Project Management for Managers
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Lecture - 38
Project Time Management- Laddering in PERT/CPM

Hello friends. I welcome you all in this session, as you are aware in previous session we discussed about lags in presidents relationship and we have seen 4 types of lags you have got start to start, start to finish, finish to start and finish to finish. Apart from this we have also seen how to draw a Gantt chart in Microsoft project.

Now let us look at another concept in network it is called laddering. Now laddering is nothing, but a process of sub dividing and an activity so that we can complete our project in somewhat lesser time. So, it is basically a technique that allows us to redraw the activity network, 2 more closely sequence project sub tasks to make the overall network sequence more efficient.

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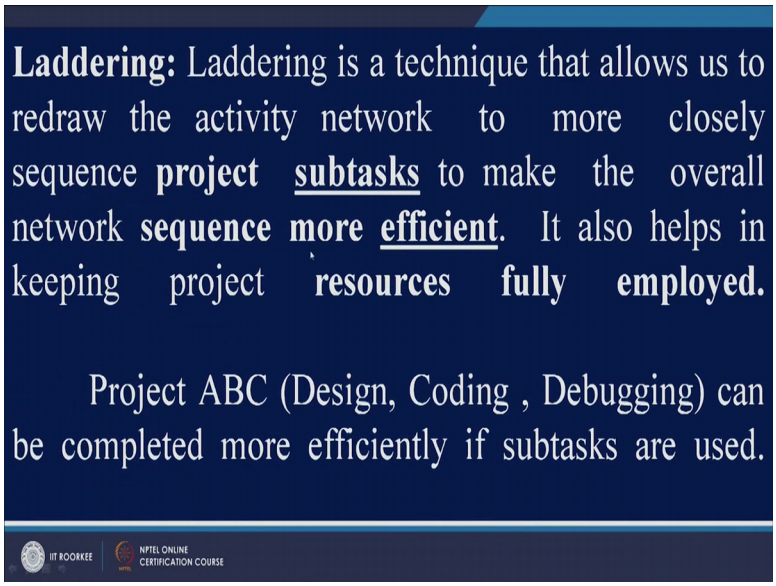

So, at the end of the day our objective here is to efficiently complete our project in time right in fact, before time right.

So, it also helps in keeping project resources fully employed. So, we will take up an example. So, there is a project which has got 3 activities, you have got design you have

got coding, and the third one is debugging. Now this is how this network looks like. So, activity a is designed, activity B is coding and C is debugging right.

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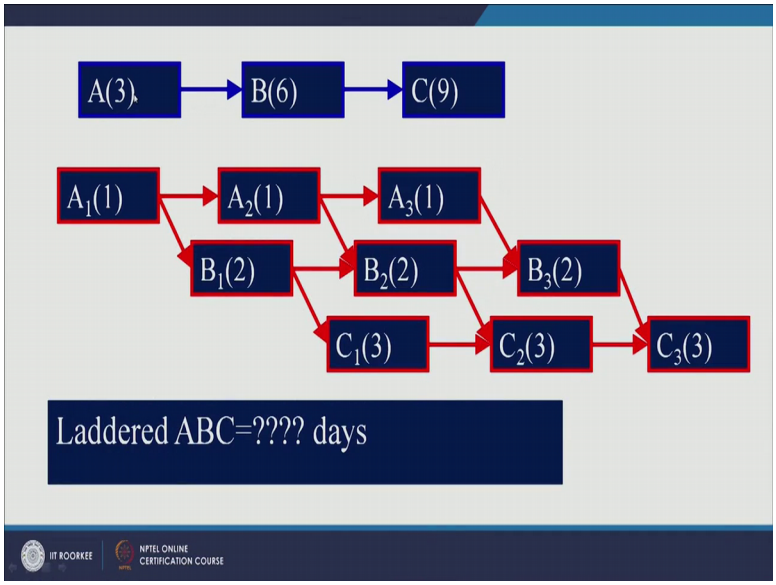

So, activity a is taking 3 minutes or 3 days, B is taking 6 days, C is taking 9 days. So, what is the total duration of this project and what kind of lag relationships are there? It is finish to start right and there is lag is 0 right similarly it is also finish to start right lag is 0.

So, tell me how much time this particular network will take for completion? It is its very simple right it is 3 plus 6 plus 9 and 9. So, total 18 days or 18 months or 18 minutes or whatever it is right now if we redraw this network this. So, we will breaking activity a into three sub activities; A 1, A 2, A 3. So, what we are doing? We are starting activity a and it is sub and it is sub task is A 1 right. So, similarly you can divide activity B also as B 1, B 2, B 3. So, divide activity B whose duration is 6 days into 3 sub tasks each has got 2 days right. So, this is 2 and 2.

Similarly, activity C; we know that we cannot complete activity C unless and until we complete activity B fully right. So, activity C can be divided into again three subtasks each has got 3 days duration right. So, C 1 3, C 2 3 and C 3 3. So, this is known as laddered A B C right and what would be the duration of this this network now laddered network how much time will be reduced just think for a while and give me answer ok.

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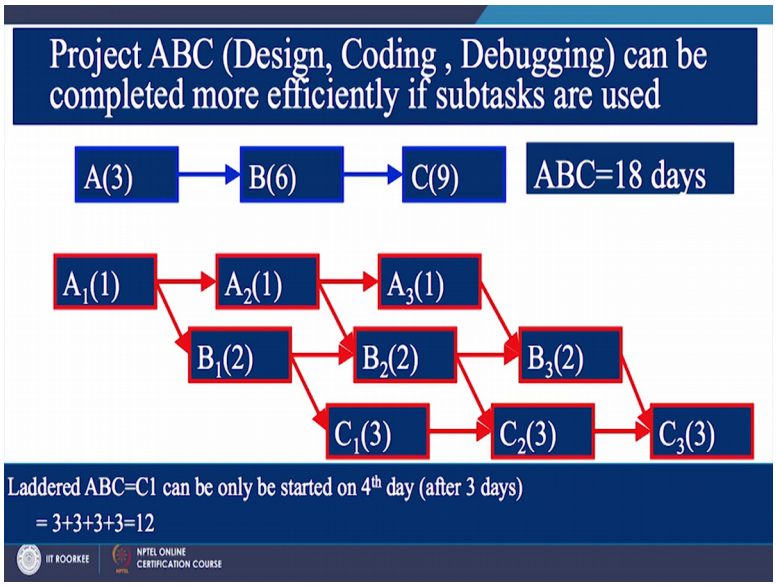

So, I hope you would have found answer and the answer is 12 days right. Initially it is taking 18 days, but when you go for laddering it is taking only 12 days. So, just see here laddered A B C is equal to C 1 and C 1 can be can only be started on forth day. In other words after three days you can start C 1, and since C 3 is taking total 9 days and you cannot start C 3 sorry C 1 before third day right on forth day only you can start C 1. So, C 1, C itself is taking 9 days and you can start it as early as on forth day right or after three days.

So, total duration would be 3plus 9 is 12 right. So, this is known as laddering a very effective method of reducing duration of the project. In fact, the concept here is on there are some activities some success activities you cannot start unless and until you completely finish preceding activity, but you may have set of activities where in you can start succeeding activity after few minutes of start of preceding activity. So, that is the situation here in this particular example. In only those situations you can do laddering otherwise you cannot do laddering.

Let us look at another concept it is called hammock activities. In fact, hammock activities are those activities which we used to summarize for some sub set of the activities. In a project you will have thousands of activities, but you will have some activities which are very small which do not generally consume much time much cost

and so on right. So, you just group those small activities together and combine them and you can keep them hanging below in the network like this right.

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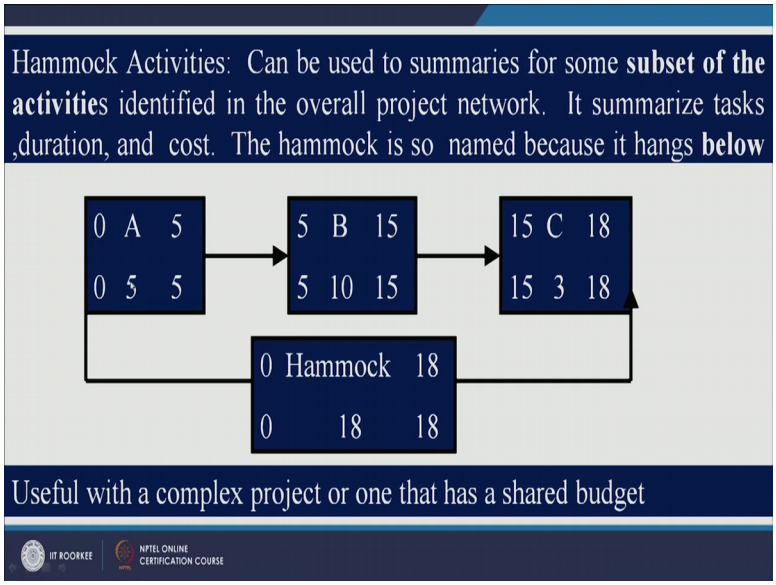

So, activity A B and C it is taking 5 days this one is taking 10 days. This one is taking three days. So, total duration is 18. So, you can just combined all of them and you know h hang over here right in a in a project. So, let me draw a network in which I will fix this particular hammock activity.

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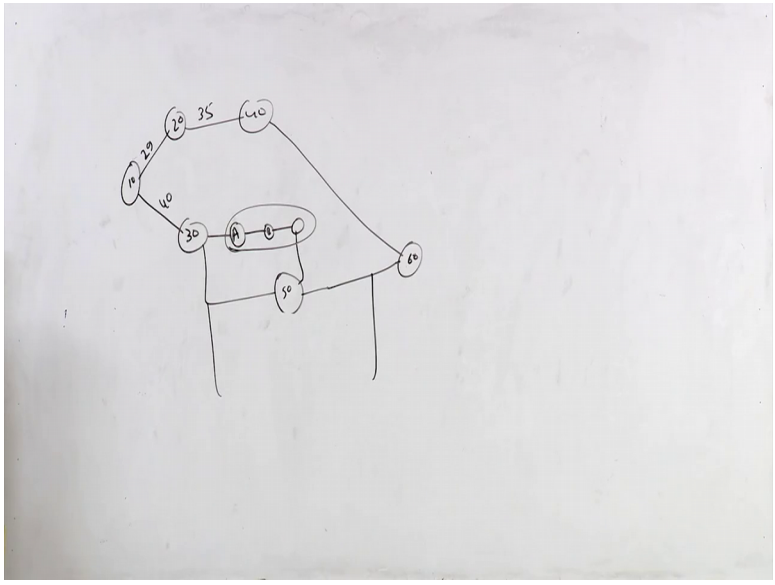

So, let us say this is activity 10, 20, 30, 40. So, let us say this is taking let us say 29 days 35 days 40 days and these are some other activities right let me put it in this way. So, this is A B and C. So, this is hammock activity this is H. So, what I have done here? I have just combined all these three activities and I have just taken one activity and I will call this is a hammock activity right.

So, this is useful with a complex project or one that has a shade budget right; so hammock activities. Let us revisit the concept of critical path we have already defined it the largest duration path is the critical path, and project cannot be completed in less than that time right.

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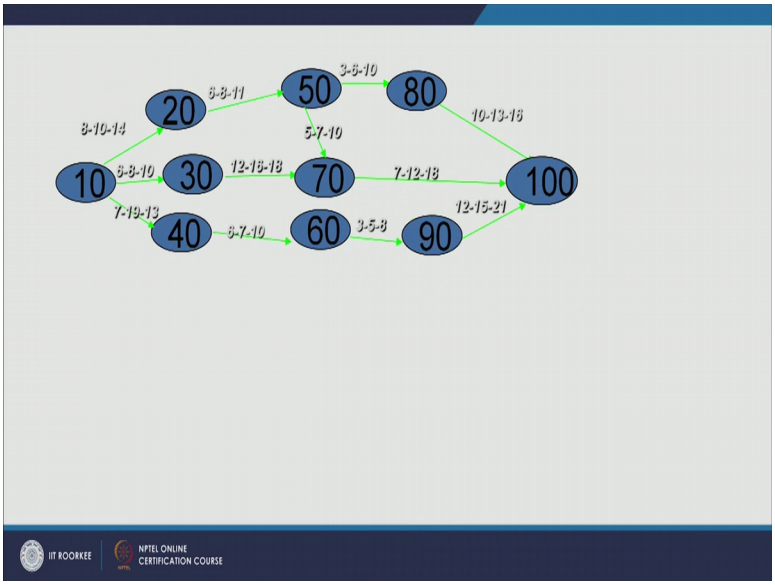

So, let us this example now this the very first example of pert network right. So, far we did find critical path and semi critical paths for C p m network right not for pert network. So, this is a pert network and we will find out critical path for this particular network. So, let me draw this network and will find out critical path. So, you have got a pert network.

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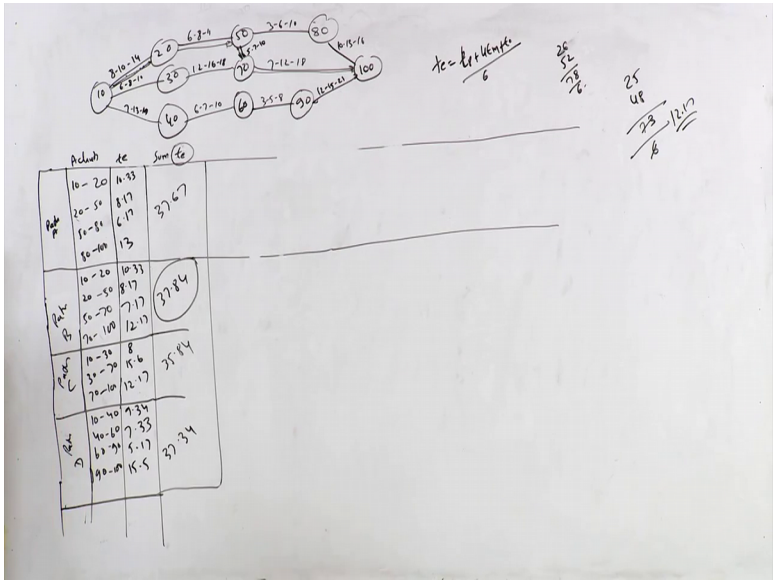

So, this is this is I need to prepare a table over here. So, I will draw this network level like this this is 10, this is 20, 30 and 40 right and the time estimates are 8 10 and 14 then you have got 6 8 and 10. So, you have got optimistic time pessimistic time, in most likely time 7 13 and 19 right anyway whatever is the value you just you need to calculate t C for this right then you have got 6. from 22 50 you have 70 and you have got 60 right.

Duration are 6 at 11 then 30 270 it is 12 16 18 6 7 10 and there is one more activity from 50 to 70 and it is duration is 5 7 and 10 then 50 to 80, 82 100 which is the last node then you have got 60 to 90 and 70 to 100. The time durations are 3 6 10 3 6 10 8 80 to 00 it is 10 13 16 13 and 16 72 100 it is 7 12 18 3 5 8 and 12 15 and 21.

So, let us try to find out critical path for this pert network right. So, how many paths are there first of all how many paths are possible. Forget about which path would be critical path, in this network there are four paths available, this is the first path right 10 20 50 80 and 100 right. So, let us call this as path a right. So, path A right then you have got path B 10 20 50 70 and 100 call that as path B right; path B then you have got 10 30 70 and 100 path C direct path right path C and finally, you have got path d right which is 10 40, 60, 90 and 100 right let us find out t e values.

So, if you look at path A, you have got activity 20 to sorry 10 to 20, 10 to 30 and 10 to 40 right only these three activities are there right. In fact, it is from 10 20 50 80 and. So, it is its 10 to 20, 20 to 50, 50 to 80 and 80 to 100 right this is path A right and since you

know what is what are these values at for this activity 10 to 20, rather than writing all these values again I am directly writing t e x expected value right. Now how to find out expected value? Expected value is what t p plus four times t m plus t u divided by 6 right. So, pessimistic time 4 times most like 4 into most likely time plus optimistic time divided by 6.

So, for this particular value this is 10.33, 8.17, 6.17 and 13. Just check whether this is correct or not check it for 80 to 100. So, 10 plus 16 26 plus13 into 4, 26 plus 52 is 8, 78 divided by 6. So, 78 divided by 6 is13 right. So, this is correct. So, t for first path similarly what are the activities on path B? You have got 10 to 20, then 20 to 50, 50 to 70 and 70 to 100. So, this is path B this is path C right; so for path B just calculate all these t values using this formula right.

So, let me write down values for path B it is 10.33, 10.33 then you have got 8.17, then you have got 7.17 and then 12.17. So, it is 110.33 8.17, 7.17 and 12.17 right. If you wish you can check one of them let us check for 70 to 80 right 70 to 80. So, 7 plus 18 it is 25 plus 48, 25 plus 48 is 73 divided by 6. So, this would be, so 12.17. So, now, you can write t e values for path C and path B, which is path C, it is 10, 30, 70 and 100 right 10 to 30 to 30 to 70, 70 to 100 right only three activities right. So, this path C, path C right and 10 to 30, it is t e is 8; it is 8 it is 15.6 it is 12.17.

So, you can check any one of them right similarly path D right this do this, and do the similar calculation for path d also right path d is 10 to 40, 40to 60, 60 to 90 and 90 to 100 right 4 activities along this path and the time durations are it is 9.34, then you have got 7.33, 5.17 and 15.5 right. Now we have calculated all t e s right for all these activities. Now you need to find out critical path which path is the critical path. So, for that what you should do? You just sum up all these values sum of all t e s along reach path right t e s. So, when you take sum of all these paths some for pathway is 37.67.

Similarly for path B it is 37.84 for path C it is 35.84, and for path D it is 37.34. So, which path is critical path. Critical path is the path which is longest path right. So, longest path is this. So, what is critical path in this case? The critical path is this one it is 10 20 al let me write like this 10 20, 20 to 50, 50 to 70, 50 to 70 then 70 to it is not 70 to it is 70 to 100 right 50 to 70 and 70 to 100 right path B ya it is 70 to 100 right. So, this is the one right.

So, what we have done in this case? Rather than calculating t e s and t n s we have just calculated t e s for all these activities and summed up all those t e s right and we said that this path is critical path right.

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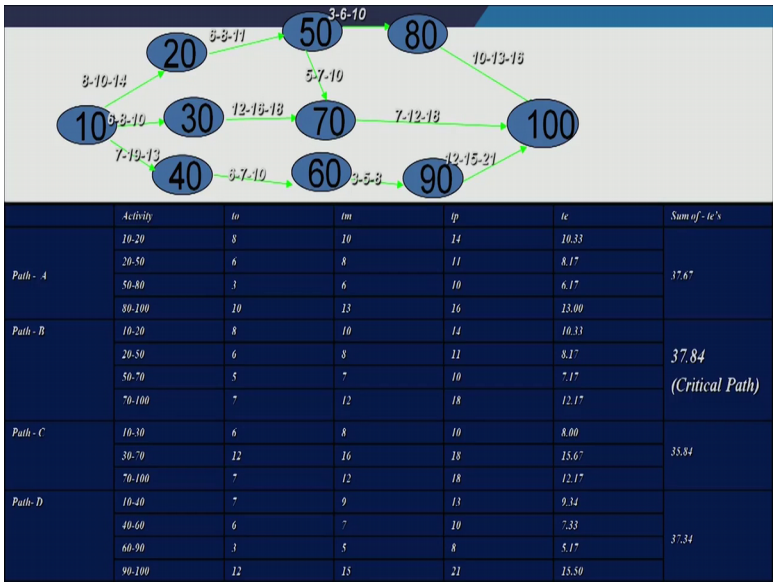

	Activity	to	tm	tp	te	Sum of - te's
Path - A	10-20	8	10	14	10.33	37.67
	20-50	6	8	11	8.17	
	50-80	3	6	10	6.17	
	80-100	10	13	16	13.00	
Path - B	10-20	8	10	14	10.33	37.84 (Critical Path)
	20-50	6	8	11	8.17	
	50-70	5	7	10	7.17	
	70-100	7	12	18	12.17	
Path - C	10-30	6	8	10	8.00	35.84
	30-70	12	16	18	15.67	
	70-100	7	12	18	12.17	
Path- D	10-40	7	9	13	9.34	37.34
	40-60	6	7	10	7.33	
	60-90	3	5	8	5.17	
	90-100	12	15	21	15.50	

So, this is one of the ways of finding critical path in pert network right. So, this is you can see answer over here. So, this is the final answer right.

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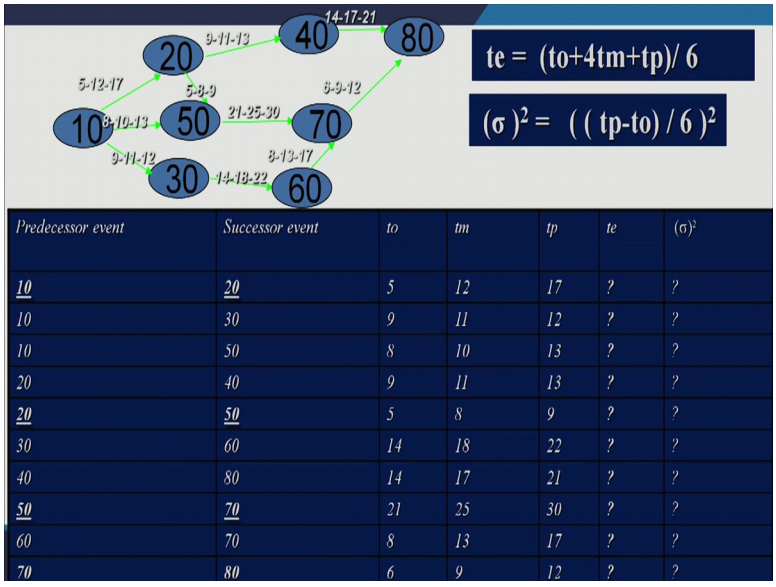

Predecessor event	Successor event	to	tm	tp	te	$(\sigma)^2$
10	20	5	12	17	?	?
10	30	9	11	12	?	?
10	50	8	10	13	?	?
20	40	9	11	13	?	?
20	50	5	8	9	?	?
30	60	14	18	22	?	?
40	80	14	17	21	?	?
50	70	21	25	30	?	?
60	70	8	13	17	?	?
70	80	6	9	12	?	?

Now, if I ask you to find out what is the variance along critical path what will you do. So, variance is it is very simple, first of all for this particular network try to find out what

is critical path, and then after finding critical path you need to find out variance along critical path. So, let us try to solve this question. So, this is a question having 8 nodes and we have to find out what? We have to find out variance along critical path. So, first of all you need to find out critical path right.

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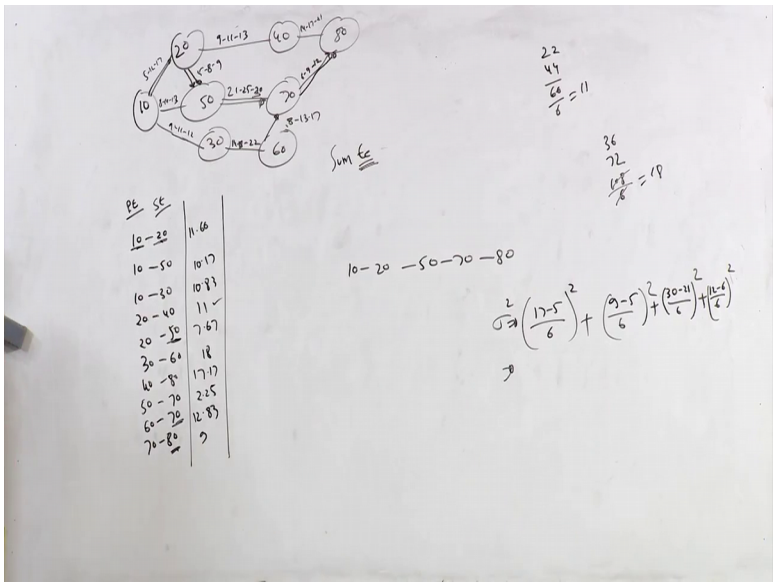

So, let me draw this network first, this is a network which is got 8 nodes, 10, 20, 50, 30 40, 80 then 70 60. So, 60 to 70, 70 to 80 and the time durations are 5, 12, 17, 8, 10 13, 8, 10 and 13 then you have got 9 11 9 11 and 12 right. You have got 9 11 13, 9, 11 and 13 40 to 80 it is 14. 14 17 and 21, you have got 6 9 and 12 this is 8, this is 8, 13 and 17 and this 14, 18, 14, 18 and 14, 18 and 22 this 21 25 and 30 right this is 30. So, you have got all these activities you need to find out variance along critical path and what is the variance, it is just square of standard deviation.

So, first of all you need to find out t e values right and what are those t e values calculate t e values first of all for this particular network and let me draw this table over here. So, 10 to 20, 10 to 50, 10 to 30 then so, this is your proceeding event, and this is your successor event right then you have got 20 to 40, 20 to 50 ya there is one more activity here 20 to 15 it is duration is 5 8 9 5 8 9 right. So, 20 to 40, 20 to 50 right then you have got 30 to 60, it is very simple how to draw this this proceeding activity and succeeding activity just take first point right first node. So, what are the out coming errors 20 50 and 30. So, just write 20 30 the next node is after 10 it is 20 right. So, what are the outgoing

activities from 20? You have got 40 and 50. So, just write 40 and 50 the next is 30 what are the outgoing activities just one right 60 the next one is 40 right what are the outgoing activities 80 right.

Then next one is 50 what are the outgoing activities 70 right then 60, 60 to 70 and finally, 70 to 80 right is not it is very simple then you need to write out write down expected values and for this you need to do calculation right. So, let me write t e values for all these events. So, for 10 to 20 it is 11.64, 11.66 for 10 15, 10 to 50 it is 10.17, 10 to 30 it is 10.83 right for 10 40, 10 40 it is 11 just check whether this is correct or not right because this is a simple value let us check for this. So, 20 to 40, 9 plus 13, 22; 22 plus 44 66 divided by 6 is 11 right is not it.

So, the value for 20 to 50 is it is 7.67 similarly for 30 to 60, it is e18 I will check this also 30 to 60. It is 14 plus 22 it is 36 plus 72 8 divided by 6. So, this is this would be 15 right for 40 to 80, 40 to 80 this value is 17.17 and for 50 to 70, 50 to 70 it is 2.25 it is 2.25 for 60 to 70 it is 12.83, 70 to 80 it is 9 right now you have calculated all these values.

Now, what either you can do. In fact, you can again come up with different paths over here right. So, you can have path a path B path C and past 4 right and just do the summation of all t e values right and take only those t e s, which are there on critical path and find out variance along that critical path or you can do one more thing you can go for t e and t l s all along all these nodes and find out which path is critical path right. So, as far as this particular question is concerned the critical path is there and let us see what is critical path it is 10 to 20, it is 10 to 20 then it is 20 to 50 it is 20 to 50 right and then 50 to 70 and finally, 70 to 80 right. So, this path is critical path.

So, you can do 2 things at this stage either calculate summation of t e s for all four five paths for this question and then take standard deviation, and then variance right. Since we know this is the critical path 10, 20, 50, 70 and 80, we also know the three time estimates for all these activities right. So, for 10 to 20, 10 to 20 right this is these are different time estimates 5 12 and 17 right. So, what is standard deviation? It is 17 minus 5 divided by 6 directly calculate variance right plus you have got 20 to 50 right 20 to 50. So, this let me draw a critical path for 20 to 50, 50 to 70, 70 to 80 right; so this plus 20 to 50 right. So, this is 9 minus 5divided by 6 plus 50 to 70. So, 30 minus 20 one divided by 6 whole square plus 12 minus 6 divided by 6 whole square right. So, this is how you will

get variance along critical path. So, you just add all these values you will get variance along critical path.

So, this is another example in which what we have done we have calculated critical path and variance along critical path. So, with this let me complete this session.

Thank you very much.