

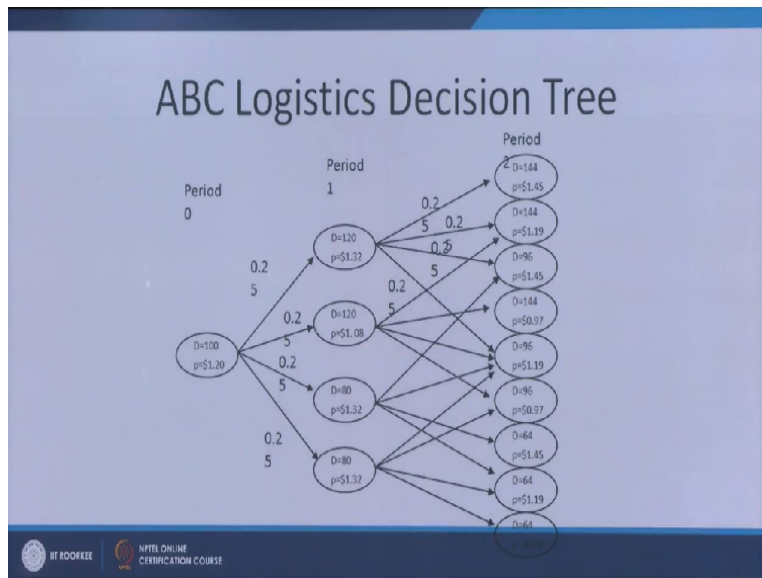
Supply Chain Analytics
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Lecture-38

Example of Using Decision Tree incorporating Uncertainty in Two Key Factors

Welcome back, in our last session we were discussing about developing the decision tree for a particular type of situations where we had the problem of developing the decision for our warehousing requirements. Three alternatives are there, and we developed the complete decision tree, which is there now for the ready reference of us.

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And this type of decision tree we finally developed in our last session. That the initial demand was 100 units, the initial price in the spot market was 1.2 dollar per square feet, and chances of increasing the demand are 20% chances of increasing the price is 10% and chances of increasing the price is also 10%, and increasing the demand, and increasing the price means the uncertainties of demand and uncertainties of the price are independent.

So I cannot say that by change of price demand is going to affect or by change of demand price is going to be affected. So these two things are independent, and therefore the probabilities of 0.25, 0.25, 0.25, and 0.25 are there for combine effect of change in demand and change in price.

And, therefore this type of decision tree is available in front of us, now when this decision trees are developed.

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ABC Logistics: Three Options

- Get all warehousing space from the spot market as needed
- Sign a three-year lease for a fixed amount of warehouse space and get additional requirements from the spot market
- Sign a flexible lease with a minimum change that allows variable usage of warehouse space up to a limit with additional requirement from the spot market

After that we need see that what are the three alternatives which are there in front of us. So the first alternative is that we can take all the requirement, we can take all the space from the spot market and for that purpose. In the spot market we did this type of calculation that for a particular situation where demand is 144 and the spot market price is 1.45, so the total cost of this type of decision where my price is 1.45.

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100% from Spot market

$P = \$1.45$
 $D = 144,000$

Cost of Space
 $Rev. = \$1.22 \times D$
 $Profit = Rev - Cost = -\$33,120$

Decision Tree:

- Root Node: $D=100, P=120$
- Option 1: $D=144, P=1.45$ (Probability 0.25) → $-33,120$
- Option 2: $D=120, P=1.32$ (Probability 0.25) → $4,320$
- Option 3: $D=96, P=1.19$ (Probability 0.25) → $-22,080$
- Option 4: $D=96, P=1.45$ (Probability 0.25) → $2,880$

Profit = $120(132-122) = -120(10)$

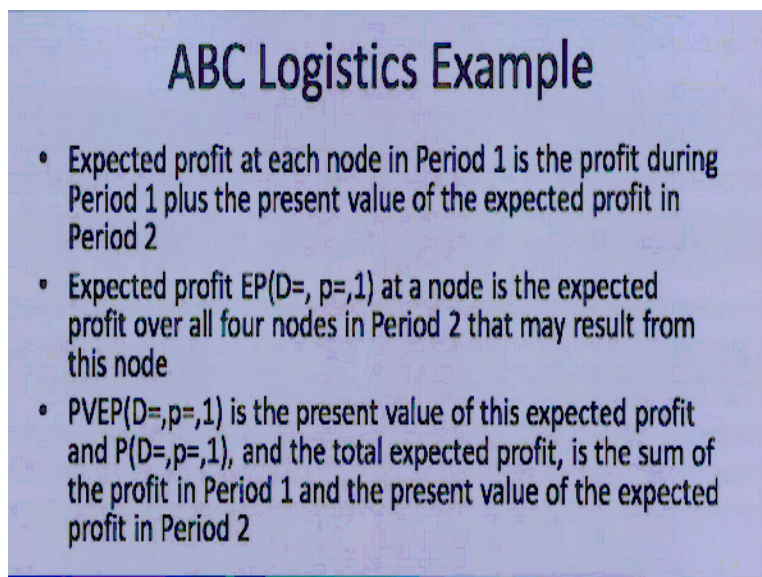
$E.P. = \sum (Prob \times Value)$

$PVEP = \frac{E.P.}{(1+K)}$

And demand is 144, 1000 units, so we calculated in the last session you remember that total cost of a space, because I am taking 100% from spot market, since I am taking 100% from the spot market, so the cost will become multiplication of this, and this is the price, so the multiplication of these two things, and then the second thing I need to calculate revenue from selling the units. So I will be selling because, I am fulfilling the entire demand.

So revenue will be the cost the selling price that is given to us as 1.22 and into the demand and then will calculate the profit that is revenue-cost this is revenue-cost and in this calculation with this data we got the value that is coming -33,120 dollars. This is -33,120 in fact this – sign signifies that it is a loss. If this type of situation comes where demand increase from original 100 to 144 units price increases from 1.20 to 1.45 and we are fulfilling our total requirement from the spot market. So we are going to get this is negative it means this is the loss we are going to have.

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ABC Logistics Example

- Expected profit at each node in Period 1 is the profit during Period 1 plus the present value of the expected profit in Period 2
- Expected profit $EP(D, p, 1)$ at a node is the expected profit over all four nodes in Period 2 that may result from this node
- $PVEP(D, p, 1)$ is the present value of this expected profit and $P(D, p, 1)$, and the total expected profit, is the sum of the profit in Period 1 and the present value of the expected profit in Period 2

Now we need to see from this particular case, this is the value we got from period 2 and with this type of state. Now we as discussed in the previous sessions from this we will go to time t_0 , and for that purpose from this is state I will come to this is state and let us how do we come to that period 1. Now expected profit at each node in period1 is the profit during period 1 + the present value of the expected profit in the period 2.

Now when I am talking of period 1 and this is 1 node, so the profit at this when I am try to determine, so what is the actual profit because of these things 120 and 1.32 is the spot market price, and the present value of various expected profits which are coming from this is state there are four states which are coming and these four states having the possibility of 25% each, so I will see the possibility of getting this negative profit possibility of getting this loss of 33,120 is 25% in period 2.

Similarly, with respect to 144 and price 1.19 I will calculate the profit with D equals to 96 profit 1.45 I will calculate the profit and the other state is D 96 and price is 1.19 I will calculate the profit. And, for all the profits the possibility is 0.25 and that will be expected profit at period 2 emerging from period 1, and then the total I will calculate the present value of that total for period 1, so that all calculation will discuss.

So, this is what I mention in the first point that expected profit at each node in period 1 is the profit 1 is the profit during period 1 + the present value of the expected profit in period 2. Now expected profit EP that is D and P at a particular state 1 comma 1 means at time period 1, and for a particular label of demand and a particular label of price. So when D is 120 price is also 1.32 that is the first state if you go back to your node books.

And see the decision diagram at a node is the expected profit over all four nodes in period 2, that may result from this node that we already explained then present value of expected profit. The next thing is present value of expected profit in our some earlier session, we have discussed about the time value of money concept and with the help of that time value of money concept, we understood that whatever is the future streams of cash flow you need to calculate the present value of those future cash flows.

And for that purpose we use the discounting rates, and in this example we have taken K equals to 10%, because we are going to calculate finally at time t_0 , so we need to determine that this is possible after time t_2 . And what will be the value of this in the present circumstances, and for that purpose the present value concept is required.

So, present value of expected profit is the present value of this expected profit and the total expected profit is the sum of the profit in period 1 and the present value of the expected profit in period 2, so this is about time period 1.

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ABC Logistics Example

- From node D=120, p=1.32 in Period 1, there are four possible states in Period 2
- Evaluate the expected profit in Period 2 over all four states possible from node D=120, p=1.32 in Period 1 to be

$$\begin{aligned}
 EP(D=120, p=1.32, 1) &= 0.25 \times P(D=144, p=1.45, 2) + \\
 &\quad 0.25 \times P(D=144, p=1.19, 2) + \\
 &\quad 0.25 \times P(D=96, p=1.45, 2) + \\
 &\quad 0.25 \times P(D=96, p=1.19, 2) \\
 &= 0.25 \times (33,120) + 0.25 \times 4,320 + 0.25 \times (-22,080) + 0.25 \times 2,880 \\
 &= 512,000
 \end{aligned}$$

Now, just to see the calculation that how this expected profit calculation has taken place for that purpose you see from node D equals to 120, and to have better understanding we have this diagram here, let us go back to that diagram and quickly. Let us have that diagram on the board, so that you can easily see that how things are moving this is D equals to 120 and P equals to initially 1.20 then you have four possibilities and let us discuss this only`

Where the D is increasing to, this what initially the 100 and then it become 120 price was initially 1.20 then it becomes 1.32, and then there are further four possibilities and these are 144 D is 144, and P is 1.45. The next is D is 144, and price has decrease it becomes 1.19 then demand decreases it becomes 96, but price increases it becomes 1.45, and then finally you have where demand also decreases, and price also decreases that is 1.19.

So, these are different alternatives which are emerging out of our tree. Now for the purpose of calculation of profit at our state 1, period 1. This is state 1 period 1. Here our demand is 120 price is 1.32, and there are four alternatives as shown on the board. Now we will evaluate the

expected profit in the period 2 over all states possible from this, so like this we have calculated the profit for the first case where it is -33, 120.

Similarly, you need to calculate the profit for this state also, for this state also, for this state also, so, that is what we have written here that 0.25 and for the situation of D 144, 1.19 D 96, 1.45, and D 96, 1.19. So these all are the four different states and then the expected profit is multiplied by the respective transition probability, that is 0.25. So that is the expected profit for all these things. Now when you have the calculation, so these values are coming like that here it is coming 4,320`

This value is coming -22,080 and this value is coming 2,880 and the probabilities are same 0.25 for all these different states, so the total expected profit is the sum of multiplying of these probability with these profits. So you need to get the sigma of probability multiplied by the profits that sigma is the total expected profit, for all these states. So, that comes to be -12000 this is -12000 that is the expected profit at period 2 from D equals to 120 and P equals to 1.32 that is state number 1 in period 1.

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ABC Logistics Example

- The present value of this expected value in Period 1 is

$$\begin{aligned} \text{PVEP}(D=12, p=1.32, 1) &= \text{EP}(D=120, p=1.32, 1) / (1+k) \\ &= -\$12,000 / (1+0.1) \\ &= -\$10,909 \end{aligned}$$
- The total expected profit $P(D=120, p=1.32, 1)$ at node $D=120, p=1.32$ in Period 1 is the sum of the profit in Period 1 at this node, plus the present value of future expected profits possible from this node

$$\begin{aligned} P(D=120, p=1.32, 1) &= \{ (120,000 \times 1.22) - (120,000 \times 1.32) \} + \\ &\quad \text{PVEP}(D=120, p=1.32, 1) \\ &= -\$12,000 + (-\$10,909) = -\$22,909 \end{aligned}$$

Now, the second thing is once you have calculated this expected profit of -12000 we need to calculate the present value this the meaning is that this expected profit is in period2 this expected profit you will get in period2. Now presently I am talking of period 1, so the present value of this

expected profit. So, I will add present value of expected profit this fraction and then this present value of expected profit is expected profit multiplied by discounting factor.

And, discount factor is represented by K in our case, so it is $1+K$ that is to be divided by. So this expected profit comes here, and it is to be divided by $1+K$, see that this is the present value of your expected profit which is expected profit divided by the discounting factor. And as we can see now that for period 1 for which we are taking where demand is 120 and the current price is 1.32 the expected profit the present value of the expected profit is coming to be 10,909.

The 12,000 was the expected profit for this data, and this is divided by 1.1 that is coming to be 10,909. So now the total expected profit for this node where demand is 120 price is 1.32 at node. At a state 1 is the sum of profit in period 1 at this node + the present value of the future expected profits possible from this node. So this is the present value of the future cash flows, and what it is going to have on its own in this particular period. So, that is coming to be here, so now in this period 1 the demand is 120.

The price is 1.32 so, using the same formula here the profit here will come, because I am going to fulfil the entire demand, so 1.32 is the cost price, and the revenue is 1.22, so this is coming to be -0.10 and therefore, this is also coming to be the loss in this particular case. So that is what shown here that 120,000 into 1.22 is the revenue factor 120×1.22 is the cost factor, so revenue-cost is the profit coming in the period 1 + the expected profit and its present value.

So that is present value of expected profit that is, so this entire value this is coming -12,000 and this is coming -10,909. So the total profit the total expected profit at node 1 of a state 1 is -22,909. This is the total profit here total expected profit here is coming to be -22,909 which includes the profit generated at this type + the present value of profits from all these future states, so that is taken get, and now you already can understand the similar calculation will flow to period t_0 also.

The same calculation will come to this state also, because this state is coming from this state when the initial demand was 0 and P was 1.20, so now when I want to calculate the total

expected profit at t_0 I need to calculate the total expected profit for all the four states, because this node is giving rise to four different nodes. There are four different nodes and in our last session we already have discussed, what those four different nodes?

So I will do the same level of calculation of determining the total expected profit for all these four nodes different values will come, and then the expected profit for this particular node will be the multiplication of this expected profit into the transition probabilities that is 0.25 for each of them, and then I will calculate the present value for period 0. So that becomes the present value of expected profit.

And then I will also calculate the profit coming from this particular node in the original period, and that is the total profit total expected profit for this particular option when I am going for 100% a spot market. So let us see that what type of table emerges, because it is a quite lengthy calculation and therefore it is not very much advisable to show the complete calculation of developing the table on the board.

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ABC Logistics Example

- For Period 0, the total profit $P(D=100, p=1.20, 0)$ is the sum of the profit in Period 0 and the present value of the expected profit over the four nodes in Period 1

$$\begin{aligned}
 EP(D=100, p=1.20, 0) &= 0.25 \times P(D=120, p=1.32, 1) + \\
 &= 0.25 \times P(D=120, p=1.08, 1) + \\
 &= 0.25 \times P(D=96, p=1.32, 1) + \\
 &= 0.25 \times P(D=96, p=1.08, 1) \\
 &= 0.25 \times (-22,909) + 0.25 \times 32,073 + 0.25 \times (-15,273) + 0.25 \times 21,382 \\
 &= \$3,818 \\
 PVEP(D=100, p=1.20, 0) &= EP(D=100, p=1.20, 0) / (1+k) \\
 &= \$3,818 / (1 + 0.1) = \$3,471
 \end{aligned}$$

So, let us see on this slides that how the table emerges, so for period 0 the starting period the total profit for the original state that is 100 units is the demand, and the initial price was 1.20 is the sum of the profit in period 0 and the present value of the expected profit over the four nodes

of time period 1, and this is how 0.25 into the profit of this + this, + this, + this is becoming the total profit.

And therefore you have the calculation of -22,909 for the first state, but for the remaining states you have these calculations 32,000, 15,000 then 21,000, and like this final calculation becomes 3,818 that is the expected profit during time period 1 but for time period 0 I need to have the present value of this expected profit.

So, I will divide by this 3,818 by the factor of present discounting factor that is $1 + k$. So, this is 3,818 divided by 1.1. It becomes 3471. So, what does it mean that this is one part, this is only one part that this is the present value of my all future cash inflows at time t_0 . But, I also need to add into this 3471. The value of profit or loss, which I am incurring in this particular period.

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ABC Logistics Example

$$P(D=100, p=1.20, 0) = 100,000 \times 1.22 - 100,000 \times 1.20 + PVEP(D=100, p=1.20, 0)$$
$$= \$2,000 + \$3,471 = \$5,471$$

- Therefore, the expected NPV of not signing a lease and obtaining all warehouse space from the spot market is given by $NPV(\text{Spot Market}) = \$5,471$

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So that is done in the next slide where this 3,471 this component is there from the future cash flows, and the present value of those future cash flow is 3,471 and the present value of my profit is in the current period. I am selling 1.22 is the revenue. 1.20 is the original cost. So, it becomes 2000 rupees is the profit 2000 dollars, so 2000+3,471, so the total amount of profit, which I am going to get is 5471. So, therefore the expected net present value of using 100% a spot market is 5471. This is the final calculation available to me.

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Trips Logistics Example

- Using the same approach for the lease option, NPV(Lease) = \$38,364
- Recall that when uncertainty was ignored, the NPV for the lease option was \$60,182
- However, the manager would probably still prefer to sign the three-year lease for 100,000 sq. ft. because this option has the higher expected profit

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Now, using the same approach for lease arrangement. So, then when lease arrangement is there in this case it is 100% spot market. Now, I want to go for the lease arrangement, thus I want to try for the second alternative.

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Trips Logistics Example

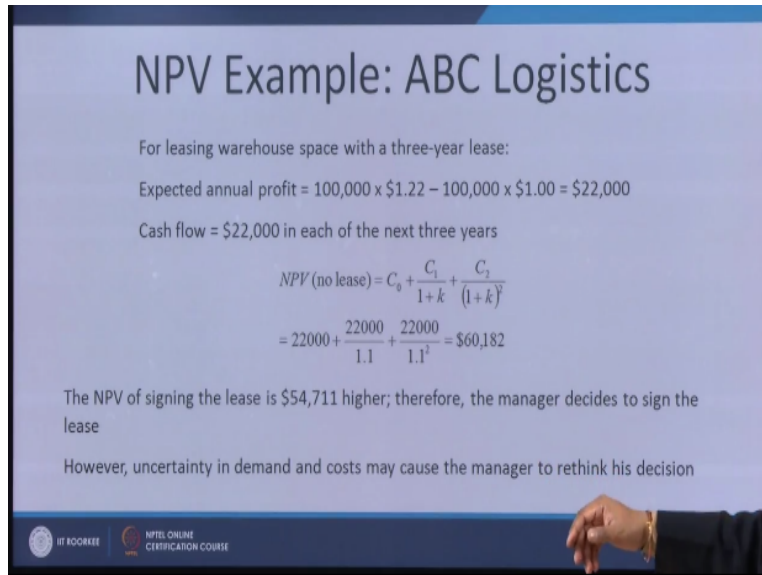
- Using the same approach for the lease option, NPV(Lease) = \$38,364
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And when I am going for the second alternative my net present value is coming 38,364. So that is the total profit which I am going to get in case of lease arrangement, If I do a lease agreement for 100,000 units initially and if I ignore the in certainty so for this type of issue if you remember in our 3, 4 sessions before the net present value was coming to the 60,182. Let me take you to those slides, where we ignore the uncertainties, and we only consider the certainties and only on

the basis of time value of money. We determine this case, so when we took that data into account.

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The slide titled "NPV Example: ABC Logistics" provides a financial analysis for leasing warehouse space. It states that for a three-year lease, the expected annual profit is \$22,000, calculated as 100,000 x \$1.22 minus 100,000 x \$1.00. The cash flow is \$22,000 in each of the next three years. The NPV calculation is shown as follows:

$$NPV(\text{no lease}) = C_0 + \frac{C_1}{1+k} + \frac{C_2}{(1+k)^2}$$
$$= 22000 + \frac{22000}{1.1} + \frac{22000}{1.1^2} = \$60,182$$

The slide concludes that the NPV of signing the lease is \$54,711 higher, leading the manager to sign the lease. However, it notes that uncertainty in demand and costs may cause the manager to rethink his decision. The slide also features logos for IIT Kharagpur and NPTEL Online Certification Course.

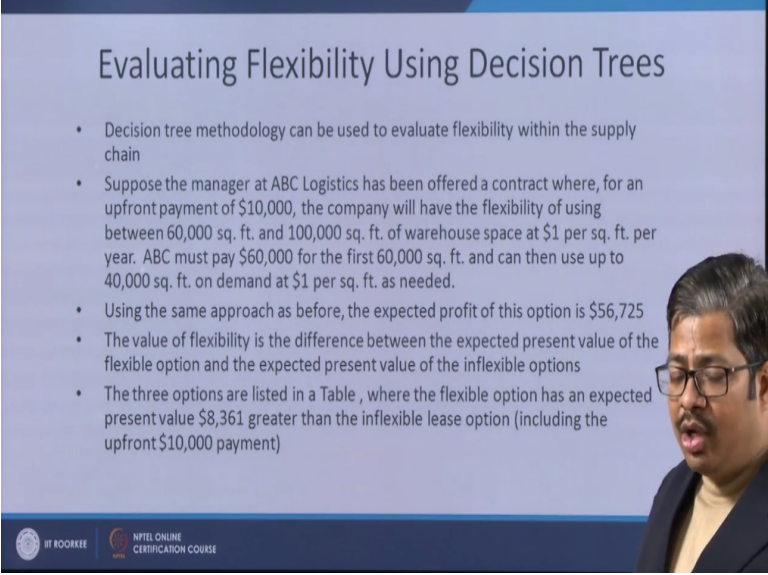
Now in this particular case, there was low uncertainty initially rate is 1.22, and the rate of lease was 1 dollar per square feet, and the profit was, annual profit was 22,000 and in this particular case the uncertainty was not there. So we were having a very fix type of cash flows and, this case gave us the NPV equals to 60,182 dollars. So, because uncertainty is there, so certainly, because of these type of fluctuations in my pattern.

In my future obviously I have to sacrifice on my profit because, I am not very sure, that what may happen, so this 60,000 reduce to 38,364, when some kind of uncertainty I have built into my situation. But considering the situation of earlier case when I am going for the spot market and in that spot market my profit was coming just 5471. So, that is much less as compare to 38,364 which is much, much higher.

So, even though uncertainty is there, but now we can see on this calculation basis, no certainty obviously, there was a clear choice that I will go by lease agreement. But even with the uncertainty the kind of data which is there, and kind of this calculation is showing to me that I should go with lease arrangement, because of and the size of the lease is 100,000 square feet so

this arrangement is very well suitable because of higher profits as compare to the spot market. The third option now, which we need to evaluate, that is flexibility.

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Evaluating Flexibility Using Decision Trees

- Decision tree methodology can be used to evaluate flexibility within the supply chain
- Suppose the manager at ABC Logistics has been offered a contract where, for an upfront payment of \$10,000, the company will have the flexibility of using between 60,000 sq. ft. and 100,000 sq. ft. of warehouse space at \$1 per sq. ft. per year. ABC must pay \$60,000 for the first 60,000 sq. ft. and can then use up to 40,000 sq. ft. on demand at \$1 per sq. ft. as needed.
- Using the same approach as before, the expected profit of this option is \$56,725
- The value of flexibility is the difference between the expected present value of the flexible option and the expected present value of the inflexible options
- The three options are listed in a Table, where the flexible option has an expected present value \$8,361 greater than the inflexible lease option (including the upfront \$10,000 payment)

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Now we want to actually in this particular case the data was supporting the lease arrangement, but it is quite possible that in some cases lease arrangement may not be that much useful. So, we want to have a combination of both these things. So, we want to have some kind of flexibility in this particular case. And when we want to have flexibility, so you can see that the condition which we are developing is so how we can use this decision tree method for evaluating the flexibility in the supply chain.

Because the whole predictive analytics, the whole real time data uses is to have flexibility in your supply chain. If you do not have flexibility in your supply chain the issue of real time data uses will not going to give you any kind of benefit. So, flexibility is a kind of you can say pre requisition for adopting predictive analytics, for adopting the real time decision making.

If your supply chain, if your ware house is a lease arrangement which is a specified space, so, there is no point in getting this real time information, because in daily basis, or monthly basis or an annual basis you are not able to change the available space. So, therefore the point is that in this particular case. We need to see that we can use the available space on the flexible basis.

And then only the usage is possible, because why I am saying that .if you go to this particular example, and when you develop this type of calculation for the lease arrangement, because I am taking 100,000 area for my lease arrangement. So, in that case when my demand is 144, when my demand is 120. These types of additional requirements I may not be able to fulfil, because, my area is only 100,000.

So, for 100,000 I will be fulfilling from my warehouse. The additional demand, I will not be able to fulfil, because I don't have space. The second issue is when I have the less demand 96, when I have less demand in that case also I will not be able to utilise my capacity to the optimum level. I have unused capacity I am giving rent for no reason

I have wasted my money because, my company is having a system of fulfilling all the demand. So, what will happen that 100,000. I will fulfil from my warehouse. And, the remaining 20, for that I will go to the spot market. Here, 100 units I will fulfil from my warehouse. And, from the remaining 44 I will go to the warehouse, so this type of arrangement when I am going for the lease kind of system.

So, this arrangement I have to follow, that a fix amount I will use from my warehouse, and the remaining I will use from the spot market. So, therefore flexibility is advocated, where in some of the cases, when you go to the continuously downward trend. Like 100 is here, and it is just 80% of the original demand. So, it will come 80, and then further in the next period it reduces to it is 80% to so it is only 64.

So, in that case you have 36,000 square feet of area which is unused. So, that is also a loss, so therefore we want to have flexibility in my supply chain, which can give me the benefit of both these options. So, we stop here in this session and in our next session we will see that how flexibility can be incorporated in uncertain environment. Because, that is the most important part where, predictive analytics can help us and we can minimize the cost of offering and it will help us in increasing our revenue. Thank you very much.