

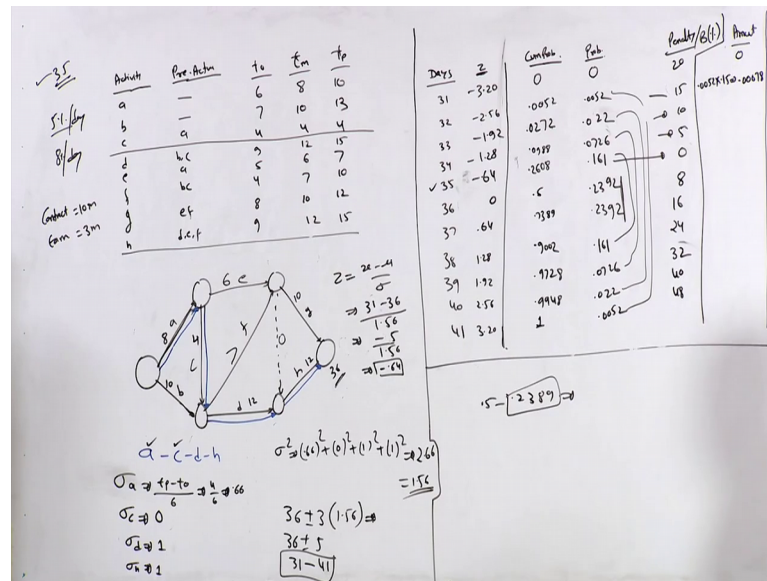
Project Management for Managers
Dr. M.K. Barua
Department of Management
Indian Institute of Technology, Roorkee

Lecture – 41
Probability Models in Network – III

Good morning friends, I welcome you all in this session. As you are aware in previous session we were discussing about probability models in networks. And we have seen how to find probability of a project with a given time. We have also seen that whenever we calculate t_e at the last node at that point the probability of completion of project is 50 percent. So, let us say for example, if t is 100. So, we will say that the probability of completion of project is 100 days. If let us say t_s which is scheduled completion time or if your client has asked to complete the project in 9ty days. Then the probability of completion of project will always be less than 50 percent keep this in mind.

If client has said that you can take 110 days then probability of completion would be more than 50 percent. Now with these information in mind we will take up 1 or 2 more examples on probability, and then we will move on to some other topic. So, the problem which we are going to take up today is this is quite an interesting one. And it is interesting because it has got you know if you complete a project before time then client is ready to give you bonus if you do not complete then there would be penalty right. So, these 2 new points would be discussed in today's session. So, we will take up an example and a example is very simple and in this example there is a project given to you and actually you are doing a project for a client.

(Refer Slide Time: 02:26)



And you have said that you would complete the project in 35 days. So, this is what you promised to the client that you would complete it in 35 days.

And the project is like this it has got several activities. So, activity a you have got a b c d e f g right. So, these are different activities. So, let us so, this column is activity column right. Now there is precedence relationship also given. So, precedence or preceding activity here is for a, a is there is no predecessors activity, for b also there is no predecessors activity right. It means a and b are starting simultaneously right. For c you have got a for d you have got b and c for e you have got a for f you have got b and c right. And for g you have got e and f and for h you have got d e and f right. So, since you have been given this precedence relationship you can draw a network. Now after drawing a network you need to have time estimates. So, as far as this particular project is concerned it is a pert projector pert network right. And you have been given 3 time estimate. So, for as far as a is concerned you have got let us say optimistic time most likely time and pessimistic time right. So, t_o t_m and t_p . So, optimistic time is 6 8 10. 6 8 and 10, 10 is the pessimistic time then 7 10 13 7 10 and 13. For activity b right. For c it is 4 4 4 right. This is for c. So, this is up to c we have completed right.

Then for d it is 9 12 15. 9 12 and 15 for e it is 5 6 7. 5 6 and 7. For f it is 4 7 10. 4 7 and 10. For g it is 8 10 and 12 and for h it is 9 12 and 15. So, these are different time estimates right. So, first of all what you should draw network of this problem. And the

question is as I said you promised to the client that you would be completing this project in 35 days right. If you complete this project a before 35 days, then you will be getting 5 percent bonus per day. If you do not complete then there would be a penalty of 8 percent per day 8 percent per day. The company will earn 3 million in total contract price of 10 million. So, contract price contract price is 10 million and company will earned 3 million. So, determine the expected earning of this particular project.

So, that is a question. Now first of all what you should do? You should draw a network for this particular problem. And the network has to be drawn carefully because you need to find out critical path and so on right. So, let us draw a network. So, this is first second third node forth fifth and 6th right. So, activity a activity b activity c activity d activity e activity g activity h activity f. And this is a dummy activity. Now if you look at this particular network, you need to write expected value of this activity a. And what we have been given we have been given 3 time estimates for a 6 8 and 10. So, how to find out expected duration of this activity? You know t_e is equal to t_o plus 4 times most likely plus t_p divided by 6 right. Isn't it? This is our formula right. So, for a what would be the t_e value? So, if you look at this is $6 + 10 + 16 + 8$ into $4 + 32 + 48$ right. $16 + 32$ is 48 divided by 6 you will get 8. So, 8 is the duration of activity a. What about activity b? $7 + 10 + 13$. So, $7 + 13 + 20 + 40 + 60$ by 6 which is 10 right.

Now, if you look at activity c, c is this activity right. This is 4 4 4 right. So, $4 + 4 + 8 + 8 + 16 + 16 + 8$ is 24 by 6 is 4. So, this is the duration of activity c. Now if you look at activity d. So, $9 + 15 + 24 + 48 + 24 + 48 + 72$ right. 72 by 6 you will get 12 right. So, for d it is 12 right. For e you have got 5 6 and 7. $5 + 7 + 12 + 24$ is 36 by 6 which is 6. So, for e it is 6 right. Now for f for f it is $4 + 10 + 14 + 28$. It is 40 2 by 6 it is 7. For activity g you have got $20 + 40 + 60$ by 6 that is 10 for g it is 10 for dummy activity 0 right. For h $24 + 48$. So, in this way what we have done? We have calculated expected time for all these activities.

Now, if you look at this these time estimates intelligently we are getting all $t_s t_m$. So, we have chosen these time estimates such that the t_e is equal to most likely time for all this activities right. Now what to, what is, what is the critical path? For critical path let us start calculating t_e right. So, t_e at this point is 8 sorry 0. At this point it is 8. This point it is 8. $8 + 4 + 12$ right. Then $8 + 6$ is 14. $12 + 7$ is 19 right. T at this point would be 24 isn't it? $24 + 12 + 36$ right. So, if you if you want you can get your critical path by

even seeing at this forward path itself right. So, we are getting maximum value from here to here right. Let me use different color pen. So, this is 8 this is 12 this is this and this is this. So, this is your critical path right. Otherwise what you can do just take t_l is equal to 36 and calculate all other TLS and critical path would be that path where t_l minus t is equal to 0 right. But In fact, since this is a small network you can easily find out critical path. In fact, there is no need of doing all this t and all those things right. Just by looking at this network itself you can you can see which is the critical path isn't it? So, let me erase all this things right.

So critical path is known right. Now since critical path is known let us find out what is variance along critical path. So, our critical path is a c d and h is our critical path right. Now what is the standard deviation along a? Standard deviation of activity a. So, let us calculate standard deviation of activity a right. Standard deviation of activity a is what? Let us t_p minus t_o divided by 6 right. So, this is 4 by 6 isn't it? It is 10 minus 6 is 10 minus 6 is equal to 4 which is 0.66 isn't it? Now let us look at standard deviation of b. So, 13 minus 7 which is 6, 6 by 6 is 1 isn't it? Standard deviation of c is a c now not no this is actually there is there is need to find standard deviation of b you need not require right.

So In fact, after a you can directly go on c isn't it? So, standard deviation of c is what? It is 4 minus 4 it is $\times 0$ isn't it? Then standard deviation of we have calculated for this for this let us calculate for d, d is this right. D is this right. So, 15 minus 9 is 6 6 by 6 is 1 and standard deviation of h is h is this right. So, 15 minus 9 again 6 1. Now let us calculate variance along critical path. So, variance along critical path is 0.66 square plus 0 square is 0 1 is just for simplicity let me write right. So, this is $1.1^2 + 2.66^2$ under root of 2.66 is you need to calculate it right. I think it is 1.41 let me give answer to this value. So, it is 1.56 it is 1.56 right.

So, this is variance and this standard deviation are known critical path right. Now if you look at this question carefully, we are saying that we will complete this project in 35 days right. Isn't it? So, 36 days this project is taking, but we have said that we would complete in 35 days. And the probability of completion of this project in 36 days is 50 percent. If we complete it in 35 days then we will get a bonus of 5 percent of this the amount which you are earning right. Similarly 8 percent is the penalty. So, let us look at what are

different possibilities here. So, we will look at this project and we will consider only those durations which are in plus minus 3 sigma limit.

So, I know that means is this right. So, 36 is the mean. So, we will complete this project in 36 days plus minus 3 sigma is this. So, is 36 plus 3 into this is approximately 4.5 are let it be 5 right. So, this is 5 right. This into this is 5. So, 36 plus minus 5 approximately you can take 5 right. So, this comes out to be 31 to 41. So, we will find out what is the total penalty and bonus for this range of days right. So, what is the probability that project would be completed in 31 days? We will calculate bonus at 31 day 32nd day third fourth and so on and after 35 you will have penalty right. So, what is the penalty on 36th day 37 38th 39th 40th 41st day right. So, you need to calculate the penalty plus bonus for this particular range of day's right. So, let us calculate that penalty and for this you need you need to look at z table. So, I will prepare a table for this and in this table first of all you have got days right.

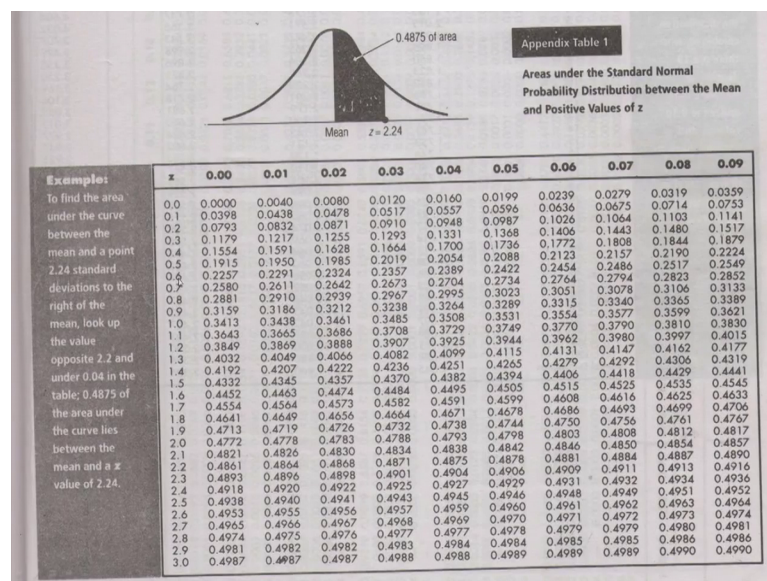
So, you are a starting from 31st 32nd 33rd 34 35 36 37 38 39 40 and 41 right. Now once you have this now you also know that at 36th day this is actually 0, z is 0. Z is 0 it means in other words the probability of completing this project is 50 percent right. So, 0.5 this is 0.5. So, let me write something called cumulative probability here. And then we will we will write down probability and then penalty, penalty oblique bonus and finally, you have got what is that amount right. So, first of all calculate all these z values all z values. So, this is how to calculate z value here how did we calculate the 0 this is z is equal to z is equal to x minus mu divided by standard deviation right. So, here x is 36 mu is 36. So, z is 0 right. So, for 35th day for 35th day what we will have? It is 35 minus 36 divided by 1.56. So, this is minus 1 divided by 1.56 and this comes out to be minus 0.64. Minus 0.64, minus 0.64 right. And for 37th day it would be plus 6, 0.64 right because instead of 35 you would be writing 37.

So, for 34th day 34th day what would be the value of z? Just write this 34, this is minus 2 and you will get value minus 1.28. Similarly for 33rd day just change this to 33 this is 3 and the answer would be minus 1.92. For 32nd day change this value this would be minus 4 and the answer would be minus 2.56. And for 31st day this is 1 this is 5 and the answer would be minus 3.20 right. So, whatever we have calculated for z values for these days we will calculate z values for these days also. Only there would be one change of sign right. So, this would be 1.28 1.92 2.56 and 3.20. So, what we will say that, the

probability of completing this project the cumulative probability of completing this project at this is 0 right. Because it is it is more than it is beyond 3 sigma limit. And In fact, you can take this value as 1 right.

So, probability of completing project in 41 days is 100 percent right. Here it is 0 it is 1 and this is 50 percent right. Now what is the probability or cumulative probability for completing this project on 35th day? So, how would you do that? For this you need to look at z table very carefully. And as I said probability of completing project is 50 percent here. And that probability will come down you know we will go down in this direction and it would be 0, it means whatever is the area under curve right. 0.64 value of z in table subtract that value from 0.5. And then just write those values here. So, let me do it for you. So, for 0.64 look at z table right. If you look at z table it is there. In fact, I will show you where is z table. So, z table is here.

(Refer Slide Time: 25:55)



It is 0.64 right. So, 0.6 and 4 is here right, 0.2389.

So, value of z area under curve at 0.64 is what? We have just checked right, it is 0.2389. So, 0.5 let me do this calculation here. So, 0.5 minus 0.2389 is equal to 0.260 right. So, what we have done here? We calculated area under curve at 0.64 we subtracted it from 0.5 right. So, probability of completing project in 35 days is 26 percent. Similarly for all other values right. 1.28 what is the area under curve? 1.28 just look at this table 1.28 and 8 that is here it is 0.3997. So, 0.5 minus 0.3997 it is point 0.0988 right. Then for 1.92 just

check area under curve at 1.92 subtract it from 0.5 you will get 0.0272. Similarly for 32nd day it is 0.0052 right. Now probability that the project would be completed in 32 days is just it is it is even 0.05 right. Probability of completing project in 34 days is just 9.88 percent right. So, 9.88 2.72 and it is even is 0.5 it is 0.05 right.

Now, what would be the probabilities in this direction? It would be more than 0.5 right. So, 0.64 look at z value in table and (Refer Time: 28:57) in 0.5, so 0.64 just look at 0.64 here, 0.6 and 4 is 2 3 8 9. So, you just state 2 3 8 9, 0.5 plus 2 3 8 9 right. So, it would be 0.7389 right and so on. So, let me write these values 0.7389 then 0.9002 0.9728 0.9948 right. Now once you have done with this what you should do? You should calculate probabilities right, which is very simple. Since you know cumulative probability you can calculate probability value very easily right. So, let say probability 0 cumulative probability is this right. So, this minus this is equal to this right. So, 0.0052 here also right.

Now, this minus this is equal to you will get probability here right. So, in this way you can right. Down all the probabilities. Let me repeat this minus this would be thus probability right. So, let me right. These values is 0.002 it is 0.022 right. Then it is 0.161 then 0.2392 then, this first second third fourth right. So, this is 0.7 just a minute. So, this minus this is 0.002 is 0.0726 then 0.161 then 0.2392 for this is 0.2392 right. This minus this 0.2392 then 0.161 0.0726 0.022 and 0.0052. So, if you see these 2 values are same and again these 2 values are same, these 2 values are same these 2 values are same and these 2 values are same right. Now I know that there is a penalty of 8 percent per day if I delay the project. If I complete before 35 days there would be a bonus right. So, let us write down what is the penalty and what is the bonus right. So, let me write over here it is penalty oblique bonus and this is amount. So, penalty oblique bonus is on 35th day 0 right because you have promised 35 days to the client.

Anything above this there would be a bonus of 5 percent per day. So, this 5 percent on 34th day 10 percent then 15 percent and 20 percent in fact, on 31st day. So, this 34th 33rd 32nd and 31st. So, let me write it here first 15 then 10 percent then 5 percent and this is 0. Similarly what would be the bonus penalty is 8, 8 percent per day right. So, this 8 percent on 36th day this 16 percent 24 percent 30 percent this 32 percent right; on 39th day 40 percent and 48 percent. Now since you know what is the probability of completion of this project and you also know the penalty and bonus in terms of

percentage, just keep in mind all these are in percentage right. So, you need to multiply probability with your penalty or bonus. So, this is 0 this is 0.0052 into 0.15 because this is percentage. So, let me write it here point. So, this is the column. So, what we are doing this is 0.0052 into 0.15 this is equal to 0.0078, 0.00078. This is your so, since lot of calculations are to be done in this question.

Let me stop here as far as this particular session is concerned. We will have remaining part of this question in next session.

Thank you very much.