this discussion to a more complicated forecasting model where we will have a trend component as well as seasonality component in built same data. Thank you very much.