Project Management for Managers
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Lecture - 39
Probability Models in Networks - I

Good afternoon friends, I welcome you all in this session. In this session, we are going to see probability models in networks. Though we have seen couple of issues related to probability when we were discussing about hillier models in fact, when we were looking at risk analysis in standalone projects, so there we have seen Hillier model. So, some of the things related to probability has already been covered, but let me tell you how probability helps you in knowing whether the duration of the project would be achieved in a particular period of time or not. So, let us get started.

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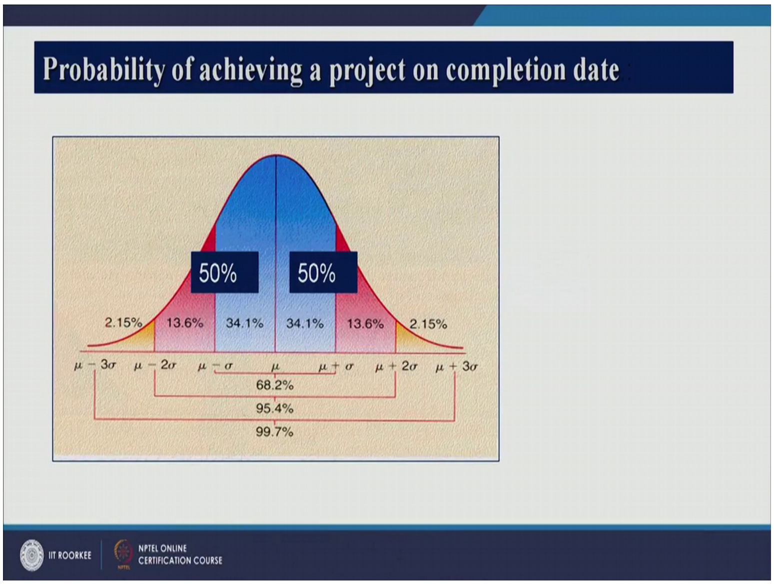

All of you would have seen this curve this is a normal curve, a bell shaped curve. This area under curve is unity, it is always one. And this is quite a symmetric curve under the central tendency of this particular curve. Central tendency is something where the whole distribution rests. So, this is mean of this curve. Since this is the symmetric one, so this is this point is actually mean mode and median; this point, this point is mean mode and median. So, this is a curve which we would be using in our calculation of duration of various projects. So, if you look at this curve, it has got two sides you have got 1 sigma 2 sigma plus 3 sigma. So, mean plus minus 1 sigma mean plus 2 sigma and mean plus 3 sigma.

This side mean minus 1 sigma mean minus 2 sigma and mean minus 3 sigma. So, we in this curve the characteristics of this curve is that within plus minus 3 sigma limits you will have 99.7 percent of the area. Within two sigma limits, the area under curve is 95.4 percentage. And within one sigma limit, you have got 68.2 percent area. So, we will use this, this information in solving couple of questions.

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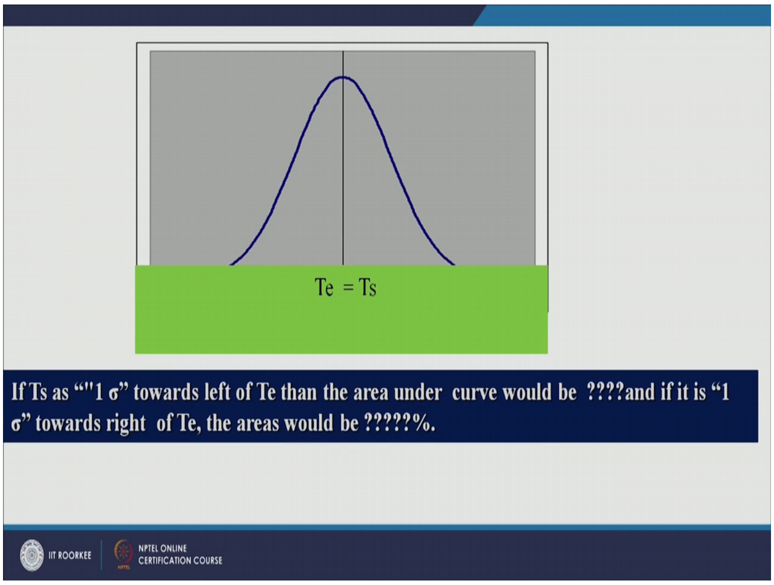

So far we have calculated T e and T l s for the networks. And let us look at and I have also said that T at the last node is nothing but the duration which a project will take for its completion. And T s is nothing scheduled completion time which would be given by your client if shelled scheduled completion time T s is not given then T e is equal to T l. Let us look at this example where in this is T e, which is also equal to T s. Since there is no difference, so we will say the probability of completion of this project is let say 50 percent. So, this area is 50 percent right this area and remaining this area is also 50 percent. So, this total area is 100 percent. Now, you should answer my question. And the question is if T s if this value is towards left of T e let say if it is here is left of t then the area under curve would be what, if this T s value is one standard deviation let say somewhere here, let me draw this is curve.

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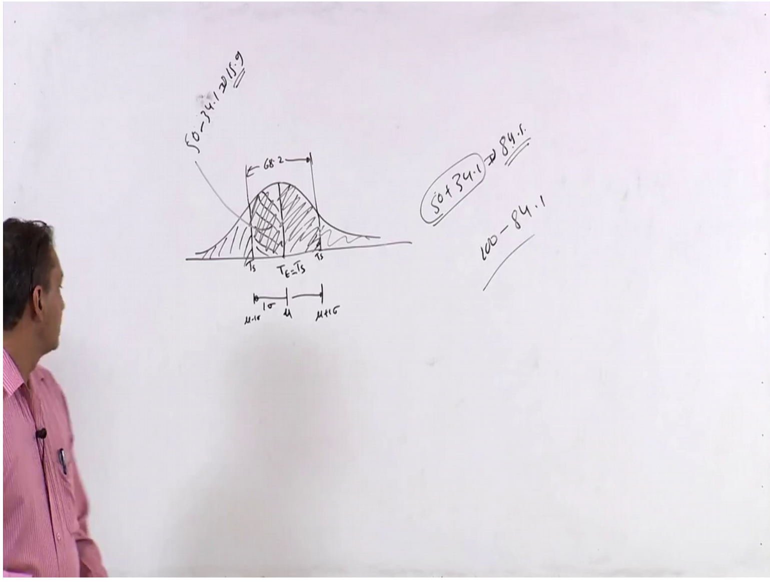

So, this is your normal distribution curve right this is your T e and at this point T s is also this right now if T s is this let say this is T s and this is one standard deviation away from here 1 sigma and this is your mu. So, what would be the area what would be this area can anybody tell me or can you tell me? I know that this total area is 50 percent and one standard deviation towards left and towards right is at 68.2. So, this is mu minus 1 sigma this is mu plus 1 sigma. So, this area is 68.2. What would be this area, this area would be 50 percent minus this area is 34.1, this area is 34.1, so 50 minus 34.1 is not it. So, we will say that this area is 15.9 this area is 34.1. So, this is how you should answer this question.

And suppose if I ask if T s is one standard deviation towards right, if T s is this then what is this area I know that this area is 34.1. So, if I want to know this area for example, right. So, 50 plus 34.1 this is 84.1 right and the remaining one is. So, 50 plus 34.1, this is 84.1, 100 minus 84.1 you will get this area. So, this is how you should be able to find out area under the curve right. So, let us move on to one more point.

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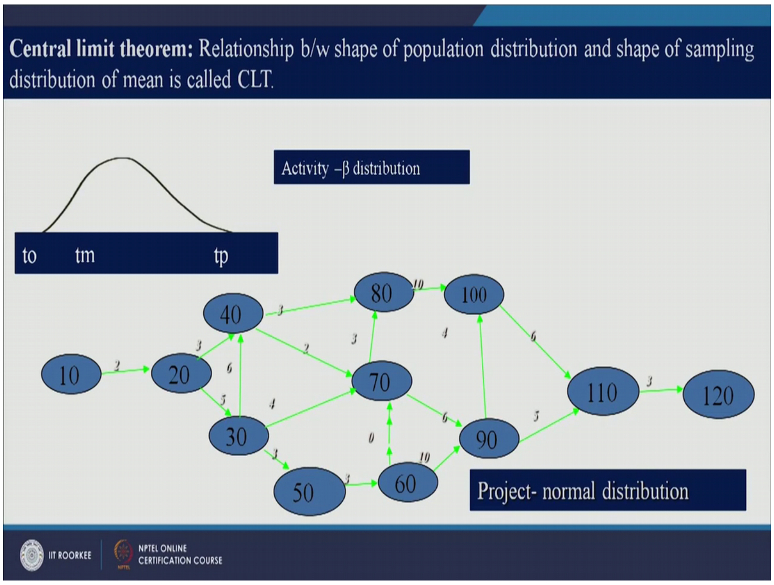

Now, there is something called central limit theorem. In fact, this theorem will help us in knowing the characteristics of the project. I have already told you that the distribution of activities is like beta distribution, it is a skewed curve right. And I have also said that the project is a combination of hundreds of activities and project has got a normal distribution. So, every activity is like a beta distribution, but at the end of the day project is having normal distribution. So, central limit theorem is actually a relationship between shape of population distribution and shape of sampling distribution. So, if we take this activity as one sample out of all those activities then the distribution of this sample would be like beta distribution and all the activities will have normal distribution.

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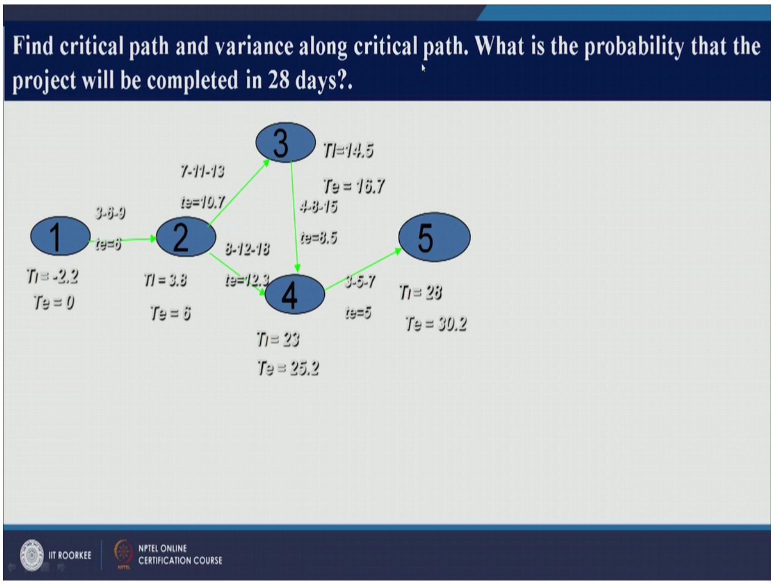

Let us look at this question find critical path and variance along the critical path. What is the probability that the project will be completed in 28 days? We want to find out first critical path, then variance along critical path, and third what is the probability that project will be completed in 28 days. Now, as I have told you that T at the last node is nothing but the project completion time. So, as far as this particular project is concerned, T is 30.2. So, the probability that project will be completed in 30.2 days is 50 percent, this is just one-half of the bell shaped curve. Now, let us find out what is the probability of completing this project in 28 days.

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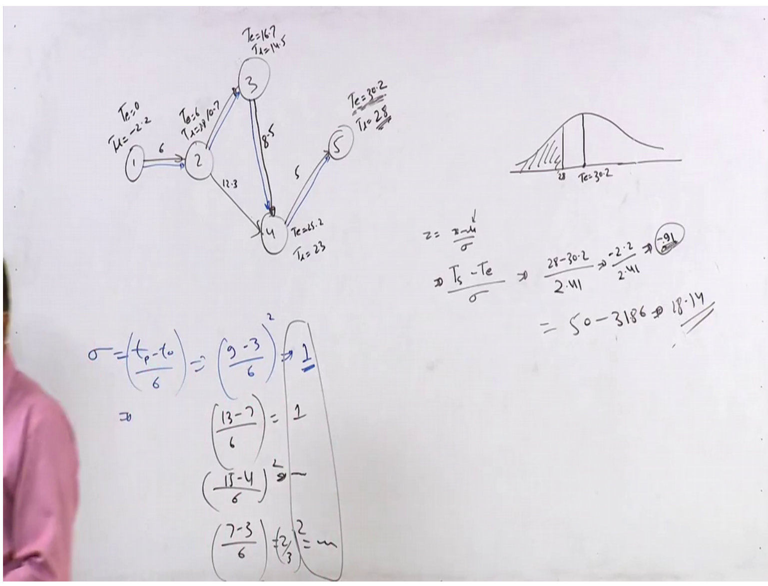

So, let me draw this network. So, you have got 1, 2, 3, 4, and 5; these are five nodes. And you have also been given different time estimates. So, this is basically a PERT network. So, you have been given three time estimates for each of these activities. So, rather than writing all these three time estimates, let me write just the expected time which has been calculated using a formula right its T p plus 4 times T m plus T u divided by 6. So, T for 1, 2 is 6; for this, it is 10.7. For 2 to 4, it is 12.3. For 3 to 4, it is 8.5. And for this, it is 5. So, we have converted a PERT network into CPM network.

So, let us start finding critical path. So, for that there are couple of methods either you just take T values at all these nodes and select the longest path, since this is a very simple network. So, you can do that. Otherwise you need to calculate T n, T l set all the nodes and then find out where slack is zero. So, let us look at this will be follow the second approach. So, T e is equal to 0, T e is equal to 6 here, T e is equal to 16.7of course, 16.7 plus 8 its 24 plus 25.2. So, T e is equal to T e this is equal to 25.2. Just read these two once again this is 1.2 plus 24, 25.2, but if you come from here then it would be 18.3. So, we have to select the higher value. And then this is T e at this node is 30.2. So, the probability that the project would be completed in this much time is 50 percent. Let me draw distribution curve also probability distribution curve. So, this is your T e which is 30.2. So, probability of completing this project is 50 percent and this is the time for that particular probability.

Now, let us find out critical path. In fact, you can easily identify it is from here to here to here to here. So, if you wish, you can calculate T l also. Let us calculate T l. In this question, we have been given what is the probability of completing is project in 28 days. So, this is scheduled completion time and let us take this as 28. Now, 28 minus 5, T l is equal 23 then T l at this point would be 23 minus 8.5. So, this would be 15.5, 23 minus let say 9, so it would be 14, it is 13.5, it is 14.5. So, this is 15 plus 8, 23. So, T l at this node is 14.5, what would be the t l at this node, it is 14.5 minus 10.7, 3.8. Then the last T l would be what, it is 3.8 minus 6. So, it would be t l is equal to minus 2.2 so, it is T l is minus 2.2, T l at this point is minus 2.2. So, what is the critical path where you have got you just calculate slake at all these nodes. So, t l minus T e a negative slake is here a negative slake here a negative slake here negative slake here and negative slake it is like here. So, this would be your critical path.

Now, the next question is variance along critical path how to find out variance along critical path. So, first of all we have found out critical path is this right 1, 2, 3, 4, 5. How to calculate variance? We know that standard deviation is this right its t e minus t o divided by 6. So, far activity one two we have got all those three time estimates. So, it is 3, 6, 9. So, for activity 1, 2, it is 3, 6, 9, so 3, 6, 9. So, 9 minus 3 divided by 6, and if you want variance you just you will get one, so variance is 1 for activity 1, 2. Similarly, for activity 2, 3, for 2, 3 what are the different time estimates it is 7, 11 and 13. So, 13 minus 7 divided by 6, so 13 minus 7 is 6 by 6 is 1, this is also 1. And for activity 3, 4, you have got 4, 8 and 15, so 15 minus 4, 15 minus 4 divided by 6 whole square. So, this is 11 by 6. So, whatever is that value, take a square of that value then finally, you have got 4 to 5 4 to 5 it is 3, 5, 7. So, 7 minus 3 divided by 6, this is 2 by 3 whole square. So, you will get value here just write down all these values you will get variance along critical path.

Now, after finding variance along critical path, what you want you want to calculate what is the probability of completing this project in 28 days. Now, this is something which you need to look at carefully. We have found T e is equal to 30.2. It means the sleep project will be completed in 30.2 days and it is probability is 50 percent; probability of completing project in less than 30.2 is less than 50 percent. So, if you want to calculate the probability of completing this project is let us say 20 days, so that probability would always be less than 50 percent, because for 50 percent this is the value, any T s - the scheduled completion time less than this will have less than 50 percent probability.

If I say what is the probability of completing this project in 40 days and that would be more than 50 percent, because if I say what is the probability of completing this project in 40 days. So, T s becomes 40. And we need to calculate this probability which would be more than 50 percent. In our question, we have been given this T s as 28. So, T s is 28 here. So, what is the probability of completing this project in 28 days, it would be less than 50 percent. But what is that value? So, for that you need to calculate z value. And z is equal to x minus mu divided by standard deviation, here mu is nothing but T e and x is T s or scheduled completion time divided by sigma. So, x minus mu x is 28 minus 30.2 divided by whatever is sigma for this. So, this you need to calculate first of calculate variance and then take under root of it, you will get sigma value.

So, let us take let me exactly give you what this variance is. So, various along critical path is 5.8. So, under root of 5.8 would be would be the standard deviation which is 2.41, so this 2.2 minus 2.2 divided by 2.41. Now, look at what is this value it would be it is less than 0.1. So, I think this minus 0.91, I think. Now this is your z value. Now look at area under curve at this point right at 0.91.

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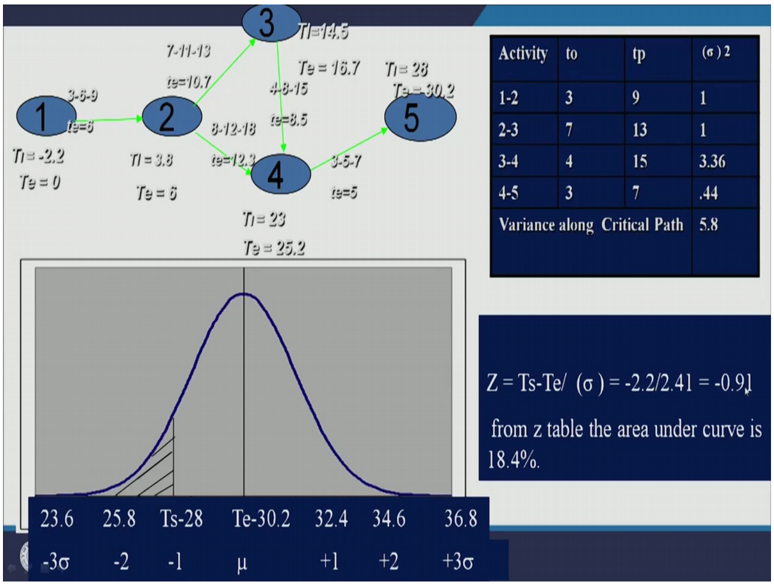

So, we will see we will see the table. So, this is z value minus 0.91.

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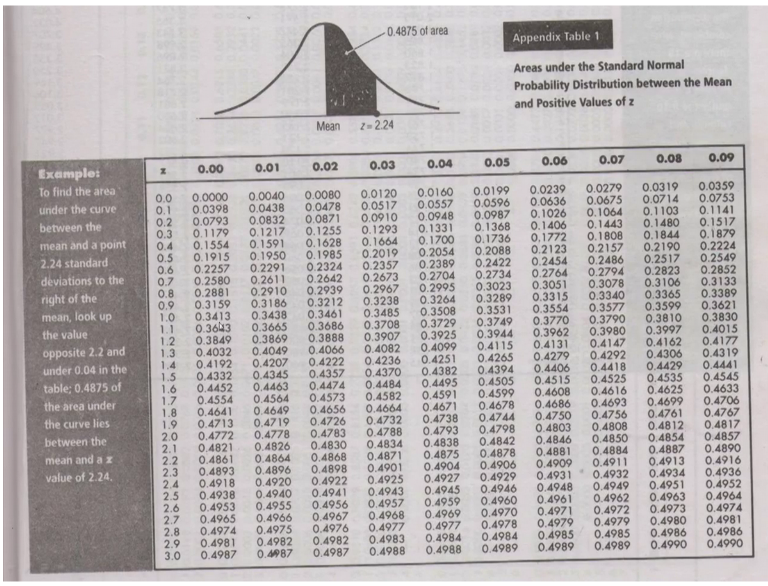

Appendix Table 1

Areas under the Standard Normal Probability Distribution between the Mean and Positive Values of z

Example: To find the area under the curve between the mean and a point 2.24 standard deviations to the right of the mean, look up the value opposite 2.2 and under 0.04 in the table; 0.4875 of the area under the curve lies between the mean and a z value of 2.24.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

And z table is this. So, 0.91 this one; it is 0.3186. So, area under curve at 0.91 is what 34. it is 0.91. So, 0.3186, so 50, it is 50 minus 31.86. So, this is right. So, this is the answer for this question, 18.4. Just see this. So, this is how you have calculate answer to a question like this.

Now, if I ask you to calculate what is the probability that this project would be completed in 22 days, this is what we have just calculated. Just a minute, so this is the question we have solved what is the probability that the project would be completed in 28 days. So, this is 18.4.

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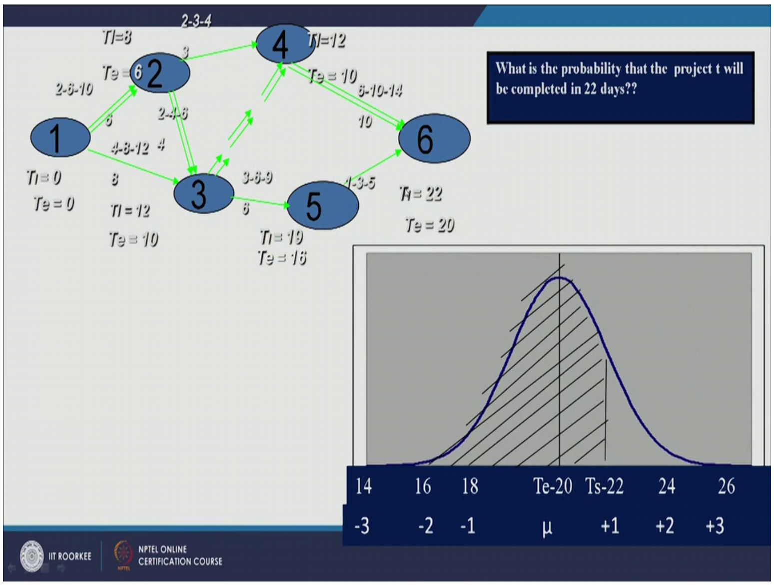

Now, let us look at this network here T e is 20. Now, what is the probability that this project will be completed in 22 days? It is very simple. So, what you need to do now, since this is more than T e value.

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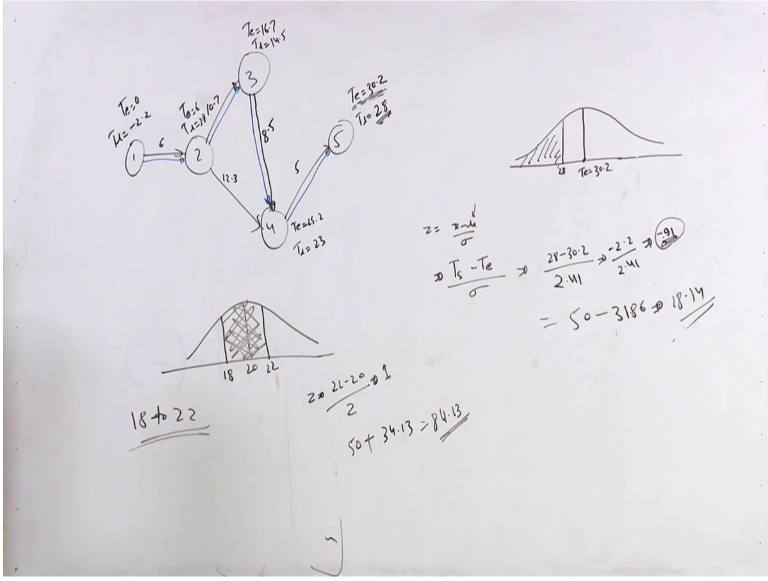

Look at this. This is your T e and T e is 20. What is the probability that the project would be completed in 22 day? So, it would be more than 50 percent. So, calculate z, which is 22 minus 20 divided by standard deviation you need to calculate standard deviation for this standard deviation is 2. So, standard deviation is 2. So, this value is 1. So, what is the area under curve when z is 1?

Look at table. So, area under curve when z is 1 is 34.13 or it is 0.3414, you just add 50 percent in this area you will get answer its 84.13. So, in this way you can solve a question related to probability. Now, you can also solve the question what is the probability that this project will be completed within 22 to 18 days. So, 18 to 22 days, in that case you need to calculate this area right. So, you would be calculating z 2 times at z is equal to 1 and 20 minus 18 divided by 2, so minus 1. So, you just add those two areas now right. So, in this way you can solve question given on probability.

So, let me summarize what we have done in today's session. We have taken a PERT network first and then we converted into CPM network. After that we found critical path using T e and T l s. And then after calculating the value of Z, we found area under the curve. And we in the first question we subtracted area under the curve from 50 percent; and in second question we added area under the curve and 50 percent. So, this how you can solve questions related to probability.

Thank you very much.