Supply Chain Analytics
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Lecture-35
Representation on Uncertainty in Supply Chain

Welcome back in our last two stations we discuss the role of analytics in the supply chain and be focused about the predictive analytics that how we want to determine that on the basis of past what can happen in the future and what will happen in the future, this is also named with lot of uncertainties. If there is a well-defined period what has happened in the past accordingly things will happen in the future.

But we all know that future is full of uncertainties and therefore we will talk of uncertainties in today's session. I can take you to the situation before 1990, in most of the pharmaceutical companies before 1990s the environment was very much certain and company is used to have dedicated capacities in the plants, the meaning of dedicated capacity was that in a part of plant you will have the entire facility dedicated for a particular kind of medicine.

But nowadays if you go to a pharmaceutical company you will find that most of the companies are having flexible capacities, because it is highly uncertain that what kind of problems will be there and problem what type of new decisions will be there and accordingly you will require new types of medicine and therefore on a very fast basis your, self search engine and accordingly the new types of medicines you need to manufacture in the same plants.

So therefore flexible capacities are required and these things are the subject matter of uncertain environment which are going to be there and we will discuss some of the specifications that how some of the tools can help us to handle the issues related to uncertain environment.

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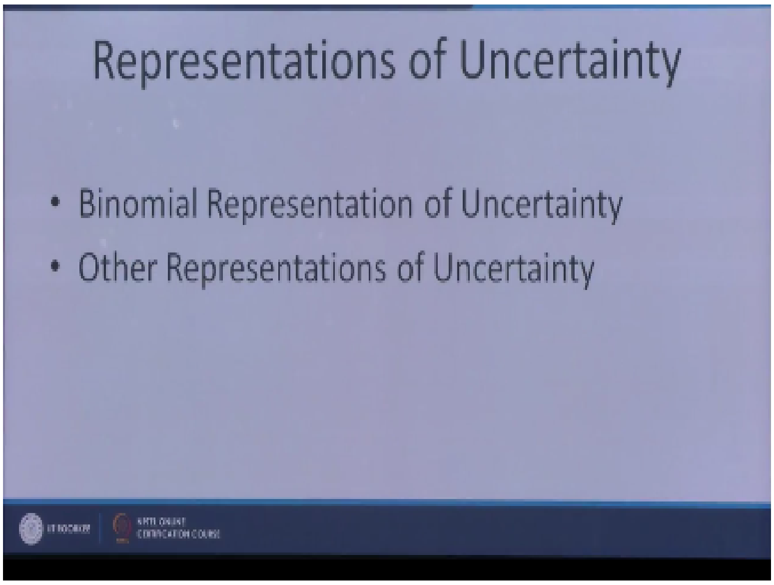

The type of uncertainty which we are going to discuss that is binomial representation of uncertainty, there can be normal representation of uncertainty also, but in this session we are going to discuss the binomial representation of uncertainty so we are going to discuss the binomial representation of uncertainty, so we are going to discuss the binomial representation of uncertainty, the binomial representation of uncertainty there can be 2 cases.

One case where we have the multiplicative uncertainty and in another case we can have additive uncertainty, this slide gives us the example of multiplicative uncertainty. Now there can be uncertainty with respect to so many factors, you are demand may be inserted in the future, you are price of the commodity maybe uncertain in the future, the exchange rate maybe uncertain in the future, the supply can also be uncertain in the future.

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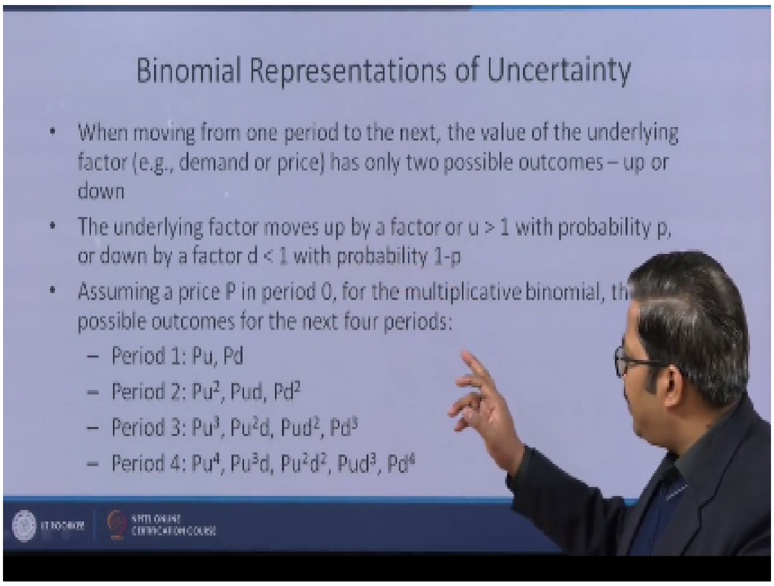

So you can take any underline underlying factor at once to understand the method of solving such type of situation, so here I am taking one underline that is price and I am representing the price with capital P in my starting time at T0 my price of the commodity is P, now price that increase or price can decrease in the subsequent period. So I am taking two factors, one is small u and another is small d, small u is greater than one that price is increasing.

If I multiplying this small u with my capitol P, so that is the increase price in the next period and small d is less than 1 if I multiply capitol P with small d the price is decreasing in the next period. So for example if price that increase by a factor of 5 % the value of u is 1.05 and if I say that price can decrease by a factor of 5% so the value of d is 0.95, so if price P is in time 0, so in time1 period one that price can have two values PU and PD.

So if P is 100 so that is 105, if Pd is there it is 95, now since it is uncertain environment the probability is also required, so the probability is small P for this u at 1-P for this D, so the chances of getting this price PU is having a probability of P and chances of having this price PD is having a probability of 1 – P. Now going further in the next period after period 1 Pu can further increase and Pu decrease by a factor of D also.

So Pu can become Pud. Similarly Pd can also increase with a factor of you, so Pd can become Pud and Pd can further decrease with the factor of d, so Pd can become Pd square, then Pu square which is there in the period 2, can further increase with the factor of you, so it can become Puq and Pu square can decrease with the factor of d, so it can become Pu square d. Pud can also increase or decrease it will increase with a factor of u, it will decrease with the factor of d.

So when it increases it because Pu square d and when it decreases its becomes Pud square and so on fpr Pd square it can also increase or decrease when it increases it becomes Pud square and when it decreases it becomes pdq and similarly from period 3 when we move further to period 4 Puq can also take two values it can increase with the factor of u, it can decrease with the factor of d, so it will take Puq to Pu4, and Puq to Pqd.

Pu square d will take Puqd and Pu square d square, Pud square will take 2 values Pu square d square and Pudq will also take 2 values, Pd4 and Pudq and you can go for period and also by doing these type of calculation each time you have two possibilities that your present underlying factors may increase or decrease and probability of increasing Rp and probability of decreasing is 1-P.

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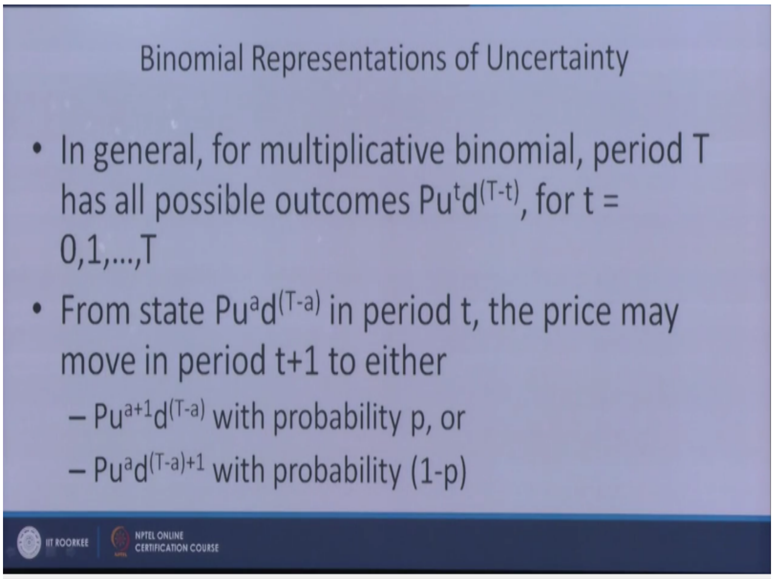

Now these are the way in which you can write on a generic basis that how things are going to change in a multiplicative binomial representation and this gives you a very general type of understanding, if you recall the previous slide that the possibility can be represented as p u to the power 2 into 2 T-t for any particular t from zero to capital The For period t for period 2, period 3, period 4 you can find all possible outcomes in this manner.

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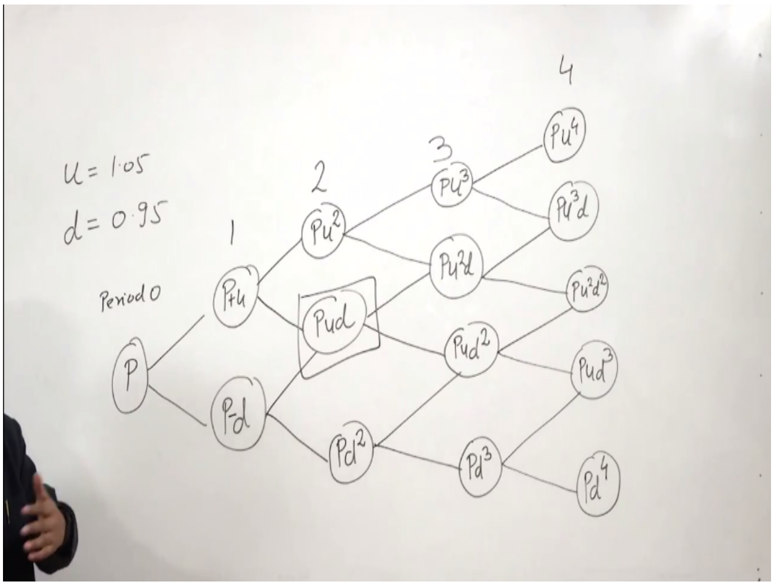

And therefore it is also very similar simple to understand that at period zero we are having this price p at period 1 price may take two values that is pu and Pd, so for that let us go back to the previous slide, so the price is taking two values Pu and Pd, further in period to see you take two values one is pu square and another is Pud when this factor is decreasing you can see on the screen also.

Similarly Pd will also take 2 values one when it is increasing it will become Pud and when it is further decreasing it becomes Pd square, going to the third period Pu square may take two values, it increases, so it becomes Puq and when it decreases by a factor of d it becomes Pu square d. Similarly pud can also take two values it can it increase and when it increases it become Pu square d.

And when it decreases it becomes Pud square, similarly Pd square can also get 2 values, it can increase when it increases it becomes Pud square and when it decreases it becomes Pdq and you can simultaneously match the development of this kind of branch diagram with uncertain values of our underline factor which we have given on the slide. Then further if I go to period Puq will also have two possibilities that is Pu4.

If it increases and decreases it becomes puqd, Pu square d also has two possibilities if it increases it become puqd and if it decreases it becomes Pu square d square. Similarly Pud square also has two possibilities, if it increases it becomes Pu square d square, and if it decreases it becomes Pu dq and similarly Pdq also has two possibilities, if it increases it becomes Pudq and if it decreases it becomes Pd4.

And you can match this diagram with the values which we got. So this is a very simple way to develop the uncertain environment this is a very simple way and just by going into the same methods you can develop this diagram.

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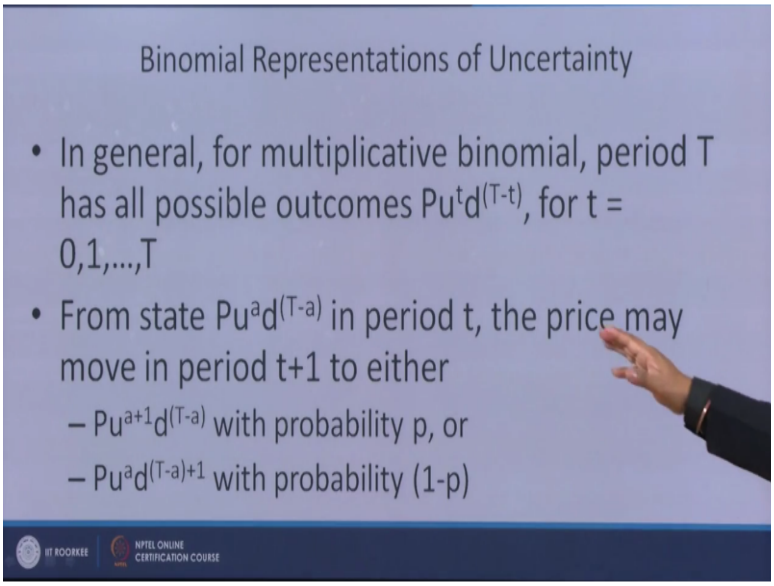

And therefore the next slide gives us the idea about how you can have a generalized representation of any state in the development of this uncertainty diagram, that is pu small t, d to the power T-t for a particular period capital The Capital T can be period 1, period 2, period 3, period 4 and in these period you have a variety of states and these formula will help us to go for all those different types of states.

There are in each of these things in a particular period from this state the price may move in increasing or decreasing order you see let say in state 2 Pud at time 2 Pud is a state, now from Pud price can increase it becomes Pu square d, price can decrease it becomes Pud square, so both these movements are possible, so this is again the generic representation of 2 possible states that the probability of increasing the price is P and probability of decreasing the price is 1-P from a particular state to the next time interval.

So this is the generic representation of uncertainly and this is the specific movement in this particular case. This is about the multiplicative uncertainty, in most of the cases in our supply chain decision we will be using the multiplicative uncertainty that uncertainty is increasing or decreasing as a multiplying factor to our underlying factor, it is also possible to have additive representation of the uncertainty.

In that case these P will have addition of the factor u and this will have negative of the d, so it is P + u the underline factor is increasing by u and underline factor is decreasing by d, so you are adding and subtracting a fixed quantity from your underline factor, so if this type of issues there it is the additive representation of the uncertainty, so similarly you can go for all subsequent period if you want to use this P + u or P – d, so all these representations will change.

And we will have a new table for representing the additive binomial uncertainty, but there are certain cases where it is not possible to have the additive binomial uncertainty, particularly in case of price, demand etc, which are very important in case of supply chain, because when you have you all can understand when you have this additive or subtractive values + u –d you can get to negative values also and it is not possible to have negative demand, negative price etc.

You can understand for example if price is initially Rs. 10 and you are talking of uncertainty that price can increase by + 2 rupees and price can decrease by - 1 rupee. So in that case it is possible that if you talk of this type of length P – d, P – 2d, P – 3d, P – 4d, P – 5d and in 11th period it will be P – 11d and at that time price will be negative, at that time you can imagine a situation where the price will be negative.

And that is not possible you cannot have a negative price, similarly demand is also there you have initial demand of the product, let say 100 units in a particular time period that in month 1 you have the demand of 100 units and there are values of + 10, - 15 that demand can change by 10 units or can decrease by 15 nits. So if I talk of this last slide there may come a time when demand make go for the negative values, so that is also not possible.

So many times you will see because our underlying factor cannot take the negative values it is not possible to use additive binomial uncertainty, so we will use in most of the cases the multiplicative binomial uncertainty, then there is another reason for that reason one reason is this that you cannot have the negative values of underline factor, the other factor is that it is not possible to have some kind of change in a very small time like for that purpose if my price of a product is 15 dollar.

In that case it is not very appropriate to say that price may change by a factor of let say 5 dollar, it is very you can say things which is not possible, it is not appropriate to say that price can change by a factor of plus minus 5 when the original price is 15 dollar. However same thing makes cells if the prices 500 dollar, if the price is 500 dollar in that case if I say that price can change by plus minus 5 dollars in a period it makes sense.

So you need to see that what is the size of my original underline factor and in that case probably you can use the additive uncertainty, but if the size of the original underlying factor is relatively small again it is not advisable it is not possible, it is not good to have the additive uncertainties because you will not have this type of fixed values which can be added or which can be subtracted from the original underlying factor.

So whenever the size of the initial underlying factor is relatively small we go for the multiplicative binomial uncertainties, so that is another important reasons, so two things are there that sometime your underline factor cannot take the negative value which probably are possible in case of additive uncertainties, so you will not take those things and when this size is also a small it is not advisable because in that case fixed uncertainties are not possible.

So uncertainties are percentage of the original values, so in that case also will go with the multiplicative type of binomial uncertainties. Now once we are clear about this representation of uncertainty binomial representation of uncertainty it also important to say that here we are only taking two possibilities that price increase by either by factor of u or price can decrease by a factor of d. So when my original price is P it can take only two states Pu or Pd.

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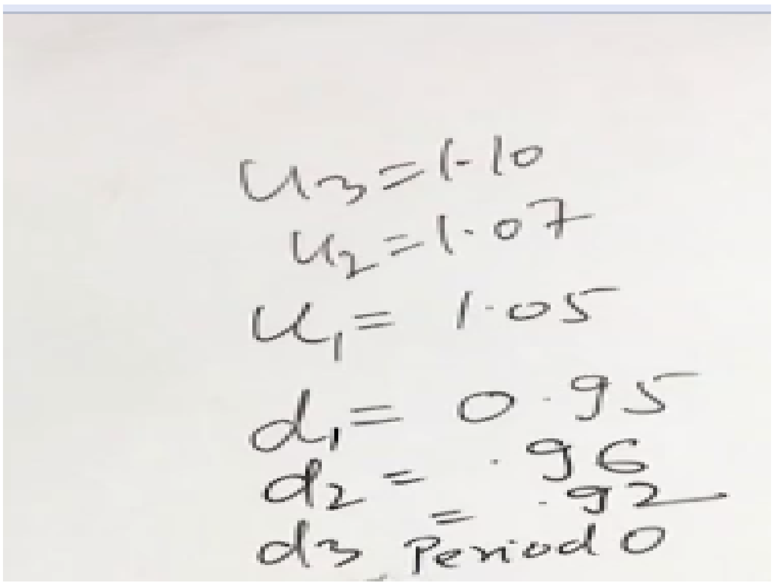

But in real life it is not so, in real life price can increase by different values, price can increase by u1, u2, u3, there are different probabilities for increasing the price from p to pu, Pu1, Pu2, Pu3. Similarly price can decrease to d1, d2, d3 there can be different underline factor, different values of d that d1 is 0.95, d2 is 0.96, and d3 can be 0.92. Similarly u can be 1.05, u2 can 1.07, u3 can be 1.10.

And you will have different probabilities for all these things that the probability of u1, u2, u3, d1, d2, d3 are different from moving P to that state, but here we are considering only two states, so there are two reasons for that one is that we are starting, we are just learning that how to incorporate uncertainty in our discussion, so for that purpose it is easy to take only 2 states, that is one answer.

The second answer is that though if your time period is slightly long then all these u1, u2, u3, d1, d2, d3 will come, but if you get a very small time period then probably u can divide that uncertainties only into two states u and d. So this type of situation can also be taken practically, it is not only just for the academic discussion, it can be taken practically also, but then we need to take the very smaller time period where you can only think of two possibilities whether u or d.

As you increase the time period you will have chances of different states with different probabilities, all those things can also be handled but that will be slightly more complicated model and then thanks to IT, thanks to computer science, these things will help us in handling those complex situations also. So if I reduce my time period if let say I am doing the determination of pricing on the monthly basis, probably u and d is fine.

If I take demand factor on a weekly basis u and d is fine, but if I take price on yearly basis then probably u1 to u3 and d1 to d3 can come, so I need to see that what is the optimum, you can say time period for taking this into my account, so that u and d only two possible states are required, so these are the initial things and when you do this binomial representation of uncertainty for fairly small time period.

And you do it for a long time you have T=n and n is fairly large, then this binomial representation of uncertainty becomes almost equal to normal distribution of uncertainty, so that is also important to understand that then it is nothing but simply the normal distribution, so we should take in better decision making better predictive analytics we should take a smaller time period and smaller time period for long time.

So that you can have the normal distribution of the uncertainty in the long run, so with this background now this is the additive representation of the uncertainty just as we have discussed if initial underline factor is P, so it can take 2 states Pu P + u, and P-d and similarly as the explanation was there for the multiplicative uncertainty P + u will again have 2 possibilities, so if it is increasing it becomes P + 2u and if it is decreasing it becomes P + you – d. Similarly P – d also has a 2 states where it can increase or decrease.

So when it is it becomes P + u – d and when it decreases it becomes P – 2d. I am not going to discuss the state 3 and period 4, I request the students that you write that what will be the different states when we are moving in the same way for the additive uncertainty and try to match your answer for what we have written on the slide that these are the different states in period 3 and these are the different states in period 4.

So please do not see these 2 lines and try to write on your own and then you can match that whether you are also getting the same kind of states for period 3 and period 4 and again the simpler way of achieving these states is to go by this type of tree arrangement that how you are getting different types of states during the different time periods, so that this will help you to achieve that tree for the additive uncertainty.

And here also you have generic representation of additive binomial uncertainty that T has all possible outcomes at a time period t, that is P + Tu - 3 – t – T x d and for this T you can have all possible States from t = 0 to capital T, so that is the way you can develop your different states for a particular time period capital T. So like for an example if I take T = 1, so your different possible outcome using these binominal representation are P + Tu – T – t x d.

And I will take 2 values of small t and these 2 values are small t are 0 and 1 capital T is one, so possibilities are P this T is 0 first x u – T is 1 – 0 x d so one state becomes P – d, the other state the next time I will take the value P = 1, so this becomes P + 1 x u – T is again 1, small t this time I will take 1 – 1 x d, so this becomes P + u and you can see that at time period 1 we have these 2 states P + u and P – d.

So similarly you can determine the states for any time period using this generic representation of additive strategy additive uncertainty, the additive uncertainty as we discussed can only be use where we have large size of our initial underlying factor, so we stop our discussion at this point and in our next session we will take a numerical example to discuss the role of uncertainty in predictive analytics. Thank you very much.