

Project Management for Managers
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Lecture – 25
Simulation Analysis

Good morning friends, I welcome you all in the session. As you are aware in previous session we were solving questions on how to calculate NPV and standard derivation of NPV and we have seen 3 example so far. We have seen one example of unrelated cash flow sorry; we have seen 2 examples of unrelated cash flow and one example of related cash flow. Let us look at one more example of correlated cash flow, and for that you would be using these 2 formulas. So, it is very simple there is no change in this particular formula as far as this one is concerned as it is; however, there is a change in this second formula right.

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Handwritten formulas and table on a whiteboard:

$$NPV = \sum_{t=1}^n \frac{A_t}{(1+r)^t} - I$$

$$\sigma(NPV) = \sum_{t=1}^n \frac{\sigma_t}{(1+r)^t}$$

t	A _t	σ
1	5000	1500
2	3000	1000
3	4000	2000
4	3000	1200

Below the table, a calculation is shown: $\frac{5000}{(1+0.06)} + \frac{3000}{(1+0.06)^2} + \frac{4000}{(1+0.06)^3} + \frac{3000}{(1+0.06)^4} - 10000 \Rightarrow 3121$

What is the change? It is just standard deviation right not the variance right and here it is 1 plus I to the power t, in earlier formula it was 2 t right. So, we will take one very simple example and we will try to solve it right.

So, there is a project, so 1 2 3 4. So, you have been given A t values directly right and these values are 5 3, 4 3, 5 3, 4 and 3000 right along with their standard deviation right. So, standard deviations are like this 15 1, 1500, 1000, 2000 and 1200 right. So, now, you

can put these values in this particular formula you will get NPV right. So, this is let us solve for n expected NPV. So, that value is you have got 5000 plus 1 plus 0.06 plus 3000 divided by 1.06 square then 4000, 1 plus 0.062 and finally, 3000 1 plus 0.06 to the power 4 minus initial investment is 10000 right initial investment is 10000 right. So, this value can be calculated easily right and for your convenience this is 3 1 2 1 right.

Similarly, you can also calculate standard deviation right just put just take the summation of all these divided by 1.06, then square q 1 to the power 4 right. So, you will get standard deviation also right. So, this we come to the end of this particular method what was this method it was Hillier model right. Let us move onto another method of calculating risk and this method is simulation analysis right. So, we have seen since several methods for example, we have seen sensitivity analysis right, we have seen scenario analysis right, we have seen Hillier model and let us look at one more simulation analysis though we did discuss little bit about simulation analysis, but we did not solve any example.

So, in simulation analysis as you are aware we apply simulation analysis in highly complex situation, when even mathematical modelling is also difficult. Generally we apply simulation when future is quite uncertain right. So, we do not know what will happen, only we know is the probability of occurrence of some events right. So, we will use the information which is available in the form of probabilities to find out answer to a question.

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Simulation Analysis

Sensitivity analysis indicates the sensitivity of the criterion of merit (NPV, IRR or any other) to variations in basic factors.

Though useful, such information may not be adequate for decision making.

The decision maker would also like to know the likelihood of such occurrences.

This information can be generated by simulation analysis which may be used for developing the probability profile of a criterion of merit by randomly combining values of variables that have a bearing on the chosen criterion.



As we have seen that sensitivity analysis and scenario analysis they give lots of information, but they are not adequate for example, in sensitivity analysis we have seen that if we decrease sales, then our NPV would also decrease, but what is the probability of that decreasing NPV. So, to calculate those probabilities we use simulation tool right.

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Procedure:

1. Model the project-How NPV is related to parameters (variables held constant by decision makers) and exogenous variables (stochastic in nature and not controllable).
2. Specify the value of parameter and probability exogenous variables.
3. Select a value, at random, from Probability Distribution of exogenous variable.
4. Determine the value of NPV for set parameter and exogenous variable generated randomly.
5. Repeat steps 4 and 5 a number of times.
6. Plot frequency distribution of NPV.



So, in simulation there when we carry out simulation there is a process procedure for it. So, first of all you need to model the project when I say modelling of the project means how you are NPV is related to different parameters right parameters means those variables which are under the control of decision makers, and there are some exogenous variables which are which cannot be controlled by decision makers right and their stock

sticks in nature and as I said not controllable, but we know the probabilities of those exogenous variables right.

So, second point in this is specify the value of parameter and probability of exogenous variables. So, from past data you can find out probability of exogenous variables right then in simulation generally we come up with random numbers and these random numbers are available in books of statistics, in books of financial accounting, project management and so on. So, we use random numbers to solve the problem. Why we use random numbers because our data are the input data are mostly random in nature right. So, the to solve that particular problem we have to use random numbers.

The fourth point is determining the NPV for set parameter we controllable as well as for uncontrollable variables repeat step 4 and 5 number of times right. So, you can have 10 runs, 15 runs, 10,000 runs 20,000 runs and so on. So, manually if you do this type of question it is very difficult, but using computer you can run several simulations right several runs for a simulation model right and then at the end of the day you can plot frequency distribution of NPV right. So, NPV the frequency distribution of NPV can take different shape right. So, we do not know what shape NPV distribution will take; it may take normal distribution shape or a triangular distribution shape or step distribution shape and so on. So, it all depends on the probabilities of exogenous variables and the random numbers.

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The slide displays the NPV formula and associated parameters. The formula is
$$NPV = \sum_{t=1}^n \left[\frac{\text{Annual cash flow}}{(1 + \text{risk free return})^t} - \text{In. Invst} \right]$$
 Below the formula, it states: Risk free return = 10% initial investment = 13000. The exogenous variable having following probability distribution

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So, we will take an example and this very simple method the simulation analysis is very simple. NPV is equal to summation of 1 to n right multiplied by annual cash flow divided by 1 plus risk free return to the power t right minus initial investment right.

Similar to what we have seen this the formula. Let us take an example wherein we have been given 2 exogenous variables along with their probability distributions, and let us take risk free return as 10 percent and initial investment in this case is 13000. So, let me give you what are the probabilities of exogenous variables, and I will also write down different random numbers which we would be using for solving this particular question to calculate NPV right. So, the question is like this. So, you have been given. So, annual cash flow is given along with probability.

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ACF

Value	Pt
1000	0.02
1500	0.03
2000	0.15
2500	0.15
3000	0.30
3500	0.20
4000	0.15

Life Cycle

Yr	Pt
3	0.05
4	0.10
5	0.25
6	0.15
7	0.10
8	0.05
9	0.03
10	0.02

Cumulative Distribution Function

Value	Cum. Prob.	Range
1000	0.02	00-01
1500	0.05	02-04
2000	0.15	05-19
2500	0.30	20-34
3000	0.60	35-64
3500	0.80	65-84
4000	0.95	85-99

Simulation Run

Run	Random No.	Value
1	53	3000
2	66	3500
3	30	2500
4	19	2500
5	31	2500
6	81	3000
7	37	3000
8	48	3000
9	90	4000
10	58	3000

NPV Calculation

Run	Life Cycle	NPV
1	9	4277
2	9	8506
3	8	929
4	9	7660
5	6	2112
6	7	4039
7	7	1605
8	7	1605
9	5	2143
10	6	66

So, annual cash flow is 1000 and it is probability is 0.12, 0.02. Let us say 0.02 right 1500 probabilities 0.03, 2000 it is probability 0.15 then 2500 probability is 0.15, same for these 2 right. 3000 probability 0.30, 3500 probability is 0.20 and for 4000 0.15. So, we will be using montecarlo simulation to solve this particular problem. So, these are cash flows and probabilities right. So, this first exogenous variable right which is not under your control right, this second one is project life cycle. So, this is let us say this is value and this is annual cash flow right first and the life cycle is second exogenous variable. So, this is in terms of years, 3 years probability 0.05 right, so this is probability right.

For fourth year it is 0.10, for fifth year it is 0.30, for sixth, seventh, eighth, ninth and tenth. So, for sixth year this 0.025, for seventh year it is 0.15, for eighth year it is 0.10, for ninth year it is 0.03 and for tenth year it is 0.02 right and let me also give you random numbers which you would be using for this particular question right.

So, the random numbers we will be using 2 digit random numbers, random numbers as I said can be generated through let us say computer or you can have let us say you can note down different chits right write 2 digit numbers on 100 chits and those number would be from 00 to 99 right because you want 100 chits right. So, put all those chits in a box and then you pick up one chit right. So, that would be a method of generating random number right. So, either you use this method or you can generate it through computer or through excel or there are random number tables available in books right.

So, will use since you have got 2 exogenous variables, so you need and these are 10 points right 1, 2, 3, 4, 5, 6, 7, 8 right 8 and 8. So, you need 16 random numbers. In fact, it how many random numbers need are depends on how many runs you continues simulation right. So, these are 16 8 values for a this cash flow and 8 values for life cycle, now you can go for 10000 runs also. So, for 10000 runs you need 20,000 random numbers right. So, in this particular question we are going for just 10 runs right. So, for 10 runs how many random numbers are needed? 20 random numbers 10 for this and 10 for this, so those 20 random numbers are like this.

So, first of all I will be writing random numbers for this one right. So, random numbers are. So, first random number is 53, 66, 30 then 19, 31, 81, 38, 48, 90 and 58. So, total 10 random numbers. So, this first second third and so on right this tenth random number. So, these are called random numbers right. So, for project life also you can have random number and this random number are 97, 99, 81, 9, 67, 70, 75, 83, 33, and 52 right. So, these are 10 random numbers for project life cycle right. So, how to proceed now? So, this, the question given to you we want to calculate what would be the NPV right. So, first of all what we should do, we should assign tag numbers to this probabilities right.

So, let us first assign tag numbers for cash flow right. So, let us say a value is 1000 1500, 2000, 2500, 3000, 2500 and 4000 right probabilities are also given. So, 0.02, 0.03, 0.0, 0.15, 0.15, point for 3000 it is 0.30, and for this is point zero and 0.15. So, first of all calculate cumulative probabilities right. So, how to find out cumulative probabilities?

0.02, 0.05, 0.20, 0.30 right it is 0.35 right 0.65, 0.85 and this is 1.0 right. Now once you are done with cumulative probabilities, let us assign tag numbers to these probabilities right. So, how to proceed? It is 00 to 01 right. What is the meaning of 1000 and probabilities 0.02? It is meaning is that out of 100 times, the value of cash flow was 1000 only for 2 times right. So, for first time we have assigned it to 0 value and for second time we assigned it to one value right.

Similarly, 1000, 1500, 2000 I hope we have missed any value 1 2 3 4 5 6 7 no. I think we do not missed did we miss any value 1 2 3 4 5 6 7 ya we did not missed right. So, what would be the tag number for this is 0 2 and 0 4 right. Keep in mind since this is 5 right if you multiply all these by 100, then this would become 5. So, this number should be 4 this would become 35. So, the end number should be 34 this would be 100. So, the ending number would be 99, this how should you assign tag numbers. So, for this 5, 5 to 19 right then 20 to 34, 35 to 64, 65 to 84 and 85 to 99. So, these are nothing, but tag numbers we have assigned right for this one right.

Similarly, assign tag numbers for life cycle also. So, let us look at this. So, life cycle in terms of years, so 3 4 5 6 7 8 9 and 10 right probabilities are given right. So, this is 0.05, 0.10, 0.30, 0.25, 0.15, 0.10, 0.03, 0.02 right and of course, you can write 0 here right. Now what is the next step? We need to find out cumulative probabilities right. So, cumulative probabilities 0.05, 0.15, 0.45, 0.70, 0.85, 0.95, 0.98 and 1.0 right; similarly now assign tag numbers for life cycle also right. So, tag number. So, this is 0 0 to 0 4, 5 to 14, 15 to 44, 45 to 69, 70 to 84, 85 to 94, 95 to 97 and 98 to 99 right.

Now, we will find out NPV and we will go for 10 different runs right. So, let us look at first run this run 1 2 3 4 5 6 7 8 9 and 10 right. So, for first run what is the random number 53? So, random number is 53; now what would be the value of cash flow for 53 random numbers? So, just look at where is 53 in this particular tag number column right. So, 53 is here it means this value would be 3000 right the next random number is 66, now look at where is this random number in this particular column in tag number column right. So, 66 is here it means value would be 3500, next 30 right 30 is here it means this is for fourth right. So, this is 2500, fourth random number 19. So, let me write other random numbers also 31, 81, 38, 48, 90 and 58.

Now, this 31 is here right. So, this is again fourth right. So, this is 2500, 81. 81 is here right it is 3500, 38. 38 is in this one right. So, this is 3000, 48. 48 is in which range it is here right. So, this is 3000, 90 is here right. So, this is 4000 and 58 is where it is here right, so this again 3000. So, what we are saying these are the values of corresponding to these random numbers, the moment you change random numbers these values will change and your random numbers will also change right. So, answer depends on how you are choosing random numbers the random number should be truly random numbers right. So, this the value right now this we have done for ACF right.

Now, let us do it for life cycle ride. So, first one, this is for ACF right. Let us do it for life cycle right I will use different color pen life cycle. So, first run right I let me write run number here right run number. So, you will got 1 2 3 4 5 6 7 8 9 and 10 right what is random number random number you have got 97, 99, 81, 09, 67, 70, 75, 83, 33 and 52 right. Now what you want? You want what would be the life cycle right in terms of years right. So, for random number 97, where this 97 lies in this column it is here right. So, life would be nine years right next can you try 99. 99 is again here. So, 99 is in this range right. So, the life would be 10 years right similarly for 81, 81 is in which range here right. So, life would be 7 years; for fourth run random number is 9, 9 is here right it means 4 years. For 67, 67 is here right it means 6 years.

For 70, 70 is here right again 7, 7 right 70 is here. So, 75, 75 is in which range here right. So, this is again 7 years. For 83, 83 is where here right, no it is here right. So, again 7 years, 33 when random number is 33, 33 is here right. So, 5 years 53, 52 is here right. So, 6 years and now you can calculate NPV using the formula which I gave you right formula is here right.

So, you use this much the all these values and calculate NPV. So, right let me also you can apply this formula and find out NPV of this particular project, so 9 10 7 4 6. So, this is perfectly all right. So, NPV would be 4 2 double 7 8 5 06 2 8 9 7 double 6 0, 2 double 1 2, 4 0 3 9, 1 6 0 5, 1 6 0 5, 2 1 6 3 and 66. So, this is how you can use simulation method for finding net present value.

So, let me summaries what we did in this particular method in simulation analysis as I said is to be used at in a situation where it is highly complex. So, you have got controllable variables some of them are uncontrollable variables for uncontrollable

variables you have their probability distributions use probability distributions to come up with tag number and then you will have to use random numbers; and random numbers we will decide what would be the answer right if you change random numbers your NPV is will also change. So, you can solve the same example by taking, now random numbers first as a let us say use these random numbers for cash flow and random numbers for this set of random numbers for life cycle and then find out how your answer changes.

So, with this let me complete this session in next session we will see some other topic.

Thank you very much.