Project Management for Managers
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Lecture - 24
Hillier Model

Good morning friends. I welcome you all in the session. As you are aware that in previous session we were discussing about hillier model of measuring risk of a project, and we have seen couple of things we did calculate expected cash flows from different projects in different years.

And we were given net cash flows and their probabilities. So, this was the problem you are looking at keep in mind that there are 2 types of situations in hillier model. You have got unrelated cash flows and related cash flow when I say unrelated cash flow means, the cash flow of for of second year does not depend on cash flow of first year. In other words there is no correlation between cash flows, right. And in case of correlated cash flows you will have relationship between cash flows of different years.

So, let us look at this example which we have solved in previous class also, but we did not solve it completely. So, a project which initial investment of thousand hundred lakhs with life span of 4 years has the following probabilistic out comes in different years right. So, net cash flow in first year is 30 lakh and it is probability is 50 percent, right. Net cash flow 10 lakh probability is 30 percent.

Keep in mind that this probability is hundred percent, right. Some of these probabilities should be hundred percent. Similarly, for fourth year mash cash flow would be 60 lakh and it is probability is 30 percent 20 lakh it is probability is 15 percent.

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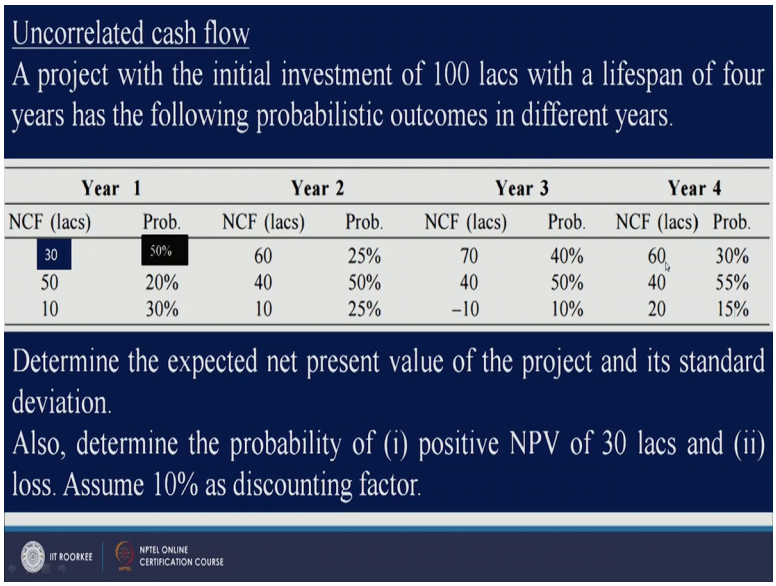

So, what you want to find out? Determine, we want to find out expected net present value of the project and it is standard deviation right. So, first we should calculate these 2 terms and then determine the probability of positive NPV of 30 lakh, right. And what is your probability of that the project would suffer loss, right? Assume 10 percent as discounting factor right.

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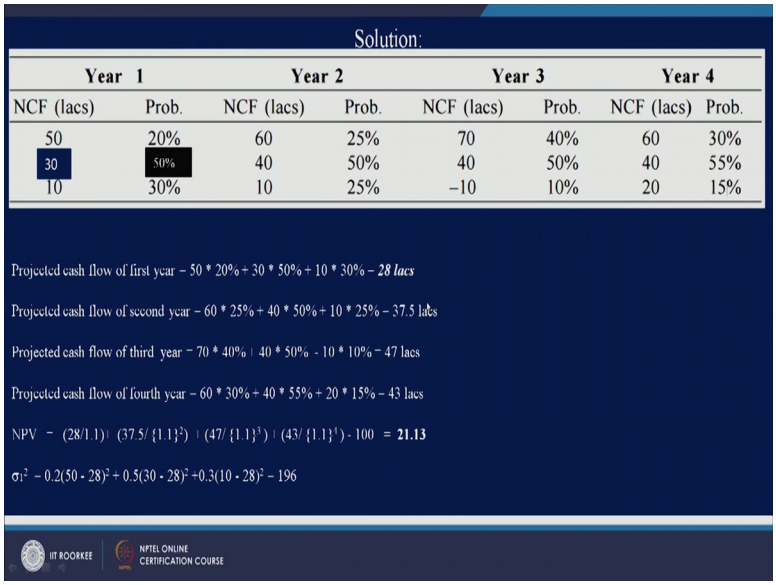

So, this is what we calculated in previous session projected cash flow for first year was 28 lakh, for second year it was 37.5, third year 47, for fourth year it was 43. And how did

we calculate these values? We just multiplied net cash flows with respective probabilities and we did at all these multiplications right. So, this how we obtained expected cash flows right.

Now, to find out NPV, NPV of this project would be this. In fact, I have written a formula over here.

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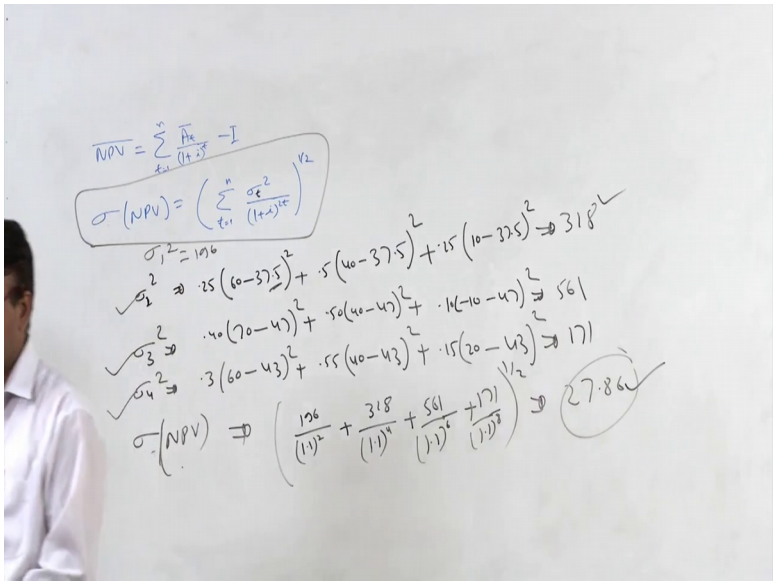

So, NPV is the expected NPV is summation of all the years t ranging from 1 to n in the case, it is 1 to 4 expected cash flows divided by 1 plus i to the power t, right minus investment. So, you need to calculate this NPV, for this you need to calculate first of all let us let us calculate this, though we have already calculated NPV for this particular example. And our answer was it is 21.13 lakh, right. Is not it? So, let us look at standard deviation, this is summation of t ranging from 1 to 4 is variance divided by 1 plus i to the power 2 t, keep in mind this is 2 t right.

So, for first year it would be 2 second year it would be 4 third year it would be 6 and for fourth year it would be 8 right. So, you have to take under root of this whole term right. So, how to find out this let us let us calculate for first year right. So, this is probability is 0.2 then cash flow it is 50 lakh minus expected cash flow right. So, this was 28 lakh right. So, it is 28 lakh, right. Whole square is not it? Then 0.5 just see these here. So, it is 0.5 into 30 minus 28, right whole square right so that the third one would be 0.3 into 10 minus 28 right. So, what we have done in this when we are calculating variance for first

year, what we are doing? We are subtracting expected cash flow from net cash flows, right. And we are multiplying by their probabilities also. So, far this first year it is 196 right. So, this would be 196, right. Then for second year how would you do it?

For second year you should start like this, first is probability 0.25, right. 60 minus what is expected cash flow of second year? It is 37.5, right. Then it is square, right. Plus what is the second probability it is 0.5, right. What is net cash flow 40 minus expected cash flow which is this will remain same, right. 37.5 square plus the next probability would be 0.25 cash flow is for second year it is 10 minus 37.5 whole square. Just add all these things you will get variance for second year, right. Similarly for third year how would you calculate? For third year first probabilities 0.4 then net cash flow is 70 expected cash flow is 47 lakh, right. Whole square plus probability 0.50 expected cash flow 40 minus 47 whole square plus 0.10, because this probability sum should be 1 right.

So, for third year it is minus 10 minus 47 whole square you will get some answer, right. We will calculate the answers as well, right. For forth year this would be 60 minus, so first of all probability 0.3 into 60 minus, what is expected cash flow for forth year? It is 43 lakh, right. Plus probability next is 0.55, right. Cash flow 40 minus 43 whole square plus point the remaining one would be 15, right. And net cash flow 50 expected cash flow 43 right. So, you just calculate these values and these values would be nothing, but first for second year it would be 318 for third year it would be 561 and 171; so 318, 561 and 171.

Now, let us calculate this value, standard deviation of NPV, right. Keep in mind you need to apply this formula now right. So, what to do for this? For this NPV is equal to, you need to look at this is what we have calculated for first year, right. For first year for first year it was there in previous slide. So, for first year it was 196, right. This is for forth year this is for third year this for second year and for first year it was 196 right. So, how to proceed now 196 plus 1 upon and what is your and discount factor here, the discount factor is 10 percent, right. Is there in question itself, right? Just see this is discounting factor 10 percent right. So, 196 divided by 1.12 into t, right. T is 1.

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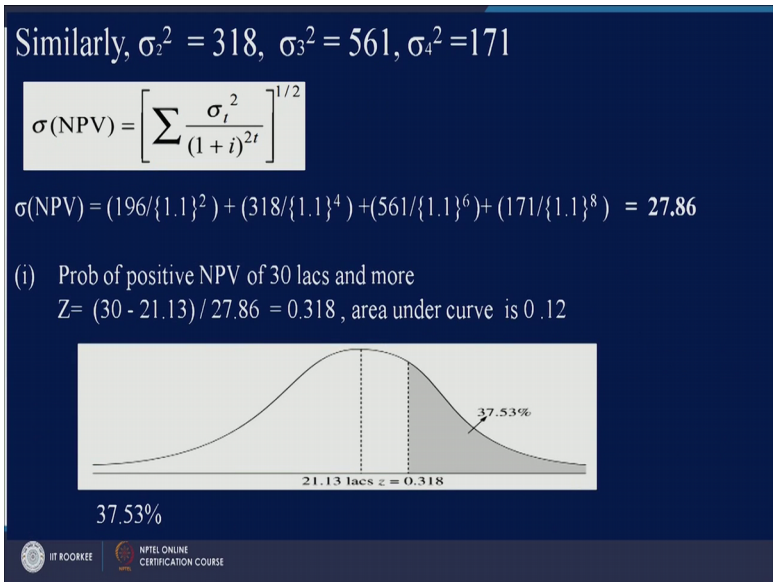

So, this square plus 318, 318 divided by 1.1 to the power now t is equal to 2. So, this becomes 4 then 561 1.1 this is 6 plus 171 1.1 to the power 8, right. Under root of this would be your standard deviation of NPV and that values 27.8 right. So, this is how you can calculate expect expected NPV and standard deviation right.

Now, the second part of this question is what is the probability of NPV? As 30 lakh right. So, we know mean and standard deviation right. So means is, means is also available and standard deviation is also available, right So, this is your standard deviation. Now what is the probability that the project? There would be positive NPV of 30 lakh and more than that, for that you need to calculate z value, right. And z value is x minus mew divided by standard deviation right. So, mean is 21.13 for this particular problem. So, how to draw a distribution for this?

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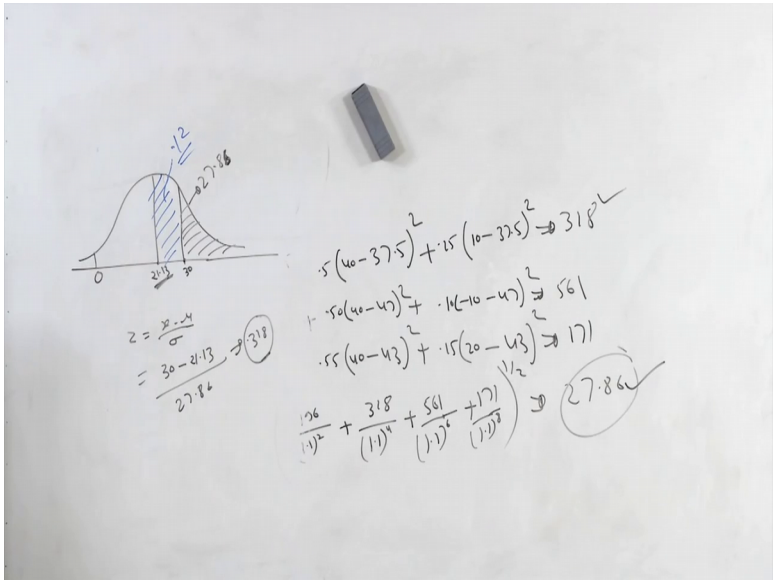

So, the distribution would be like this. Mean, mean is 21.13 right. And standard deviation is just what we have calculated here it is 27.86, right. 27.86 now calculate z value because what is the probability that it would earn 30 percent, 30 lakh rupees right. So, this is 21.13 and this is 30 right. So, when I say, if I ask you a question what is your probability that the project would earn, or what is the probability that the project would earn and this much NPV?

Now this is always at 50 percent right. So, you can always say that it is the probability that project would earn 21.13 lakh is 50 percent right. So, beyond this side it would be more than 50 percent this side it would be less than 50 percent right. So, our question is; what is the probability that the project would earn a positive NPV of 30 lakh and more right. So, what is the area under this curve right. So, for that you need to calculate z is equal to x minus mew divided by standard deviation right.

So, x is nothing, but 30 right. So, 30 minus 21.13 divided by 27.86 this is equal to 0.318. Now look at the z value the z table to know what is the area under curve? And z is equal to 0.318 right. So, let us look at z table, and area where z value Is 0.318.

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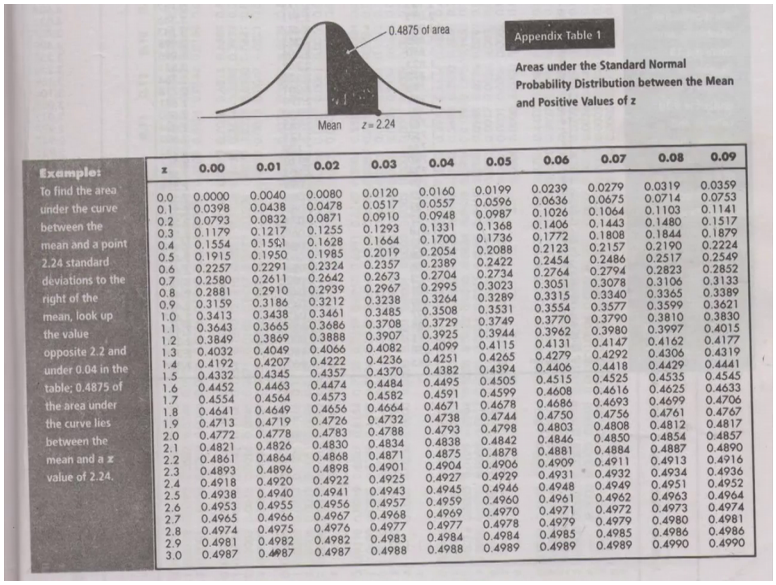

0.4875 of area

Mean $z = 2.24$

Appendix Table 1

Areas under the Standard Normal Probability Distribution between the Mean and Positive Values of z

Example: To find the area under the curve between the mean and a point 2.24 standard deviations to the right of the mean, look up the value opposite 2.2 and under 0.04 in the table; 0.4875 of the area under the curve lies between the mean and a z value of 2.24.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

So, 0.31, 0.31 is 0.1217 and 0.32 is 0.12 double 5 right. So, it is somewhere between these right. So, for 0.318 the area under would be somewhere between these 2, right. And that area is 0.12 let us take that value 0.12. So, 0.12 right So, area under this, let me use a different colored pen, this is 0.12, right. And this is 50 percent right. So, what is this what is the probability that it would earn more than 30 lakh?

You just subtract this value from 50 percent, right. And that would be at around 37.53 percentage right. So, this how you should calculate, because we know this area is 0.12 and this area is, this area is 0.5 minus this value. So, you will get 37.53.

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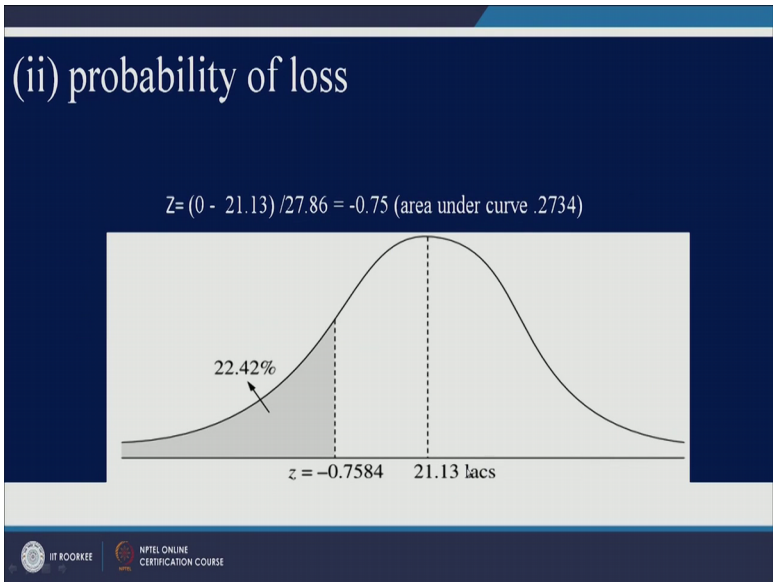

Now what is the probability of loss for this particular project? Take x is equal to 0, right. Again calculate z value which is now minus 0.75. Now you need to look at area under curve. So, minus 0.75 look at z table 0.7 and 5 is this right; so 0.2734 it is whether positive or negative side, right. When it is negative side you just take this side. If it is positive this that would be this side right. So, when I say 0 meet me somewhere here, right. And area under curve would be 0.75. So, this is 0.27384. This is area under curve 0.2734.

Now, since you want to know this value, because this area is 0.2734, right. Just keep in mind this area is 0.2734 you want to know this area. So, 0.5 minus 0.2734 you will get 22.24 as your answer; so probability of making a loss as for as this project is concerned is 22.24 percentage. So, this is how you can solve a question, if you if you want to calculate mean the expected NPV and it is standard deviation right. So, far we have seen a case of perfectly correlation cash flow.

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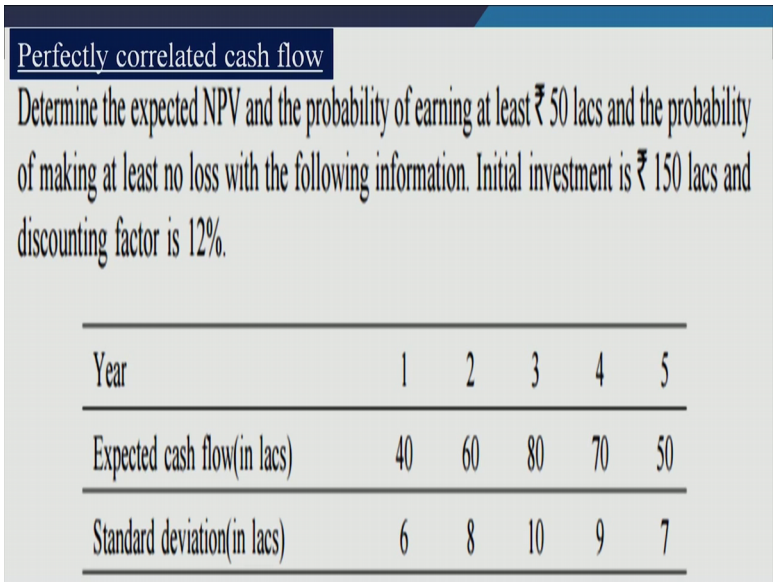

Let us look at look at another example wherein cash flows are correlated, right. When I say correlated means the cash flows are almost similar in different years right. So, this the question and you need to solve this question. Now this is a question wherein you have already have been given expected cash flow, you need not calculate right; so not been given probabilities. So, rather than calculating you know expected cash flow from net cash flow and probabilities, you have directly being given in this question expected cash flow plus standard deviation is also available right.

So, the question is like this. Determine the expected NPV and probability of earning at least 50 lakh and probability of making at least no loss with the following information, right. And initial investment is 150 discounting factor 12 percent right. So, you need to apply the formula which I just wrote on board you can again see that formula.

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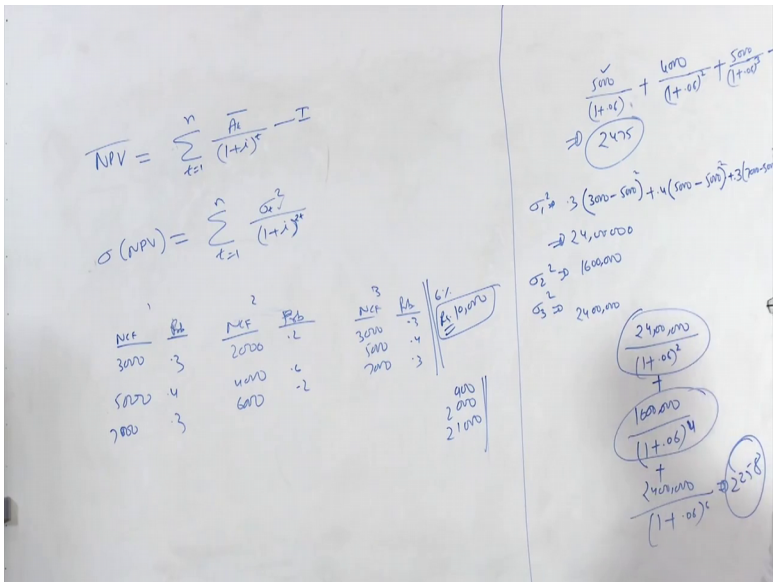

So, this is expected NPV is equal to summation of t ranging from 1 to n expected cash flow 1 plus i to the power t minus initial investment, right. This is what we have calculated, right. And standard deviation of NPV is summation of t ranging from 1 to n 1 plus i to the power 2 t, right. This is i this, right. This is t right. So, the difference between previous formula and this formula is that this is just t, it is not 2 t. In earlier formula it was 2 t right. So, let us solve a question, or if you if you want you just solve this question.

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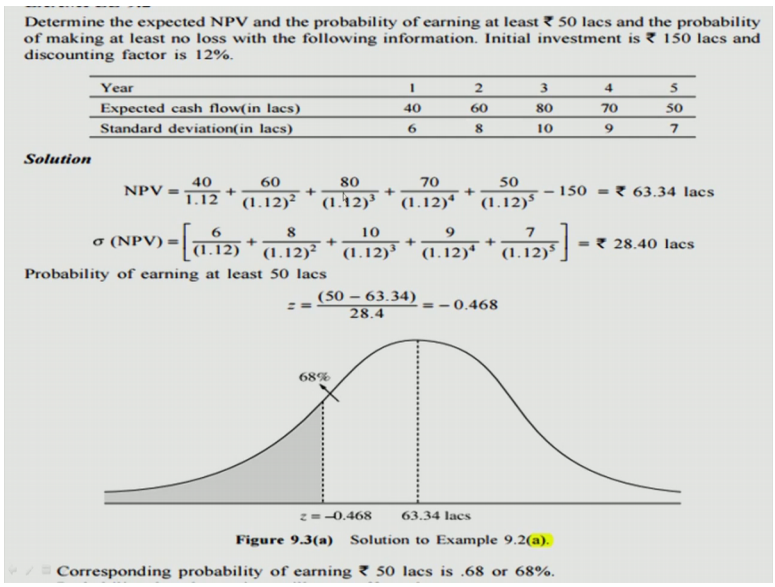

Determine the expected NPV and the probability of earning at least ₹ 50 lacs and the probability of making at least no loss with the following information. Initial investment is ₹ 150 lacs and discounting factor is 12%.

Year	1	2	3	4	5
Expected cash flow(in lacs)	40	60	80	70	50
Standard deviation(in lacs)	6	8	10	9	7

Solution

$$NPV = \frac{40}{1.12} + \frac{60}{(1.12)^2} + \frac{80}{(1.12)^3} + \frac{70}{(1.12)^4} + \frac{50}{(1.12)^5} - 150 = ₹\ 63.34 \text{ lacs}$$

$$\sigma(NPV) = \left[\frac{6}{(1.12)} + \frac{8}{(1.12)^2} + \frac{10}{(1.12)^3} + \frac{9}{(1.12)^4} + \frac{7}{(1.12)^5}\right] = ₹\ 28.40 \text{ lacs}$$

Probability of earning at least 50 lacs

$$z = \frac{(50 - 63.34)}{28.4} = -0.468$$

Figure 9.3(a) Solution to Example 9.2(a).

Corresponding probability of earning ₹ 50 lacs is .68 or 68%.

So, for this question NPV is 40 by 1.2. 60 by 1.12 square 80 by 1.12 cube, right. Because t is you need to just increase value just t by 1 every time right.

So, NPV for this project is 63.34 lakh, right. And standard deviation of NPV is 6 divided by 1.12, because you have been given standard deviation; so this 6, 8, 10, 9 and 7 and since you can also calculate 1.12 to the power 2 cube to the power 4 to the power 5. So, this becomes 28.4 lakh, right. Now this is this is this case is of uncorrelated, right: Uncorrelated cash flow. This example is not of correlated cash flow.

So, will do this the example in a way which we did earlier example right. So, this is 28.4 and the probability of earning at least 50 lakh is x minus mew divided by standard deviation right. So, standard deviation is this mean is this. So, what is the probability that the project would be earning at least 50 lakh? Just calculate area under curve at z is equal to this and finally, the answer for this question is 68 percent right. So, the probability that this project would earn at least 50 lakh rupees is 68 percent.

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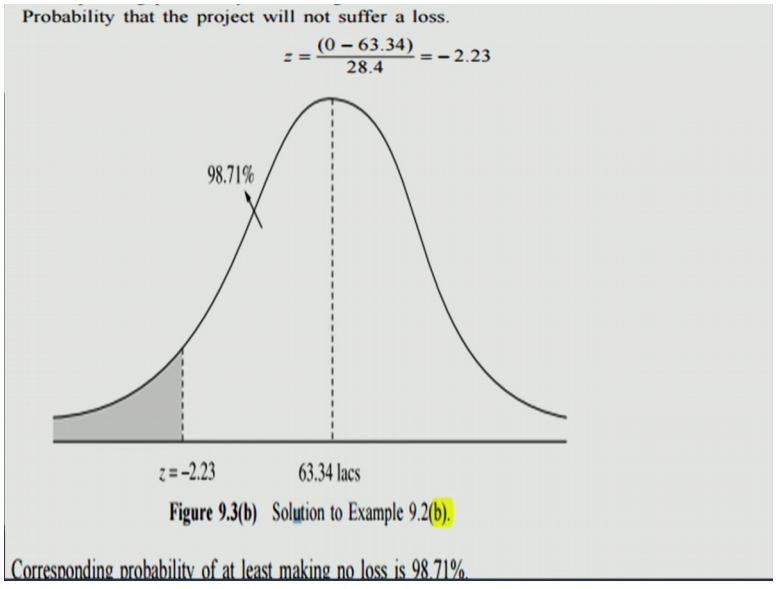

And the probability that the project will not suffer a loss or is you just put x is equal to 0 minus mew divided by standard deviation. So, z value is minus 2.23 and area under curve is 98.71 right. So, the corresponding probability of at least making of making no loss is 98.71 percentage right.

Now, we will solve 2 more questions related to this hillier model. Again will take one case of uncorrelated cash flow and the other case would be of related cash flow right. So, let me write those 2 questions and then will work out those questions. Questions are very simple. So, first of all we will do it for un uncorrelated cash flow, and for uncorrelated cash flow this formula becomes 2 t, right. And question is like this. So, net cash flow net cash flow, in the first year with probabilities are given. So, this is 3000 0.3, 5000 0.4, 7000 0.3, right; this for first year, right. For second year, for second year you have got net cash flow 2000, 4000 and 6000. Probabilities are 0.2, 0.6 next would be 0.2.

Then for third year net cash flow and probabilities; so net cash flow 3000, 5000 and 7000. Probabilities are 0.3 0.4 remaining would be 0.3 right. So, now, and the discounting factor is 6 percent for this question, 6 percent and initial investment is also given, this rupees 10,000, right. This is your initial investment. Now you need to work out this question. So, first of all calculate this value, right. NPV expected NPV how would you do it? Just try for this question. So, you need to calculate expected cash flows first; for all these years right. So, how to proceed? Let us solve this question. So, expected cash flow is known to you this is this net cash flow into probability. So, this 900 this is 2900, 2000 and 2100 right. So, this becomes how much 900, 2000. So 2900, plus 2100 since 5000 divided by 1.06, right to the power 1. Is not it?

So let it be like this. Second expected cash flow it is 400, 2400. So, this becomes 2800 plus 1000, 1200. What is that value would be? Just calculate it is 400, 2400 then So, this is 2800 3600. I think this is, right. Let me check whether this write or not this is actually 4000 there is something wrong. So, this is this is this is 4000 this is 4000 summation of this is 4000 this 2800, 3000 and 4000, yes. 4000 divided by 1.5 square, right. The third one would be, 900 2000, right. Then 2100 this is 5000 divided by 1 plus 0.06 minus initial investment is 10,000.

So, this value would be you just calculate and that value is actually this is 2475, 2475, right. You can calculate this you will get this answer; so this how you can calculate NPV. Now standard deviation: how to calculate standard deviation? Using this formula right. So, first of all you need to calculate this value is not it? So, how to find out this value? We did it in previous example right. So, sigma 1 this is the variance of first year is, how to proceed? There is a probability 0.3 into 3000 minus 5000. What is this 5000? Expected cash flow in first year plus 0.4 into 5000 this value net cash flow minus 5000

plus 7000 before that probability; so 0.3, 7000 minus 5000 square, square, this square of this, right. Ok.

So, this i swhat we were solving sigma 1 square is equal to this and the answer to this is 240000, right. Double 0 tribal 0 right? So, this is; what is the variance of first year, right. Similarly you can calculate variance of second year and third year, right. Let me write those values for your convenience and those values are this 1600000 and 2400000 right.

So, with this we will solve the remaining part of this question in next session, right. Because we did not find out s for this value, right. We just calculated these values. Otherwise let me finish in this session itself. So, you have 24 divided by one point 1 plus 0.06 square plus this plus 1600000 divided by one point 1 plus 0.06 this is 4, right. 2 into 2 4, right plus. So, this plus 24 1 plus 0.06 6, right and this when you add all these things your final value would be 2258; this is your standard deviation of the project and NPV you have already calculated which is 2475 right. So, this is a case of uncorrelated cash flows and Hiller model right.

So, with this let me finish this particular session we will have next session and in this next session we talk about correlated cash flows.

Thank you very.