Project Management for Managers
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Lecture - 40
Probability Model in Networks- II

Good morning friends. I welcome you all in this session. In this session we will discuss a numerical which is based on probability models. So, let us look at a network which has got total 11 activities.

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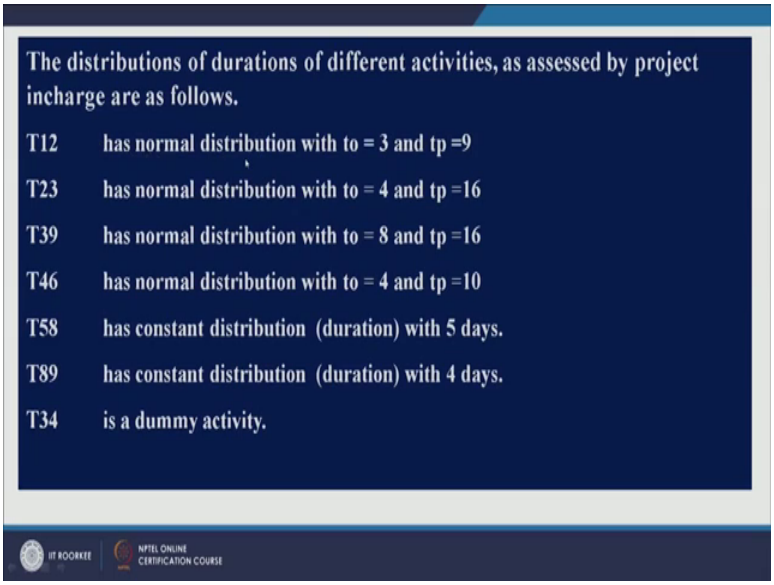

And these are different time durations of those activities: activity 1 2 as normal distribution with optimistic time as 3 days, pessimistic time 9 days: activity 2 3 again a normally distributed activity with optimistic and pessimistic times given here: activity 3 9 normally distributed with optimistic and pessimistic times.

Similarly you have got activity 5 8 has a constant duration. So, when you have got constant duration of an active it will not have any standard deviation right. So, the constant duration of this activity number 5 8 is 5 days. Similarly activity 8 9 again constant duration it is duration is 4 days activity 3 4 is a dummy activity in this project.

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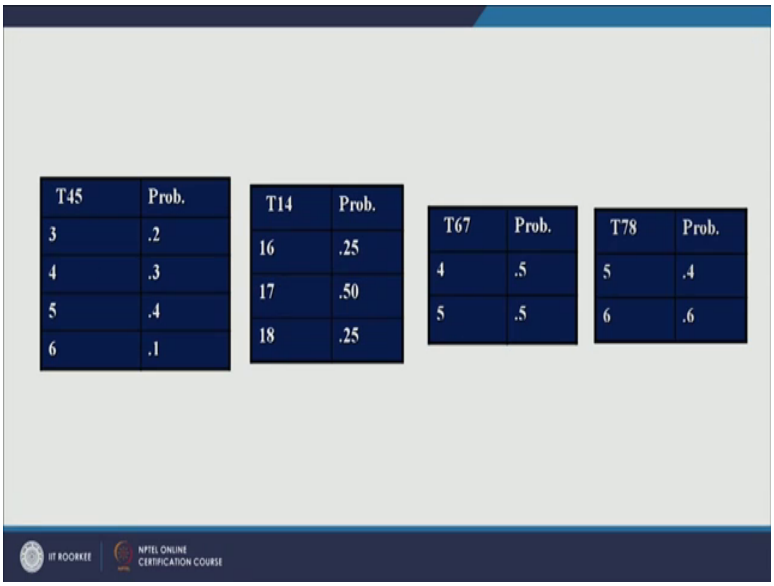

Let us look at the time durations of other activities in this network. So, there is an activity T 4 5 right. So, this is activity T 4 5 and the probability that it would take 3 days is 0.2, it would take 4 days 0.3, it would take 5 days 0.4 and it would take 6 days 0.1. So, the sum of these probabilities is 1, right. Similarly for a activity 1 4 6 7 and 7 8 right.

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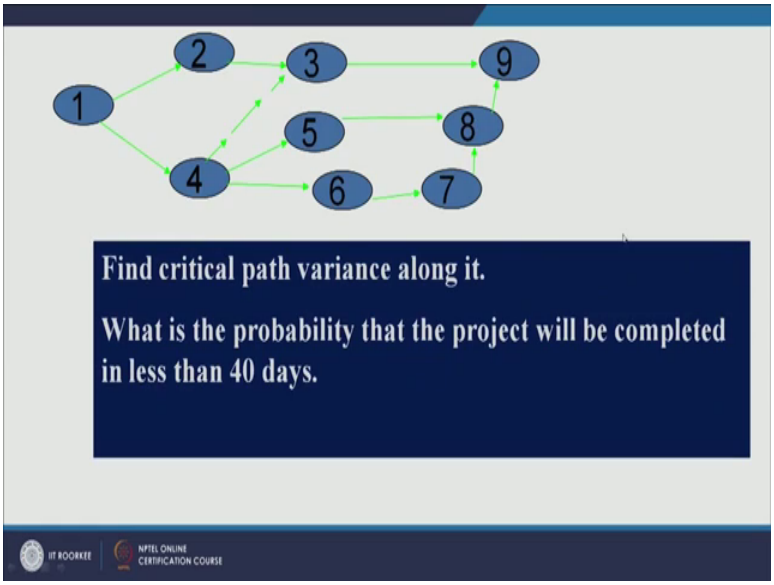

Now, this is the network for which we have just discussed different time durations right. So, what we have to do in this question find critical path and variance along critical path. This the first part of the question. The second part is what is the probability that the

project will be completed in less than 40 days? So, we will start solving this particular network and we will find out what is the probability that the project would be completed in less than 40 days right. So, let us just started and we will find out first critical path right.

So, first of all you need to calculate the expected duration of all the activities right. So, if you if you look at the first activity which is activity 1 2, activity 1 2 right. So, optimistic time 3 days pessimistic time 9 days. So, how to find out it is mean time are expected time. So, as far as activity 1 2 is concerned you have got activity 1 2 and we know that it is optimistic time is it is 3 days pessimistic time 9 days right.

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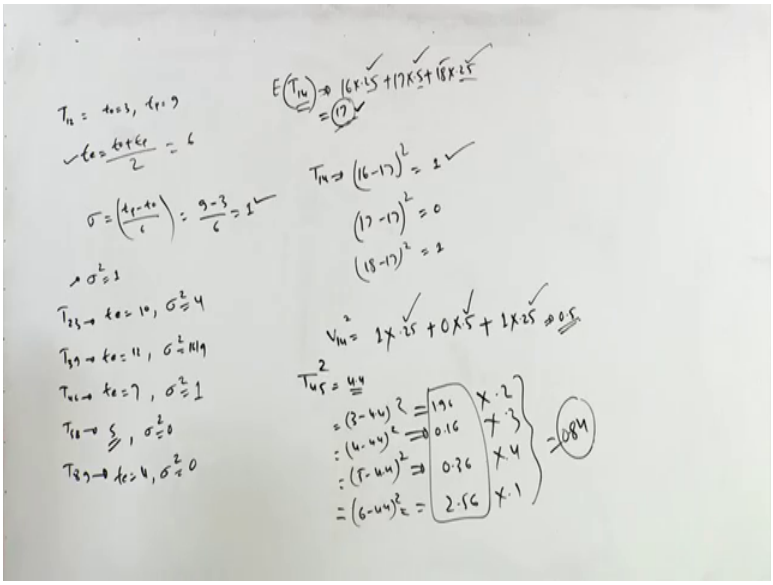

So, it is expected time T e would be just take the average of these two. So, the formula is T o plus T p divided by 2. So, this is 6 isn't it? Now you need to calculate variance of this activity also. So, for variance first of all calculate standard deviation and the formula for this is T p minus T o divided by 6. So, if you look at T p T p is 9 minus 3 divided by 6 this is one standard deviation is 1. So, standard variance is also 1. So, in this way you need to calculate expected time as well as variance of all the activities even in the network right.

So, similarly since you know how to calculate T e and standard deviation of 1 2 you can do it for activity, let us say 2 3. So, for activity 3 let us means is let us say T e for this is 10 and variance is 4. I am not going to calculate how did I get this you can easily find out

right. For activity 3 9, T 3 9, T e expected time is 12 and variance is 16 by 9, right; or 4 by 3. For activity 6 T 4 6 it is expected time is 7 and variance is 1 right. Now if you look at the activity 5 8, for activity 5 8 it is duration is it is fixed and the duration is 5 days. So, for activity 5 8 it is duration is fixed which is 5 days. So, it is T e is also 5 and it is variance would be 0, because standard deviation is also 0. Similarly for activity 8 9 T 8 9 it is expected time T e is 4 and variance is 0.

Now, since we have calculated expected time and variance of several activities, but we did not calculate so for expected time and variance of those activities for which you have been given probabilities along with their duration's right. So, let us take first activity which is activity number T 1 4, right; activity 1 4, right.

If you look at activity 1 4 then it is durations are given for activity 1 4, look at this it will take 16 days it is probability is 0.25 it will take 17 days probability 0.7 it will take this probability as 0.25 right. So, we will calculate the T e value for activity 1 4, right. Or expected time of activity 1 4. So, what you can do here is, you just multiply the durations and probabilities right. So, expected time for activity T 1 4 is equal to it is 16 into 0.25 plus 17 into 0.25 plus 18 into 0.25 right. So, 18 is the duration 0.25 is the probability, keep in mind that this sum of these probabilities should be 1 right.

So, this value is 17. So, expected time for activity 1 4 is 17 days. Now let us calculate it is variance, right. And for variance what you need is this. Since you know this time expected time and you have also got the probabilities; so first of all what you should do let us calculate standard deviation, for activity 1 4. So, for this you have got duration is 16 days expected time 17 days right.

So, 16 minus 17 whole square, then you have got 17 days minus 17 is it is expected times. So, again square of this then the third duration for this activity is 18 days right, so 18 minus 17 whole square. So, what we have done we just subtracted expected time from their durations and will take square of that value right. So, if you calculate these values then this is 1, right. This is 0 and this is also 1 right.

Now, after calculating these values you just multiply these values with the probabilities, which are given in the question itself right. So, what you should do. So, let us look at variance of activity 1 4 is equal to 1 into this 1 into it is probability 0.25, then 0 into probability 0.5 then 1 into again 0.25. So, this probability 0.25 which is here, this

probability 0.7 which is here, it is probability 0.25 which is here, right. If you solve this equation then you will get 0.7. So, this is the variance of activity 1 4, right. This was your expected time. So, this how you should calculate expected time and variance of those activities for which you have been given durations along with probabilities right.

Similar to this calculation you can do you can calculate expected time and variance for other activities for example, if you look at activity 4 5 for activity 4 5 activity 4 5, right. This is you need to first calculate the expected time and the durations are like this, so 3 4 5 8. So, just multiply these 2 values. So, this is 0.6 this is 1.2, this is point this is 2, right. Not 0.2 this is 2 and this is 0.8. So, if you add all these values it would be 4.4, right. This is 4.4, right. Now what to do next since you have got mean value 4.4. So, 3 minus 4.4 whole square plus 4 minus 4 whole square then 5 minus 4.4 whole square and 6 minus 4.4 whole square right. So, you will get 4 different values right. So, let me write those values. So, this is 3 minus 4.4 whole square then you have got 4 minus 4.4 then you have got 5 minus 4.4 whole square then you have got 6 minus 4.4 square right.

So, this is 1.96, this is 0.16, this is 0.3 6 and this is 2.76, right. Now since you have got these values you just multiply these values with the probabilities right. So, you can just write it like this 0.2 into 0.3 into 0.4 into 0.1. So, if you take sum multiplication of all these values and summation of that you will get that value as 0.84. So, this is the variance of activity this is the variance of activity 4 5, right. In this manner you can calculate the expected value and variance of all other activities, right.

So, let me directly write the values for other activities; so similarly for activities 6 7 right. So, for activity 6 7, for activity 6 7 mean is 4.7, 4.7 and it is variance is 0.25 right.

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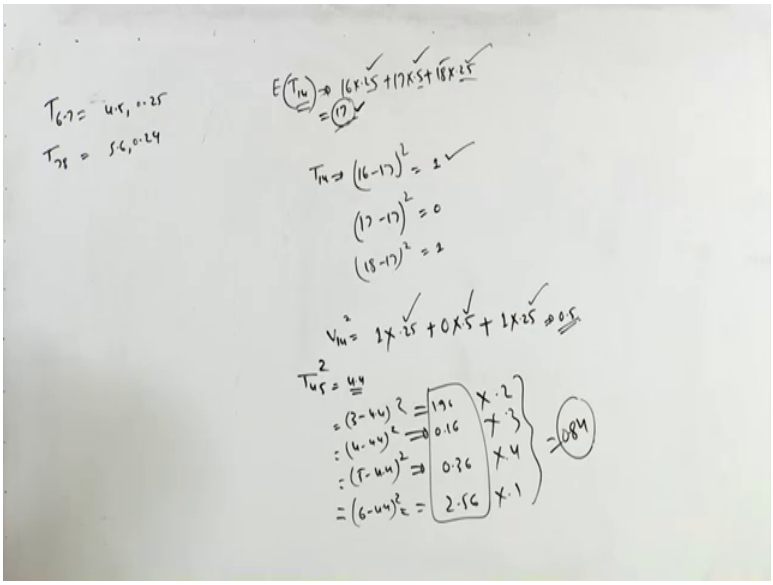

For activity 7 8 it is mean is 5.6 variance is 0.24, right. Now we have calculated expected value as well as variance of all the 11 activities, right. Now we will calculate the critical path right. So, for that you need to draw a network first right. So, if you look at network would look like this. In fact, it is already there in slide.

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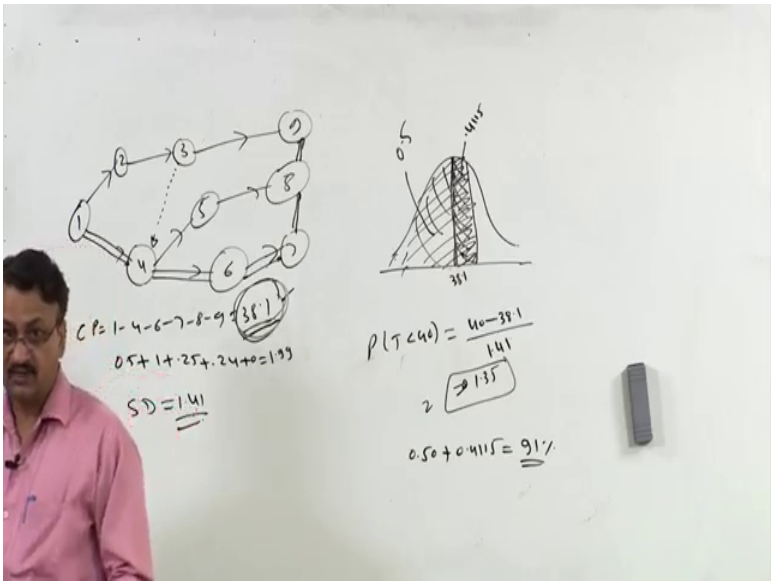

So, let us look at this for your first event, this is your second event, this is your fourth event, this is third, this is fifth, this is sixth, this is your ninth event, you have got 5 to 8 , 6 to 7, 7 to 8 and 8 to 9. So, this event is the last event, right. And these are different

activities, right. If you have got a dummy activity also it is 4 to, In fact this is like, In fact it has to be like this, right 3 to 4, right?

If you look at their now you can write their expected values as well as their variances, right on all these activities. So, if you look at the duration of the project it would be looked it will look like this right. So, if you look at the critical path. This is your critical path it is 1 4 6 7 8 and 9. So, this is your critical path and the total duration of this critical path is 38.1. So, critical path is 1 4 6 7 8 and 9 and it is value is 38.1, right.

And if you look at the variance along critical path, variance along this path, so variance we know that a variance of activity 1 4 is 0.5. So, you just add all those variances. So, if you look at the variances then it is 0.5 plus 1 plus 0.25 plus 0.24 plus 0 this becomes 1.99. So, when this is the variance standard deviation is 1.41.

So, the first part of this question is what the first part was what is the critical path and what is the duration of this is projects. So, this is critical path 1 4 6 7 8 9 this is your total value of the critical path this is your standard deviation, right. First part we have done. If you look at the second part of this question, the second part is what is the probability that the project would take less than 40 days?

Now we know that the probability that the project would be completed in this much time is 50 percent, right. This is what we have already seen in previous couple of sessions, right. Now we have to find out what is the probability that it would be completed in less than 40 days. So, this is your mean value, right this 38.1, right. And what is the question what is the probability that it would take less than 40 days right. So, we have to calculate this area, right.

Now, we already know that the 50 percent of the area is 0.5. So, from here to this point is 50 percent, right. Now we need to calculate only this much area now for that you need to calculate z value and what is the z value for this question. So, the probability that the project would take less than less than 40 days is equal to this right. So, this is 40 minus this x minus mu, right. It is 38.1 divided by standard deviation which is 1.41. So, you will get z is equal to 1.35.

Now look at the area when z value is 1.35. So, I have already talked about how to look at the area under curve. So, if you look at z value, when z value is 1.35 then that area is

point this area is point it is 0.4115 this area right. So, we already know that from here to here this area is 50 percent and this area is 41.15. So, total is it is 0.5 plus 0.4115. So, this is approximately 91 percent.

So, what is our finding? Our finding is that the probability that this project would be completed within 40 days is 91 percent. So, this how you can solve a question in which you have got activities having different time estimate. Some of them are no normally distributed, some of them I got fixed duration I got some of them are dummy activities. So, this is an example wherein you have been given 5 nodes and 5 activities. So, activity 1 2 is normally distributed and stands for normal distribution right.

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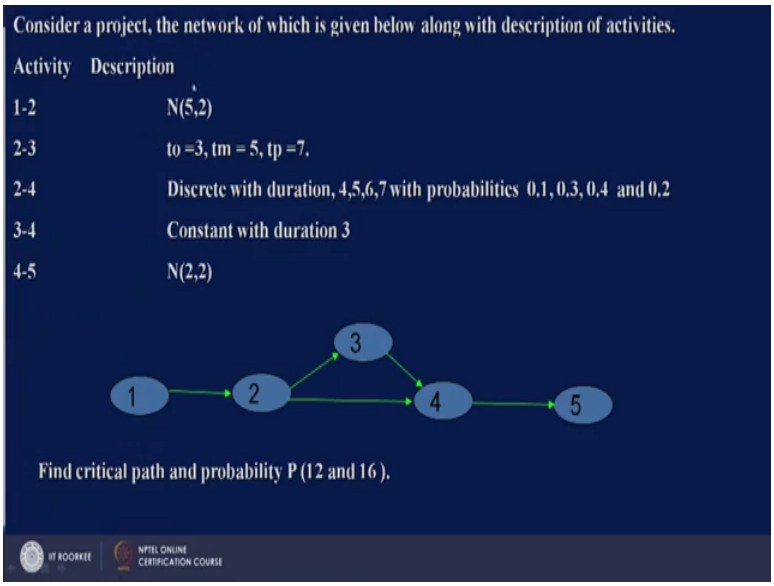

So, this is an activity which is normally distributed and this is mean and this is standard deviation. So, you need not calculate mean and standard deviation separately.

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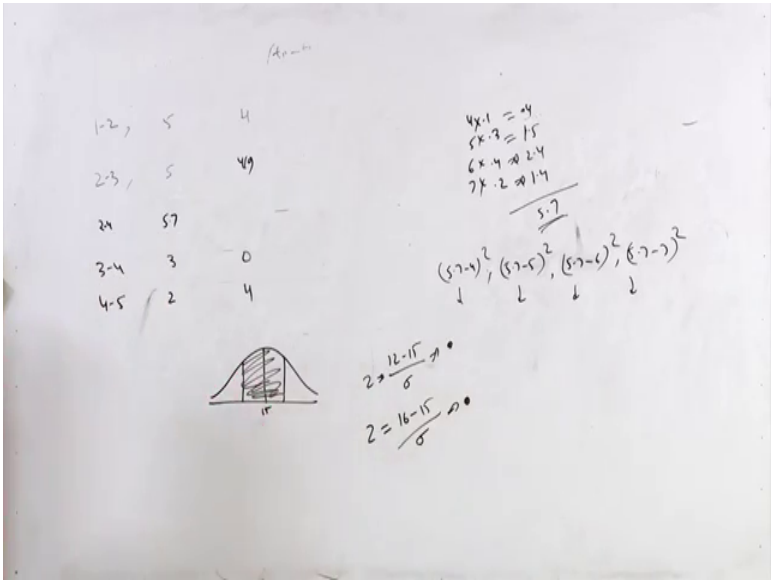

So, for activity 1 2 for activity 1 2 means is 5 and of course, standard deviation is 2, right. And if you want very variance is 4, right. 2 3 it is it is it is an activity having 3 time estimates right. So, mean would be what you need to apply that formula right. So, what is that formula it is 3 plus 4 into 5 plus pessimistic time 7 divided by 6 right. So, this is 7 plus 3, 10, 20, 30. 30 by 6 means 5. And what would be the standard deviation it is 7 minus 3 by 6 whole square right. So, 7 minus 3 it is 4, by 6 it is 2 by 3 16 by 9, right. Sorry it is 4 by 9.

Calculate once again for activity 2 3 it is 7 minus 3 this is 4 by 6 it means 2 by 3 4 by 9, right. For activity 2 4 for activity 2 4 which is a discrete activity having different you know time durations along with probabilities. So, the probability that this activity will take 4 days it is probability is 0.1 it will take 5 days probabilities 0.2. So, you need to find out mean, right; the way in which we have just calculated in previous example. So, let me calculate mean first right.

So, it is 4 into, it is 4 into 4 5 6 7 multiply with their probabilities. So, it is 0.1, 0.3, 0.4 and 0.2. Now it is not like this it is 0.3 point 0.1, 0.3 point yeah it is 10, did the total should be 1 right. So, it is 6 plus 0.4 right. So, this is what 0.4, 1.5, 2.4 1.4. So, this total is what 9 5.7 right. So, this is 5.7 is the mean variance you need to again do all those calculations right.

So, it is 5.7 minus 4 whole square yeah let me do it here, right. It is 5.7 minus 4 whole square plus 5.7 minus 5 whole square plus 5.7 minus 6 whole square plus 5.7 minus 7 whole square not plus. So, just put comma over here. So, you calculate all these values and multiply these values with these probabilities, right. And then you will get variance over here, right. This is what we did in previous example.

If you take activity 3 4 it is constant. It has got constant duration 3 4, it is duration is 3 variance is 0, right. Because duration is fixed activity 4 5, it is normally distributed it is normally distributed with mean and standard deviation. So, means is this standard deviation is 2 variances is 4 isn't it? Find critical path and probability of comprising completing this project between 12 and 16 days. So, how to solve the question like this? So, let us solve this question. So, far e 1 to it is 5 and 4, right. Variance is this 2 3 5 and 4 by 9 for 2 4, 2 4 it is it is 5.7.

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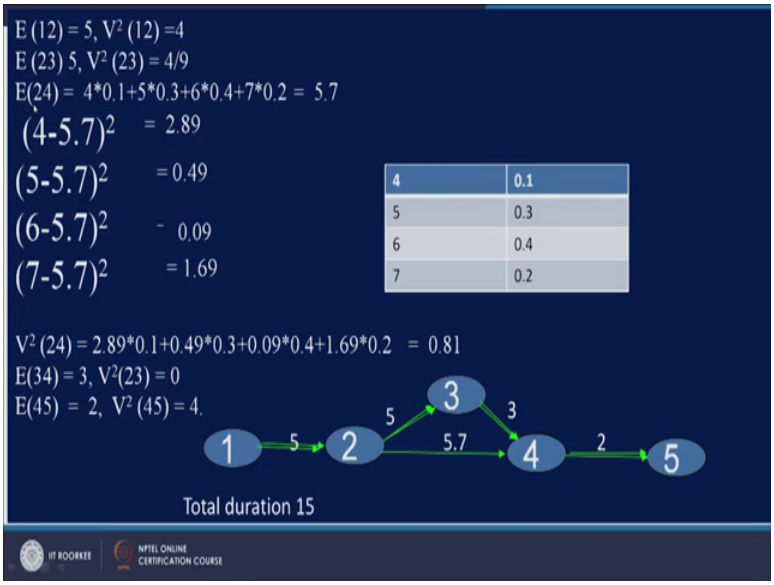

It is not 5.9. In fact, I must have done some mistake over there, right. It may be correct also, but there is not much difference right. So, it is point 5.7 or 5.9 does not make any difference right.

Now, the variance is 0.81. So, this is what I have done there it is it is it is point 2 point 4.54, minus now this is actually it is mean is what first of all you need to calculate mean. So, mean is 5.7. So, 5.7 minus 4 whole square either you write 5.7 here or here, does not make any difference right. So, just add all these values after multiplying with these

probabilities right. So, this value comes out to be 0.81, right. And for activity 3 4 it is duration is fixed variances is 0 for 4 5 also it is normally distributed. So, mean is 2 variance is 4, because standard deviation is 2 right. So, in this way you have got the duration of this particular project. So, critical path is 1 2 3 4 and 5 and the total duration is 15 days, how come 15: 5 plus 5 10, 13 plus 2, 15 days right.

Now, probability that the project will be completed in 50 day 15 days is 50 percent. Now we have to calculate: what is the probability that this project would be completed within 12 to between 12 to 16 days. So, that part you need to calculate what you need to do is let me do it. In fact, since you have got. In fact, you need to know the standard deviation along critical path.

So, since this is 15, since this is 15 you need to find out this area and what you need to do 12 minus 15 divided by whatever is standard deviation along critical path then z is equal to 16 minus 15 divided by again standard deviation. So, whatever is the value here look at area under curve whatever is the value here look at area under curve and add those 2 areas right. So, you will get probability of completion of this project between 12 and 16 days.

So, with this let me complete here thank you very much for attending this session.

Thank you.