Project Management for Managers
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Lecture – 42
Probability Model in Networks – IV

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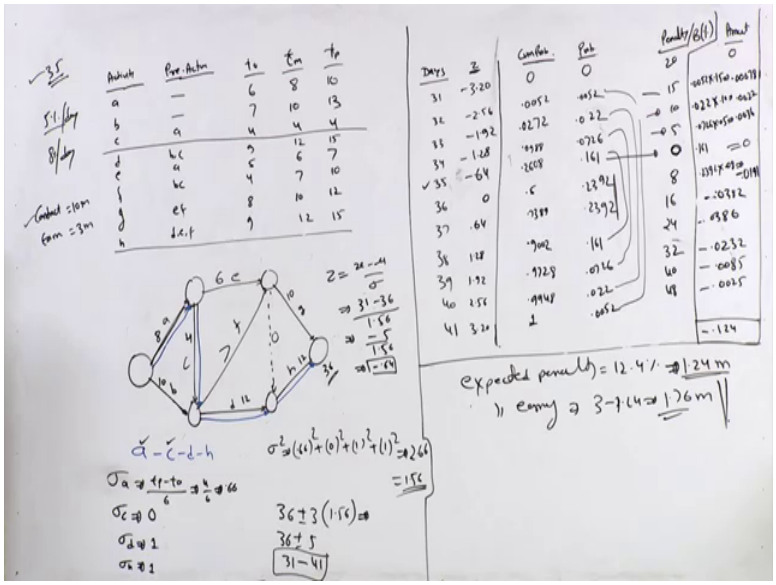

Hello friends, welcome you all in this session. In previous session, we were doing this question. So, we will complete this question first and then we will take up another question. So, let me summarize what we did so far, there is a network in which these are different time estimates the critical path is a, c, d and h and the probability that this project would be completed in 36 days is 50 percent. Now, when you hand over this project to your client, if you submit this project to the client before 35 days, because you promised to deliver this project to client within 35 days. If you submit before 35 days then you will get a bonus of 5 percent per day, but if you do not submit or you if you do not hand over project to the client then there would be a penalty of 8 percent per day. Total contractual amount between yourself and client is 10 million and you are expected to a 3 million.

Now, what we have done after finding this value 36 days, which is the mean value mean completion time of the project. We have taken what is the probability of plus minus 3 sigma deviation from 36 days, so 36 plus minus 3 standard deviation. And how did we

calculate this standard deviation of this project, first we calculated a standard deviations of each, each of these critical activities right a, c, d and h and then we calculated variance of this project which is 2.66 and the standard deviation is 1.56. So, after calculating standard deviation of the project what we did we calculated z values. So, z value at 36th day is 0; at 35th day minus 0.64, minus 3.20 on 31st day, 3.20 on 41st day and then we calculated cumulative probabilities; and from cumulative probabilities we found probabilities.

In fact, you can do the other way around also. In fact, you should do other way around first of all you should find out probabilities by looking at appropriate table and compute cumulative probabilities and then in this column we have got penalty and bonus. So, if you deliver project on 35th day, there is no penalty no bonus. 5 percent bonus, if you deliver it on 34th day; 20 percent bonus, if you deliver on 31st day; 48 percent penalty if you deliver on 41st day. So, what is the amount the penalty amount is probability multiplied by this percentage of penalty or bonus, so 0.0052 into 0.15, why it is 0.15, because this is in terms of percentage. So, what would be this value now 0.022 into 0.10, so this 0.0022 it s very simple, then 0.0726 into 0.05. So, this is in terms of percent. So, this is 0.05, 0.05. So, this would be 0.0036, the next one would be 0.161, of course, this is 0, then 0.2392 into 0.08 right 0.08 this would be point minus 0.0191, why this minus because this penalty. So, penalties are will have all negative amount.

So, for next the next amount is. So, let me not write these values now 0.2392 into 0.16 I will directly write what is the penalty amount. Since, you know how to compute right. So, the next value is 0.038182 then 0.0386 of course, all these are minus right minus then 0.0, then minus 0.0085, minus 0.0025. So, when you after calculating all these values just end this column. So, it would be minus 0.124. So, expected penalty 12.4 percent of what of this amount. So, the penalty amount is this is equal to in other words 1.24 million. When this is the penalty, what is the expected earning 3 minus 1.24. So, this is 1.76 million. So, the answer to this question is what was the question we wanted to find out what is the expected earning for this project. So, expected earning is 1.76 million, so this how you can solve a question like this.

Now we will work out one more question, which is similar to this, but with some other data set. So, in this question, we have been given different probabilities and again different duration. So, let us take one more example on probability models related to

networks. The example is quite an interesting one. And we will see how to solve this example. There is a company a construction company, which is to bid for a highway project and the company has identified that the probability of getting highway project depends on time it quotes in its quotation.

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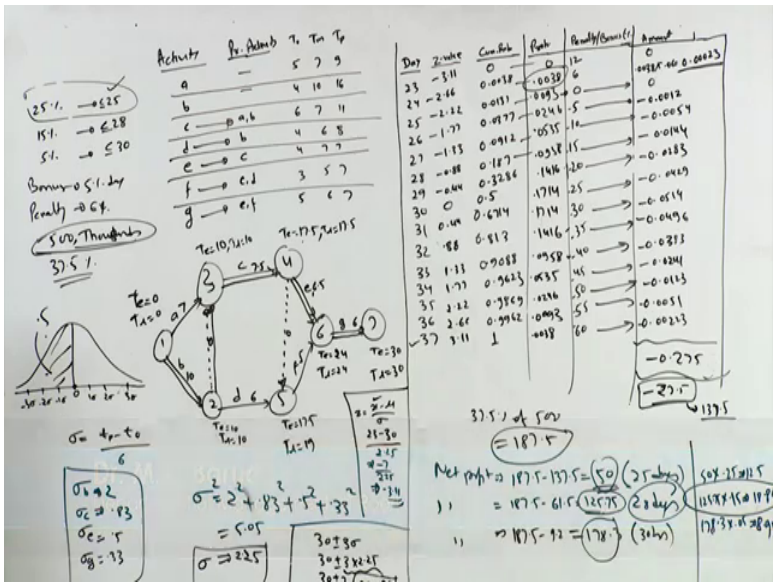

So, let us say the probability that the company will get that particular project a highway project is 25 percent; if it quotes that the project would be completed in less than or equal to 25 days. Now, the probability that the company will get this project is 15 percent if the company quotes the completion duration is 28 days. And the probability that the company will get that particular project is 5 percent, if company quotes the duration is less than or equal to 30 days. And if you quote more than 30 days then it has no use right. So, we have to decide what should be the duration company suit quote coat. So, that it gets the tender right. Apart from this there are couple of other information. So, whatever time you quote if you finish it before that then you will get a bonus of 5 percent per day. So, let it be like this bonus is 5 percent per day. I will tell you 5 percent of what little later.

If the company quotes let say for example, 25 days and if it completes in let say 23 days then there would be 10 percent bonus. Similarly, if company quotes 25 days and if it completes in 26th day, if it completes on 26th day then there would be a penalty of 1 day and that penalties 6 percent, so penalties 6 percent. Apart from this the total quoted price

is 500, 1000, 500, 1000, 500, 1000 and the margin is expected to be 37.5 percent. So, this profit margin is 37.5 percent of this value. You can easily calculate what would be the profit. Now, you have been given a project with precedence relationship. So, there are different activities in this particular project.

So, let me write those activities here. So, activity, precedence activity, preceding activity, so this be first activity and you are given some preceding activity and your been given three time estimates, optimistic time, most likely time and pessimistic time. Now, when you have let say these are different activities a, b, c, d, e, f and g. So, preceding activity of a is none; for b also none; a and b are the preceding activities for activity c. It means you can start activity c, once you complete activities a and b. For d, it is b; for e, it is c; for f, it is c and d; and for g, you got e and f. And when you draw this particular network for this problem, you will have to use couple of dummy activities also.

So, we will draw that network also. Let us write time estimates. So, this 5, 7, 9 for a 5, 7, 9; for b, you got 4, 10, 16, 4 10 and 16. For c, you have got 6, 7 and 11, 6, 7 and 11. For d, you have got 4, 6, 8, 4 6 and 8; for e, it is 4, 7, 7 4 7 and 7; for f, 3, 5, 7, 3, 5, 7 and finally, 5, 6, 7 right. Now, from this particular precedence relationship you can draw the network for this particular question and let us find out what is the total completion time for this particular project right. So, the network let us draw this network. So, you have got different nodes let say this is one, this two this three you have got fourth fifth sixth and seventh right. So, there are 7 nodes and these are different activities.

And if you look at this and this is your activity a, activity b, activity c, activity d, activity e, activity f and g, this is your dummy activity, this is another dummy activity. Why did we draw this dummy activity, because we know that c is the activity and predecessors are a and b. So, for c, a is preceding activity and similarly b is also preceding activity for this activity c. So, you can see that b is this. Now, we have drawn this particular network. Now, let us find out what are the expected time durations of these activities. So, for activity a, what is T e value, we have already seen t is what it is to plus 4 times T m plus T p divided by 6, this is the formula we have already seen right.

So, for a let us calculate the expected value right. So, this 5, 5 plus 4 into 7 plus 9 divided by 6, so this is 28 plus 14, 28 plus 14 is 42 divided by 6, so this is 7. So, the duration of these activity is 7. Similarly, you can calculate durations of other activities.

So, for b, what would it would be, so this would be say 40 plus 26, 60 by 6 it means 10 right. So, for b, it is 10. Similarly, for other activities you can also calculate durations.

So, let me write these values this 7.5; for d, it is 6; for f, it is 5; for g, it is 6; for e, it is 6.5. So, activity e and its duration is 6.5 and of course, these two are dummy activities, so zero durations. Now once you are done with this particular calculation, you need to find out how much time this project will taking in its completion. So, let us find out T e and T l values set all these nodes. So, early start time is 0. For this T e is 10 right for this T e is 10 why 10 because this dummy activity. So, as I said we need to calculate T e in forward pass right and we take the maximum value. So, at this node from this one this particular node T e would be 7, and from here it would be 10. So, we will take the higher value. So, we have selected 10 right.

Similarly, here it would be this is 10 plus 7.5 - 17.5 right 17.5 plus 0 is 17.5; from here it would be 10 right 10 plus 6 - 16, so 16 and 17.5. So, we will take 17.5. At this node this is 17.5 plus 6.5, so 18 plus this is 24 and for this is 30. So, we will say that the schedule completion time is 30. Since we have not been given any other schedule completion time. So, we will take T l as 30, this is we have already seen earlier. Now, we need to calculate T l values for this particular network. So, T l at this point is 24, 30 minus 6 is 24. Similarly, you can calculate all T l value. So, at this point T l is 19; at this point T l is equal to 17.5; at this point T l is equal to at node number 3, this is 10 here. At node number 2, T l is 10 and here it is 0. Now, you need to calculate critical path. So, critical path is this path right. So, this is 10, this, this and this.

So, what are the different critical activities you have got activity b, c, e and g, these are four critical activities. Now, let us calculate the standard deviation for all these critical activities and what is these standard deviation is standard deviation we already seen it is t p minus t o divided by 6. So, standard deviation of activity b right standard deviation of activity b is t p minus t o. So, this is 16 minus 4 -12 by 6, it is 2 right. So, standard deviation for activities b is 2. Similarly, for activity c, what it would be 11 minus 6 – 5, 5 divided by 7, so 5 divided by 7,s you will get 0.83. Similarly, for activity, you have got e; for activity e, it is 0.5. Similarly, for activity g, it is 0.33.

So, once you calculate all these standard deviations of activities along critical path, you need to calculate now standard deviation of the critical path. So, let us calculate standard

deviation of critical path. So, for this what you need to do is let us calculate variance first. So, this is 4 right, this 2 square right, 2 square plus 0.83 square plus 0.5 square plus 0.33 square. Now, this becomes 5.05 and standard deviation is standard deviation is 2.25. So, this is our standard deviation. So, we are saying that this project would take 30 days time with standard deviation of 2.25 days.

Now, let us move further and calculate how much duration should we quote in our quotation. So, that we get this particular project. So, we will have to do calculations at three levels. First level, we will quote 25 days and we will see what is a total penalty or total bonus; similarly, for 28 days, and for 30 days. So, first of all, we will calculate it for 25 days. I have already told you that the area under the curve area under normal curve is within plus minus 3 sigma, it is 99.73 percent. So, approximately let say this area is 99.73 and this is your zero point right and this area is generally we say that area under curve is unity are one right. So, this area is 0.5, and this area is also 0.5, is not it. So, this how we say that total area under curve is 100 percent.

So, let us look at what is the possible duration of this project because we know that this is standard deviation. So, this is your mean value, which is 30, so 30 plus minus 3 sigma is what 30 plus minus 3 into 2.25. So, let us take this value is equal to 7 though this not exactly 7, but let us take it 7. So, you will have 30 plus minus 7, it means 30 minus 7 – 23, and 30 plus 7 - 37. So, this is the total duration possible 23 to 37 days. So, you can complete the probability that you will complete this project within the range is 99.73 percent.

Now we will calculate different penalties and bonus. So, as I said you can complete this project in 23 days and maximum you can complete in 36 days. So, let us look at this is day. Let us say 23rd day 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36 and 37. Now, when you let us also calculate z value. How to calculate z value for 23rd day, I have already told you, let me write this formula once again what is what is z value it is x minus mu divided by standard deviation right. So, x value is 23, so 23 minus mu, mu is this 30 divided by 2.25, so this is minus 7 divided by 2.25. So, this comes out to be minus 3.11. So, this is minus 3.11.

Similarly, for what would be the z value for 24th day, just put here 24. And what would be the z value for 37th day, just put 37 here. So, 37 this would be just a plus 3.11 right.

So, in this way you can calculate all z values. So, let me write these z values. So, this is minus 2.667; for 25th day, it is minus 2.22. For 26th day, it is minus 1.778; for 27th day 1.33; for 28th day minus 0.889; for 29th day, it is minus 0.444; for 30th day it is 0, because 30 minus 30 is 0. Similarly for 31st day you have got. Now, you just change the sign. So, instead of minus 0.44 you will have plus 0.44. So, this let me write it 0.444, 0.889, then 1.33, 1.778, 2.22, 2.66 and finally, 3.111. So, once you calculate this side, the other side can easily be calculated. So, z values are this.

Now, you need to find out what is cumulative probability. So, let me write here. This is cumulative probability. Now, you should look at z-table. What is the in fact, if you look at this particular curve as I said that in this curve this area is 100 percent. So, what will say this is 0 value, this is 1 sigma, let me redraw this curve. So, this is your 0 value, this is one sigma two segment and this three sigma right three sigma two sigma and one sigma, so the minus 1, minus 2, minus 3. And similarly this side you have got one sigma two sigma and three sigma right, so this all these are plus right. So, no need to write plus sign right, so 1 sigma, 2 sigma, 3 sigma. So what we say that any value which is which is in this region. The area under curve would be zero right and when we move towards this side the area under curve will increase and at this point area under curve would be 50 percent. So, any area under curve beyond minus 3 sigma would be zero.

So, let us look at and this value you can take it from probability table z-table. So, let us take this is zero value. When you check z value in 2.66 column, so let say when z value is 2.66, then you will get a value whatever value you get you just subtract it from 0.5 because this area is 0.5. So, you can do this. So, whatever is let say it 2.66, you will get I think 0.4961, if I remember correctly. So, at 2.66 area under curve is 0.4961 minus 0.5, so you will get a value which is 0.0038. In fact, there is no need to calculate area under curve up to you know these three decimal points. In fact, 2.66 will do not a problem; do not calculate z area under curve at minus 2.667 only up to two digits would suffice. In fact, you can delete all these. For easy calculation, you can remove third digit after decimal point right. So, this would be your cumulative probability.

And as I said when you moved towards this side the area under curve would increase right. So, let us let me write you know remaining cumulative probabilities. So, here you will have 0.0131, 0.0377, then 0.0912, then 0.187, then 0.3286 and then z is equal to 0

this is 0.5. As I said z is equal to 0, area under curve is 50 percent. Then this is 0.6714, point eight one three at 1.33 it is 0.9088, then this is 0.9623 and 0.9869, 0.9962 and 1.

Now, it is not necessary that when you look at z value, you will get these cumulative probabilities. You just look at the z value at 2.66 subtract it from 0.5, whatever value you get you just write that value right is not it. These values I have written on the basis of you know you are having one more digits on all these z values. So, these values are on the basis of those particular z values, but you have to write only those cumulative probabilities which you seen z table. Now, since you know these are cumulative probabilities you can easily calculate probabilities.

So, let me I know that area under curve when z is 3.11, it is 1 right or 100 percent. So, the probability would be let say probability would be for this particular 37th day, this 1 minus this. So, this would be 0.0038, this minus this; similarly this minus this and you just keep on calculating cumulative probabilities sorry probabilities. So, this is 0.0093, 0.0246, 0.0535, 0.0958, then 0.1416, then 0.1714, then 0.1714, then you just write these values again from here onwards right. So, let say 0.1416 is not it then the next value would be there right 0.0958 then 0.0535, 0.0246, 0.0093 and 0.003 and finally, 3 8 and this is 0. Now since you know these are the probabilities.

Now, let us calculate whether you would be getting bonus or there would be penalty right. So, penalty oblique bonus as I have said we are doing calculation for this particular quotation, when we quote 25 or less than 25 days. So, when we complete this project in 25th day, there would not be any bonus or any penalty, but if a complete it on 24th day right and then there would be a bonus of 6 percent; and if we complete it on 23rd day there would be bonus of 12 percent. So, all these are in terms of percentage. If you do not complete it, it on let say 25th day then there would be penalty and penalties at the rate of 5 percent per day. So, from 26 days so this 5 percent, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55 and 60, so these and you can always write one more thing because all these are penalty. So, you can put minus sign here right all these are minus right and of course, this plus 6 percent and plus 12 percent.

Now, what is the amount of penalty or bonus. So, amount would be let look at this initially it is 0 into 0.12, so this is 0; then it is 0.0038, this is the probability 0.0038 into 6 percent right it means its 0.06. So, this value is the amount and that value is let me write

that value it is 0.00023 right. So, this is the this is the amount of bonus right 0.0023 of course, we will multiply it with the actual amount. So, these are different amounts. So, for this the next time it would be this zero how did we get this zero this 0 into 0.12, this is 0.0038 into 0.06 and the third one value would also be 0. Why zero, because zero into this right and similarly you can calculate all other values.

So, for 5 percent let me write what is this amount. So, 5 percent what it would be 0.0246 into 0.05. So, let me write it in this way. So, this is minus 0.0012. For 10 percent, it is minus 0.0054; for 15 percent, it is minus 0.0144; for 20 percent, it is minus 0.0283. For 25 percent minus 0.0429; for 30 percent, it is minus 0.0514; for 35 percent minus 0.0496; for 40 percent, it is minus 0.383; for 45 percent, it is minus 0.0421; for 50 percent, it is minus 0.0123. For 55 percent, it is minus 0.0051; and finally, for 60 percent, it is minus 0.00223. So, you just state all these values. Now, this comes out to be minus 0.275. So, when you see this is negative value it means it is a penalty.

So, let us calculate what is the net expected profit. So, this is 0.275 percent is the penalty. So, the total expected penalty is this much, this much right. When you convert it into in terms of percentage, it becomes minus 27.5 percent. So, this is your penalty 27.5 percent. In other words, this is best penalty is nothing but equal to 137.5 lakh 137.5 thousand rupees why how did we get this value. This is because our total you know the quote price is 500, thousand and penalty is 27.5 percent. So, total penalty is 137.5 thousand rupees right. So, this is a penalty. And what is our profit margin it is 37.5 of this 37.5 of 500, thousand, so that profit would be 187. So, let us calculate is 37.5 percent of 500 would be 187.5. So, this is your profit and this is your penalty.

So, let us calculate net profit net profit is equal to its equal to 187.5 minus 137.5, this is 50,000. Similarly, this is net profit for 25 days; for 25 days. When we quote 28 days, all these values will change right.

So, let me directly right net profit for second option the net profit for let say for 28 days it would be, so net profit for 20 days would be 187.5 minus 61.5. So, this is 125.75, this for 28 days. And finally, for 30 days, the net profit would be 187.5 minus nine point two thousand which is equal to 178.3 thousand is for 30 days. Now, we have been given we have calculated all these net profits, but we did not use anywhere this particular information which is given in the question.

So, let us calculate the resultant expected income. So, resultant expected income is 50,000 in first case. So, 50,000 into 25 percent, so this is 0.25, so this becomes it is 12.5, 125.75, 125.75 into 0.15, 15 percent. So, this becomes 18.86. And finally, you got 178.3 into 0.05 right. So, you will get 8.91, this is 8.91. So, what is your solution how many days would you quote in your quotation, so that you get this particular project the maximum value is this. So, you should quote 28 days. So, in this way, you can solve this particular question. So, with this let me complete this session.

Thank you very much.