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Lecture - 10 Methods of Project Selection (MCDM -I)

Hello friends, welcome you all in this session. In previous session, we discussed project selection techniques like linear programming we have also seen integer programming. Let us look at couple of other multi criteria decision making techniques which are available for project selection; when I say multi criteria it means you need to select a project after considering several criteria. For example, criterion maybe a qualitative one or maybe a quantitative one to give you an example let us say if you want to select a vendor and let us say there are 4 vendors available. Vendor a is ready to give you material at less price, but does not deliver material in time on the other hand vendor b delivers material in time, but charges more money.

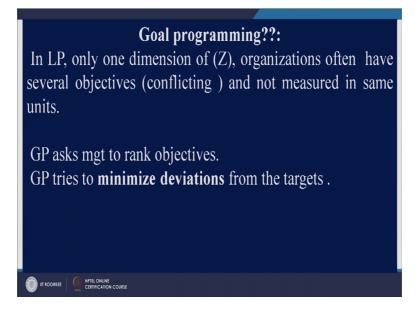
On the other hand let us say if you have got vendor c, he is ready to give you more credit period, but charges very high for the raw materials and d he does not give you credit period, but charges less money. So, there are 4 vendors and there are several criteria now which vendor is to be selected. So, a situation like this can be handled using MCDM techniques and there are several MCDM techniques we will look at couple of those techniques, a very prominent technique is multi attribute utility theory then you have got AHP it is called analytic hierarchy process we will see this technique in detail.

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We have gave got fuzzy set theory, we have got case based reasoning there is something called DEA data envelopment analysis, you have got smart it is called simple multi attribute rating technique. We have got gaol programming we will see goal programming in detail apart from AHP, there is something called electre method it is elimination choice translating reality, you have got promethee the ninth MCDM technique it is preference ranking organization method for enrichment evaluation. You have got simple additive weighting and the last one is topsis. We will also see this technique in detail it is extends for technique for order preference by similarity to ideal solution. So, this is topsis. So, we will look at couple of these techniques.

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Let us first look at goal programming. It is extension of linear programming. In linear programming what we have done we converted the problem into linear programming model and then we solved it using graphical method for the example which we had taken. In goal programming what happens you have got several goals in LP you had only one goal there was one objective, but in goal programming you have got multiple objectives and in real life situation if you look at organizations do have multiple objectives for example, they want to use their resource in efficient manner, but they also do not want too many inputs to be used. For example, let us say they want to minimize waste they also want to maximize profit. So, organizations want multiple objectives to be optimal optimized.

So, as I said in LP you have got only one objective, but in goal programming or GP you have got multiple objectives. So, what happens in goal programming management will give different priorities to different goals for example, profit is to be earned this much, but at the same time we want to have these many units of products x 1 are product x 2. So, you can have 3 goals right and management will give priorities to these goals. In GP the objective is always to minimise deviation from the target. So, whatever is the target we always try to minimize the deviation, let us say if the objective is to maximize profit. So, of course, we would not like to maximize profit sorry we would not like to minimize

profit, but we want to minimize deviation if profit is not achieved. Let us say if profit is not achieved by 100 rupees. So, we will try to minimize that hundred rupees value.

So, let us look at this example we will take the, we will continue with the same example which we did using LP. So, you have got two products mobiles and laptops, you have got profit per unit of mobile 120 rupees profit per unit of laptop 90 rupees, these two processes these two products go through two processes assembly.

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E	Ex:			
		Mobile	Laptop (lt)	
	Assembly	6	3	90hrs
	Finishing	3	6	72hrs
	Profit	120	90	
t	f profit is Rs120 pe he best combination forofit of Rs 2100.		1 1	• '

And finishing you have got 90 hours of assembly and you have got 72 hours of finishing, 6 hours are needed by one unit of mobile in assembly process.

Similarly, 6 units of or 6 hours are needed to make one laptop in finishing process. So, this is your problem and we want to determine the best combination of mobiles and laptops to realize a profit of 2 1 double 0. Now here the goal is what we want minimum profit of 2 1 double 0 in earlier case when we solve this example using LP what was our objective can you recall, in previous case our objective was to maximize total profit, but here we want profit of rupees 2 1 double 0. So, let us look at how to formulate this problem.

This is a single profit single goal problem and the goal is to achieve a profit of rupees 2 1 double 0. So, in goal programming we will have some more variables for example, in linear programming technique there were only two variables x = 1 and x = 2. So, x = 1 was

number of units of mobiles and x 2 was number of units of laptops. So, apart from those two variables we will add few more variables here, and let us call them as D u and D o when I say D u it is it is the amount by which profit goal is under achieved, and D o is amount by which profit goal is over achieved.

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Amount by which profit g ize Z = Du (underachie 120x1 +90x2+Du-Do	vement of profit							
6x1+3x2<=90 3x1+6x2<=72 x1,x2,Du,Do>=0 6x1+3x2+s1=90 3x1+6x2+s2=72 x1,x2,s1,s2>=0	$c_{\theta} = \begin{bmatrix} c_{j} \\ Basis \\ 1 \\ D_{u} \\ 0 \\ x_{1} \\ 0 \\ x_{2} \\ \overline{c}_{j} \\ \overline{c}_{j} \end{bmatrix}$	x ₁ 0 1 1	x ₂ 0 0	51 - 50/3 2/9 - 1/9	5_2 $-\frac{20}{3}$ $-\frac{1}{9}$ $\frac{2}{9}$ $-\frac{20}{3}$ $\frac{20}{3}$	D _w blue 1 terms 0 terms 0 terms 0 terms 0 terms 1 terms 0	12 will be the	2 6 6
using simplex method, x1 980, underachieved by R		=120, 1	Means	actual	profit e	arned is	Rs= 2100)-

So, there are only 3 possibilities in this question either you will achieve a profit of rupees 2 1 double 0 or you will not achieve it or you will achieve more than it. So, under achievement and over achievement have been expressed by these two variables D u and D o, and in goal programming as I said the in previous slide we always try to minimize deviations from the target and our target is what to a profit of rupees 2 1 double 0 right. So, let us look at how to formulate a model. So, minimize a Z is equal to D u which is under achievement of the profit goal.

If we are over achieving a profit that is more than welcome right it is good thing for the organization. But if we are achieving profit goal then that then by what amount you have under achieved that profit goal that is to be minimized and which is D u in this case right. So, minimize D u subject to constraint is 120×1 the 120 is the profit per unit of mobile plus 90 x 2, 90 is profit per unit of laptop plus D u minus D o is equal to 2 1 double 0. So, if your profit is let us say profit is 20; 200 instead of 2 1 double 0; that means, you have over achieved profit by 900 rupees. So, that under achieved then that under achieved value is to be subtracted from remaining 3 variables here right.

If we have under achieved profit let us say by 100 rupees, and if your total profit is let us say 2000 rupees it means you have under achieved by 100 rupees so that is to be added in this equation in left hand side. So, this is your constraint on profit these two constraints will remain as it is these are capacity constraints, you have got 6×1 plus 3×2 less than or equal to 90; 3×1 plus 6×2 less than or equal to 72. Now of course, these variables are not negative. So, you have got one non negativity constraint $\times 1$ and $\times 2$, D u and D o all have to be equal to or greater than 0. Now when you convert this problem these two especially these two constraints into standard form then you need to add slack variables here right. So, here s 1 and s 2 are known as slack variables.

A slack variable is a variable which we add to make these inequality constraint into equation form. So, this left hand side is less than right hand side. So, we have added something in left hand side right. So, this is 6×1 plus 3×2 plus s 1 is equal to 90; then 3 $\times 1$ plus 6×2 plus s 2 is equal to 72. Now this is a problem which we would be solving using a simplex method, this is extension of graphical method when the number of variables are let us say even if the you have got two variables you can apply simplex method, but when number of variables increase the complexity in graphical method increases. So, it is good to use simplex method. So, we will have solution by simplex method and I am directly giving you solution by simplex method.

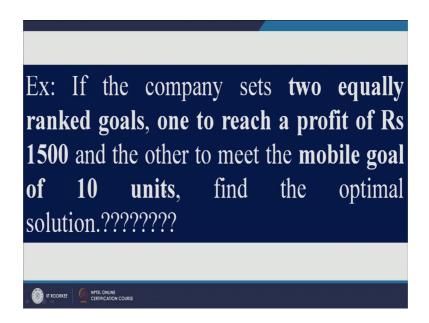
So, this is your model and this table is giving you optimum solution. So, if you look at this particular table then you have got let us say this basis; basis means these are basic variables. So, the variables under basis column are known as basic variables this b column these are the values of basic variables here this is your decision variable x 1, decision variable x 2, s 1 and s 2 are slack variables which we have added to covert inequalities into equations and then you have got D u under achievement of the profit object to and this is D o over achievement of profit objective.

So, this is c j is nothing, but now this coefficients of these variables in objective function equation. So, if you look at this objective function equation has got just one variable which is just D u and it is coefficient is one right. So, this is one we have written all other coefficients are 0. So, you do not have to x 1, x 2, s 1 and s 2. So, you just put them 0 over here right. So, 0 0 and 0 right this c B are nothing, but the coefficients of basic variables right. So, this if you look at how to interpret this particular table or this solution. So, if you look at x 1 is equal to 12 it means we are making 12 mobiles, x 2 is

equal to 6 it means we are making 6 laptops. So, 12 mobiles 6 laptops would give you how much profit? Can you can you get the any answer to this question you have got 12 mobiles and 6 laptops profit per mobile is 120. So, 120 into t12 is plus 90 into 6 you would get a profit of rupees you just multiply these two terms.

And if you look at this table this this is D u, D u is under achievement of profit which and it is value is 120, it means profit goal is under achieved by 120 rupees it means what how much profit you are actually earned you earned profit of 1980. So, this value is you can obtain this value by putting these values in your in this equation right. So, 120 into 12, 90 into 6 plus 120 minus 0 you will get 2 1, you will get you will get 2 1 double 0 if not this then you will get this value right. So, this is the example on goal programming and the in this case there was only one objective right.

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Now, let us look at the let us extend the same problem. If the company sets two equally ranked goals one to reach a profit of 1 5 double 0 and the other to meet the mobile goal of ten units find the optimum solution. The same problem the will the problem is same right the same problem, but now the profit goal is to achieve a profit of 1 5 double 0 rupees plus to achieve a mobile goal of 10 units right. So, you should produce 10 units of mobile phones and we should have this much profit right.

So, how to go ahead with this question how many variables will be here? In LP there were two variables in graphical method right in goal programming in previous case there

were 4 variables right x 1 and x 2, D u and D o. D u was for under achievement of profit and D o was for over achievement of profit. Here you will have one more for mobile goal you will have under achievement of mobile goal plus over achievement of mobile goal.

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Solution:											
Dup= Amount by which profit goal i	s ui	ndera	chie	ved							
Dop= Amount by which profit goal i	s ov	verac	hiev	ed							
Dum= Amount by which mobile goa	l is	unde	rach	ieve	d						
Dom= Amount by which mobile goa											
Minimize Z =Dup+Dum		cj OR	0	0.0	0	0	00	0 10	0 1	0	0
ST 120x1 +90x2+Dup-Dop = 1500	G	Basis	x1 0	x2 3/1		s ₁	s ₂ 0		D_{op} D_{ur} $-\frac{1}{120}$ -1	Dor	6 5/
x1+Dum-Dom = 10	0	x1		/4					$-\frac{1}{120}$ $-\frac{1}{120}$ 0	0	12 25/2
$6x1 + 3x2 \le 90$	0	SI	0	-3/2		1			1/20 0		12
3x1+6x2 <= 72	0	\$2	0	15/		0		. /		0	69/
x1,x2,Dup,Dop,Dum,Dom >=0	Z, .		0	0		0	0	- 1/40 0	0 0	0	12
····,···	ē,		0	0		0	0	D I	0 1.	0	P
After using simplex method, $x1=25/2$	2 1	Dom	= 5/	2 Me	ear	ns r	profit	tooal	of $R_{s}=1$		al b.f.s.
achieved, since both Dup and Dop de								goui		000	
	0-110	ռեր			nal	tat	<i>.</i>				

So, two more variables to total 6 right, so D u p. So, this is very simple D u means under achievement of p is profit goal right.

So, among by which profit goal is under achieved D o p amount by which profit goal is over achieved, Dum under achievement of mobile goal right among by mobile goal is under achieved, Dom among by which mobile goal is over achieved right. So, as I said in goal programming we always minimise deviation from target right. So, we will just sum up these two under achievement goals right. So, D u p plus Dum right and what would be our profit constraint? It will remain same except just this particular value. So, 120 x 1 plus 90 x 2, plus under achievement of profit goal minus over achievement of profit goal, so 15 double 0, then you have got x 1 which is number of units of mobile phone right. So, x 1 plus Dum minus Dom is equal to 10 and these two are capacity constraints right and this is non-negativity constraint. So, let us try to solve this question and I will give you direct solution.

Here is the solution. So, you have got 4 basic variables here right. So, here instead of D o r this is nothing, but mobile phone right Dom right you can make this correction. So, if

you look at here x 1 is equal to 25 by 2 it means 12.5, and x 1 is what number of units of mobile we are making 12.5 units of mobile phones are have we achieved our mobile goal. Yes, we have achieved we have achieved our mobile goal and. In fact, we have over achieved it by how many units this one right this is Dom right this is not Dor this is Dom. So, we have over achieved our mobile goal by 2.5 units, if you look at these are slack variables let us not discuss these things. So, what we are saying at the end of the day what about profit goal did we achieved profit goal yes or no.

Yes we did achieve our profit goal because under this column under basis column you do not have any under achievement of profit goal value variable right. So, x 1 is equal to 20.25 by 2, x 2 is yeah it is not there it is 0. So, profit goal is 15 double 0. So, how to get this 1 5 double 0? It is 120 into 22 by 2. So, 60 into 25 which is equal to this since both as I said D u p and D o p do not appear in final table it means they are 0. So, this is a case wherein we were having two goals right.

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Analytic Hierarchy Process (AHP)

- Analytic Hierarchy Process is a multi-criteria decision making (MCDM) technique was developed by **Saaty** in 2000 year.
- The analytic hierarchy process (AHP) is also a structured technique for helping people deal with organizing and analyzing **complex decisions**.
- AHP is also a measurement theory that priorities the hierarchy and consistency of judgmental data provided by a group of decision makers.
- The AHP provides a comprehensive and rational framework for structuring a problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions

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Let us look at another multi criteria multi criteria MCDM technique, multi criteria decision making technique it is called AHP, it was developed by Thomas I Saaty in the year 2000. AHP is also structured technique for helping people deal with organization and analysing complex decision. In fact, the decisions of organizations are always complex why they are complex because there are many stake holders in world. In fact, it is not necessarily that the organizations decisions are complex, but there are several

government decisions which are also complex. So, in complex decision making you know when you have got complex situation you need to make decision in fact, more carefully.

So, AHP is measurement theories that prioritise the hierarchy in consistency or judgement data, provided by a group of decision makers. So, what do we generally in AHP is we collect data from experts and we collect data on several variables and we will and we take their opinion and then after doing necessary calculations we come up with our findings. So, it is it provides a comprehensive and rational frame work for structuring a problem, we need to structure our problem in AHP in different levels of format. So, you have got level 0, level 1 and level 2 and so on right.

So, in this case we will have several criteria and each criterion can have several sub criteria. So, the first thing which we do in AHP is establish the hierarchy structure.

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Establishmer The relative values 1, 3, 5	nt of pair-wi importance o 5, 7, and 9.	ise compariso	ht computation on matrix s is rated using	
The relative values 1, 3, 5	importance o 5, 7, and 9.	•		a scale with th
Equally Preferred	Moderately Preferred	Strongly Preferred	Extremely Preferred	Absolutely Preferred
I	3	5	7	9
2, 4, 6, and 8	indicate inter	mediate value.		
			2, 4, 6, and 8 indicate intermediate value.	2, 4, 6, and 8 indicate intermediate value.

Then we have got various hierarchy elements weight computation. In AHP we use a scale called saaty scale saaty scale is 9 point scale and we asked our experts to rate any two pair of variables on this 9 point scale and we use let us say 1 3 5 7 9, where one is two variables are equally preferred. 9 means they are absolutely preferred similarly 3 5 and 7 and 2 4 6 and 8 indicate intermediate values. So, this is nothing, but a saaty scale. So, this is how we will be getting our input data. So, you will have a matrix like this

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 $A = [a_{ij}] = \begin{array}{ccc} C_1 & C_2 & C_n \\ C_1 & a_{12} & \cdots & a_{1n} \\ C_2 & 1 & a_{12} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{array}$ Where $a_{ij} = 1$ and $a_{ij} = 1/a_{ji} = 1, 2,n$. $A = \begin{bmatrix} c_{1} & c_{2} & c_{n} \\ c_{2} & c_{n} \\ c_{2} \\ \vdots \\ c_{n} \end{bmatrix} = \begin{bmatrix} c_{1} & w_{1}/w_{1} & w_{1}/w_{2} & \cdots & w_{1}/w_{n} \\ w_{2}/w_{1} & w_{2}/w_{2} & \cdots & w_{2}/w_{1} \\ \vdots & \vdots & \vdots \\ w_{n}/w_{1} & w_{n}/w_{2} & \cdots & w_{n}/w_{n} \end{bmatrix}$ Where $W_i / W_i = a_{ii}$

Then we also check a something called consistency index and consistency ration in AHP, and then we finally, go for overall hierarchy weight commutation.

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• Eiger	n value and	eigen vecto	or calculatio	n				
• Cons	sistency test	$\lambda_{max} =$	$\sum_{j=1}^{n} a_{ij} \frac{W_j}{W_i}$					
CI (C	onsistency	Index) =	λma	ax - n				
CR (0 ✓ H	Consistency Random ind	Ratio) = ex values w		- 1 given by S	aaty where			
n=1	n=2	n=3	▶ n=4	n=5	n=6	n=7	n=8	
R1=0	R1=0	R1=0.52	R1=0.89	RI=1.11	RI=1.25	RI=1.35	RI-1.4	
iii. Over	all hierarc	hy weight o	computatio	n				
		OURSE						

So, this is the technical details which we will skip and let me tell you an example of related to AHP.

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Supply Chain Performance Fivaluation of Automotive Industries	Level 1: Ultimate Goal
Quality Delivery Cost Flexibility Innovation	Level 2: Criteria
Figure : Hierarchical structure to evaluate the supply chain per evaluation of automotive industries	formance

Let us say there is a supply chain, and the supply chain performance evaluation of one automobile industry it depends on several criteria. So, in other words what are the factors which affects supply chain often or to automobile industry. So, the factors which we have gathered are you have got quality you have got delivery we have got cost flexibility and innovation. So, our ultimate goal is which is level one is to measure supply chain performance, and these are different criteria through which we are measuring supply chain performance.

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	Quality	Delivery	Cost	Flexibility	Innovation
Quality	l	3	1	1/3	9
Delivery		1	1/3	1/5	6
Cost			1	1/4	7
Flexibility				1	8
Innovation					1
Pair-wise con highest fre		natrix of res or all criteri		(Either cons	ensus or

So, this is hierarchy structure to evaluate the supply chain performance evaluation of automotive industries. This is the initial matrix in which we would be getting data from experts. So, let us say you are asking to an expert how do you rate quality and delivery, how do you compare quality and cost, how do you compare quality flexibility quality and innovation. So, let us say experts says 3, it means that for that particular expert deliveries 3 times important than quality, but cost and quality are equally preferred, the importance given to cost and qualities same.

When we ask him to compare flexibility and quality he says that flexibility is one third time important than quality it means or in other words quality is 3 times important than flexibility. When we ask him to compare quality and innovation he says innovation is 9 times more important than quality similarly you can have other pairs also for example, let us say delivery and innovation. So, innovation is 6 times important than delivery, but flexibility is just one fifth time important than delivery and so on. So, this is how you can compare a matrix like this right. So, this is pair wise comparison matrix of respondents.

Now, each now let us say if you have collected data from 3 respondents. So, one fellow said 3 on quality and delivery, the second fellow said 5 and the third fellow said 9 so what to do in that case. So, there are several ways to write this particular value. So, either go for consensus you can ask all those 3 experts to come to a consensus if they are coming to a consensus that would be the best. So, let us say if all of them a have set it is three. So, you just write 3 otherwise you can go for highest frequency value let us say if you take this one. So, first fellow said quality to innovation is 9, second fellow says 5 third fellow says 9. So, frequency is 9 and the highest frequency is nine. So, just write 9 over here right.

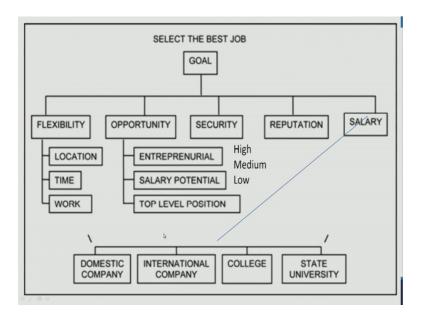
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Respondents		CR	Respondents	CI	CR	
	0.07749	0.06918		0.08364	0.07467	
	0.07986	0.07131		0.06853	0.06119	
	0.06862				0.02926	
	0.09214	0.08227	R ₁₂	0.08052	0.07189	
	0.08884	0.07933		0.10024	0.08950	
R ₆	0.09095	0.08120	R ₁₁	0.08556	0.07640	
	0.11118	0.09927	R ₁₅	0.04887	0.04363	
	0.10366 Table : Cons	0.09255 istency index (CI	${ m R}_{16}$) and consistency ratio (0.09692 CR) of all respond	0.08653 cnts	
				CR) of all respond		
) and consistency ratio (CR) of all respond	ents tk	
	Table : Cons Criteria) and consistency ratio (Weights of Criteri	CR) of all respond a Ru	ents 1k	
	Table : Cons Criteria Quality = C ₁) and consistency ratio (Weights of Criteri 0 2937	C(R) of all respond a Ray 2	ents 1k	
	Table : Cons <i>Criteria</i> Quality = C ₁ Delivery C ₂) and consistency ratio (Weights of Criteri 0.2937 0.1995	(IR) of all respond Ran 2 3	ents 1k	
	Table : Cons Criteria Quality = C_1 Delivery C_2 Cost = C_3	istency index (CI) and consistency ratio (<i>Weights of Criteri</i> 0.2937 0.1995 0.1533	(IR) of all respond Ran 2 3	ents tk	

So, I will not go into details of this particular problem, but you have got let us say in this case there are 16 respondents. So, you have you have got consistency index and you have got consistency ratio. Now this is your final table. So, you have got these 5 criteria these are different bits and these are different ranks. So, we will said that the highest weight is for flexibility. So, we say flexibility is most important for supply chain performance quality is at number 2. So, we say quality is second most important criterion for performance of supply chain.

Similarly, rank 3 delivery rank 4 cost and rank 5 innovation. So, I will I will give you one more example related to AHP let us say if you are selecting for a job. So, you have completed your degree and you are looking for a job. So, what kind of things you would look in a job. So, definitely you would like to have a one of the criteria would be salary right. So, salary can be again you can have if it is a domestic company you will have some salary structure, in international company you will have some other structure in college you will have different structure and so on right.

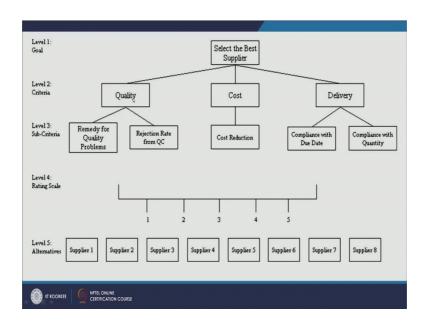
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So, salary can be divided into 4 different sub criteria right similarly flexibility. So, what is the location of the job, what are the timings and what work you are supposed to do and then you have got opportunity to grow in that organization. So, you have got what are the entrepreneurial capabilities you would develop in that organization what is the growth as far as salary is concerned, then you have got what is the; what is your probability of reaching at top level, what kind of security you are looking for. Let us say if you are female, female student and if you are looking for a job which where the timing is from let us say 8 p m to let us say 4 a m in the morning. So, would you like to take up that kind of job? So, security what kind of security you want high medium or low and how reputed that organization is.

So, you will look at all these criteria and you divide each of those criteria into sub criteria and then take appropriate decision I will give you one more example let us say if you want to select supplier.

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So, you can have different criteria like quality cost delivery, in quality you can have two sub criteria and delivery you can have again sub criteria. So, you can have different levels right. So, this is your final objective level 1 level 2 and so on right. So, this is how you can prepare a hierarchy.

So, with this let me stop here. In this session we have seen two methods goal programming and AHP; in next session we will see how to solve a problem using AHP.

Thank you very much.