Project Management for Managers
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Lecture - 48
Crashing of Networks- I

I welcome you all in this session. As you are aware in previous session we were discussing how to crash a project. So, we were solving this question.

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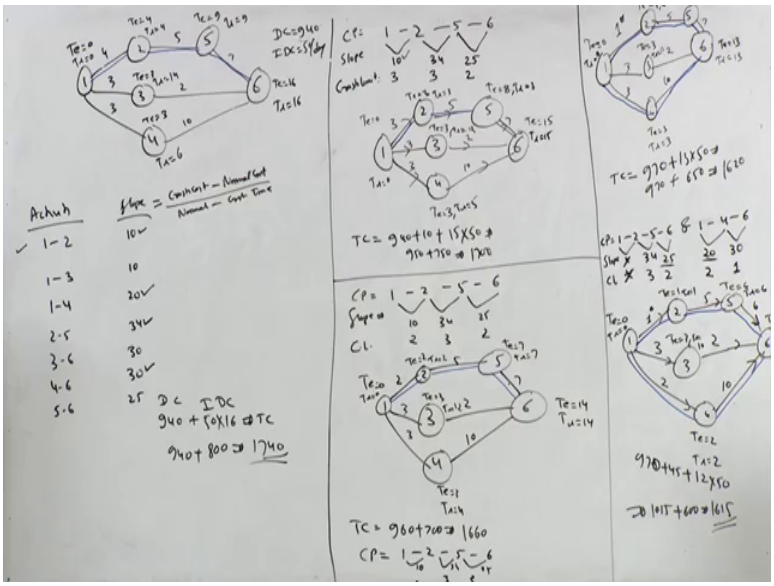

And in this question we have seen that the initial duration of this project is 16 days and initial total cost is 740. After that we selected critical path, and along critical path we selected that critical activity, which had least slope right. And we reduce the duration of that activity 1 2 from 4 to 3 days right. And now the total duration is 15 days. So, and the total cost would be what here it is 1700 right.

Now, let us look at what to do next. Now again critical path is 1 2 5 and 6, what is the slope? Write all the slopes for activity 1 2 this 10 2 5 34 5 6 25. Crash limit, crash limit is now 2 here right, because we have already you know reduced it by one day right; c limit 3 crash limit 2. So, let us reduce this activity by one more day and let us see what happens. Of course, this would become 14, cost will again direct cost will increased by 10 rupees that is 960 and this will come down by 50 rupees right. In fact, here itself I know that what would be the total cost in next iteration right. Let us solve this right.

So, this 1 2 3 4 5 6, one more point here the earliest start time is T e. For this node is 3 by mistake it was written 4 here right. So, you have got 2 here now. All others durations will remain same; so 3 3 5 2 10 and 7 right. So, I know that T e is 15 here right. Now it would be 14. So, T e is equal to 0 T e here is 2 T e here is 7 T e is 3 T e is 3 right. Of course, T l is also known at this node. So, this is 14 T l is 7. T l is 2, T l is 0, T l here is 14 minus 10 it is 4 and T l here is 14 minus 2 is 12.

So, except at these 2 nodes at all other nodes T e and T l's are same right. So, this path again remains is critical path. Total cost you can find out, total cost is what? Total cost is now 960 plus 700 right. So, this would be what you just calculate and write down that total value right. Right now what? Again reduce the duration of this activity by one day right. So, critical path is this activity with least slope is again 1 2. So, let me draw that network. So, I hope that we have not we did not get any other critical path, yes there is only one critical path so for right.

So, this is 14. Duration is 14; now let us look at what happens next. So, let me write critical path here, critical path is 1 2 5 and 6 slope minimum is 10 and 34 then 25 and crash limit is 1; 1 3 and 2 right. So, I can make a duration of this activity now from 2 to 1 right; so this 1 2 3 4 right. Now this duration is 1 and one is star, star means you cannot further reduce it is duration right.

So, this is 3 3 5 2 and 10 this is 7 right. So, T would be 13. So, T at this point is 0, T e here is 1. T e here is 6. T e here is 3 T e here is 3. T l is 13 T l is 3 just see this point. Now for the first time we are getting T l and T e had this node equal right. What about T l here? It is 11. 13 minus 13 minus 2 this is 11. And T l at this point would be same 1 T l at this point 0. So, T and T l are 0 at node 1 right. Now let us look at which path is critical path. So, you have got 2 critical path here, this is your first critical path, and this is your second critical path right.

Now what to do? You would like to reduce duration of this particular project, and what would be the total cost is at this point, total cost is you have got 970 plus 13 in to 50. So, this is 6 how much this? This is 650 plus 970 right. So, this would be total how much this 1000 16, 20 right. So, if you look at every time the cost is total cost is reducing by 40 rupees, why 40 rupees? Because direct cost is increasing by 10 rupees and indirect cost is decreasing by 50 rupees every time right.

So, that is why that the net change is of 40 rupees every time right. Now you have got 2 critical paths, what to do now? So, let us write critical path you got to 1 2 5 and 6 right. Write here so, critical path is 1 2 5 6 and second critical path is 1 4 and 6. You cannot crash this activity. So, let us look at what is slope first of all. So, there is no need you cannot you should not right. Slope and crash limit for activity 1 2 right. So, this is exhausted right. So, this is slope whatever is slope you cannot reduce it is duration right. For 2 5 what is what this the slope? It is 34 crash limit is 3 right. For 5 6 what is, what is the value of slope? 25 crash limit e is 2.

Now for activity 1 4 what you what is the value of slope for activity 1 4? This is 20. For activity 4 6 what is the value of slope? Is 30. Crash limit crash limit for activity 1 4 can be found out from here right. So, this is 3 minus 1.

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Example: Find optimum schedule.

Activity	Normal		Crash	
	Time	Cost	Time	Cost
1-2	4	100	1 (up to 1 day)	130
1-3	3	140	1	160
1-4	3	200	1	240
2-5	5	100	2	200
3-6	2	50	1	80
4-6	10	150	9	180
5-6	7	200	5	250

Indirect cost = 50 per day.

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So, 2 is the crash limit for activity 1 4, for 4 6 what is crash limit? For 4 6 it is 10 minus 9 that is 1. Now what to do? See is you want to reduce duration of this project from 13 days to either 12 10 or whatever it is right.

Because we do not know at what, at what time duration the cost would be optimum right. So, you can you have to take a decision. First of all first decision is that to reduce the duration of the project. You will have to reduce both the critical paths simultaneously right. So, what you can do? You can first decision first option is what? You either reduce activity 2 5 activity 2 5 or 5 7, any one of these 2, and 1 4 or 4 6 right. So, for this first of

all again the least cost slope will help you, and reducing activity in selecting critical activity. So, the least path here is that the least slop here is 20 and for this 25 right.

So, select activity 5 6 activity 5 6 this is for 1 4 and 4 6. So, this is 2 5 and 5 6 not 5 7 right. So, what to do here? You have to select activity 5 6 because it has got least slope activity 1 4 it is got least slope. The cost will increase by how much? 25 plus 20, 45 days right. So, direct cost will increase by 45 days and the duration of the project will come down by one day. So, indirect cost will decrease by 50 rupees again the total cost will decrease 5 rupees. So, let us go for this option. So, what we are doing? We us reducing activity 5 6, so let me first draw network. So, this one already exhausted view to reduce 5 6 5 6 is where 5 6 is this right. So, this is now 6 and 1 4 1 4 is where 1 4 is this. Now this is 2 all other durations will remain same 5 2 and 10.

All these activities are in this direction right. So, this is 6. Let us find out what is the total duration. So, T e is T e is 0, T e is 1 T e is 6 T e is we will calculated it later on right. So, T e is here is 3, T e is 2 right. So, from this side T e is 12 from this side this is also 12. Now let us find out anyway just calculator T l also right. So, T l is equal to 12 T l is equal to 6 here, T l is equal to 1, T l is equal to 0 and T l is equal to 2 and T l here is to 12 minus 2 is 10 right. So, again you have got 2 critical paths right. What is the total cost? Total cost would be In fact, this 970 plus 970 plus 45 plus 12 into 50.

So, this is 45 1 0 1 5 right isn't it? And 600 this is 1 6 1 5. Total duration is 12 and total cost is this. Now what to do? Shall be further and decrease duration of this activity this particular project, because here we have got 2 critical paths right. And the total duration is 12. You can do one more thing; see is your got 2 critical path here and there is still possibility of reducing it is duration.

So, what you can do? You can further select any to any critical activity from this path and any critical activity from this path; again you have to keep in mind the least slope concept right. So, here let me again write critical path ok.

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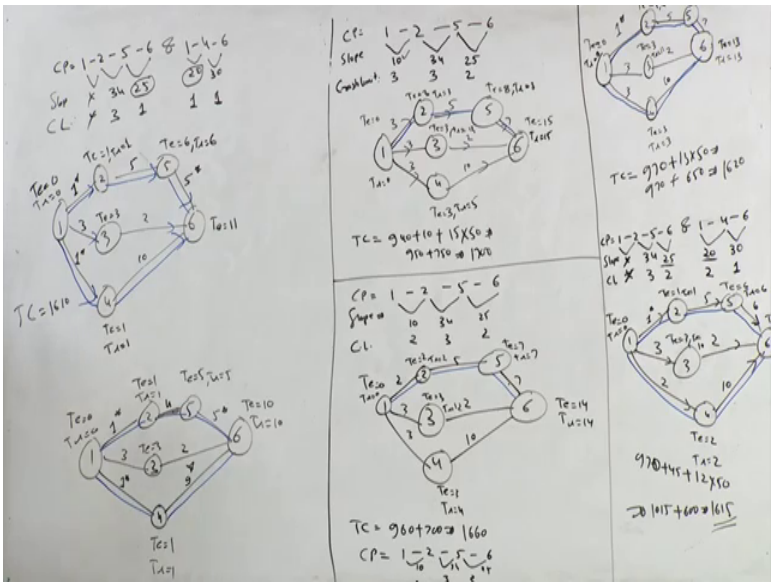

Let me write here. So, critical path is you have got 1 2 5 and 6, and you have got second critical path is 1 2 4 right. No is 1 4 6; so 1 4 and 6 right. So, let us look at slope 1 2 no point and this crash limit 5 6, 2 5 what is the slope? 34 for 5 6 what is what is the slope? 25 now it is crash limit is 1 for 3 4 it is still 3 right. For activity 2 5 it is still 3 right, but this is now one for 5 6 right. And for 1 4 slope is 20 crash limit is 1 for 4 6 what is slope 30 crash limit is 1 right.

So, what decision will you take now? Select 5 6 because it is got the least slope from here and 1 4 from here which is got least slope right. Cost will decrease by sorry, to a direct cost will increase by 45 rupees indirect cost will you know that will decrease by 50 rupees. So, let us draw that network right. 1 2 3 this is 4. So, this 1 2, this is 3 4 5 6 right. You to reduce duration of activity 5 6, 5 6 is this right.

Now, this duration will become 5, and activity of 1 4, 1 4 is this now this will become 1. All others will remain same right. So, this is 1. In fact, now on the other durations will remain same. So, this is 5 this is 2 this is 10 right. And this is 3. Let us find out T e and T l values. So, T e is equal to 0 here T l is equal to 1 T l is equal to 6 sorry, it is T e right. T e is equal to 1 T e is equal to 6 T e here is 3 T e here is 1. And T e here is 11, isn't it? T l 6 T l 1, T l here is 1 right. T l here is 0. So, again you have got to critical path, this one and this one right.

Now the total duration is 11 days. And if you look at the total cost at this point, what would be the total cost at 11 days? Total cost would be 1 0 1 6 1 0 this would be the total cost right. Now can you further reduced duration of this project? Is it possible? You got you have already crashed to the maximum limit of these 2 right. So, this you crashed limit is achieved crashed limit is achieved here also crash limit is achieved right. So, you cannot reduce this critical activity this critical activity and this critical activity. Now what is the option available? If you want if you are asked to reduce duration of this project what would you do? First thing is you have to reduce these 2 activities simultaneously right.

And that and that is the only solution available or that is the only option available right. So, you got these 2 these 2 options available; so 2 5 and 4 6. So, for 2 5 this is the slope and for 4 6 this is the slope right, isn't it? So, you can reduce activity 2 5 and 4 6. By one day, but when you do that you are cost will increase by your total direct cost will increase by 64 rupees. Your total cost direct cost will increase by 64 rupees and your indirect cost will decrease by 50 rupees.

So, this cost will increase by, how much? 16 rupees isn't it? No 14 rupees So that option is not is not advisable, but if you want you can still do that you can make one more final network just for your convenience. So, total cost for this is 1 6 1 0 right. For eleventh day the total cost is 1 6 1 0 right. Now just final network 2 3 4, this is 5, this is 6. So, what we are doing we are reducing 2 5 2 5 is where? 2 5 is here. So, this is 4 now right? 2 5 is now 4 and 4 6 4 6 is this is now 9. And all others will remain same right. So, this is exhausted, this is exhausted, this is 3, this is also exhausted, this is 2. Calculate T e value 0 T e is equal to 1, T e is equal to 5, T e is equal to 10. Either from this side or this side this is one

So, T e is 3 here. So now, simply you can write T l this 9 T l is 5 T l is 1 T l is 0 here and T l is one here right. So, again this is your critical path. So, what you have done here? You have reduced activity 4 6 and 2 5. So, for 4 6 also you have achieved it is crash limit right. Now what? Can you further reduce it is duration, is it possible? Just think for a minute. Can I further reduce duration of this project? Yes or no? No it is not possible. Because to reduce duration of this project I will have to reduce activities along, I will have to reduce critical activities along these 2 critical paths. And I have got only this critical activity available for you know reduction, but there is no critical activity

available on these paths. So, I cannot further reduce this duration. So, this is the last the minimum duration which is possible in this project, but what about cost; cost as gone up in this case.

So, what is optimum solution to this question? What is optimum solution to this question? You need to draw you can. In fact, prepare a table over here. So, you can prepare a table where in you can write down all the cost and So, when your duration is 16 days, direct cost is something indirect cost is something and you have got total cost right.

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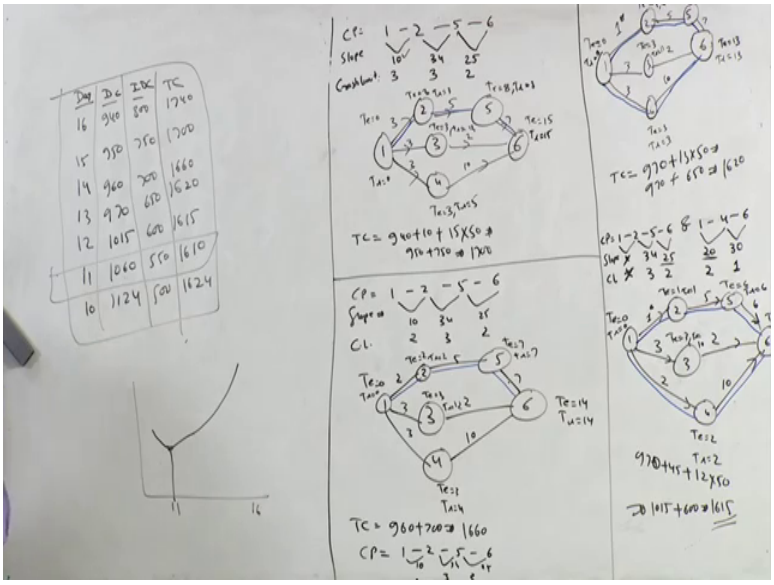

So, 16 days this was the direct cost and indirect cost was 800. So, total cost 1 7 4 0, when duration was 15 days 14 days 13 days 12 days 11 days 10 days. So, direct cost was 950, then it was 960 then 970 then 1 0 1 5 1 0 6 0 and 1 1 2 4. Indirect cost of course, 750, 700, 650, 600, 550 and 500. This is 550, total cost is this is 1700 this is 16 1660 right. Then 16 20 then 16 15 this is 16 10 and this is 16 2 4.

So, if you look at this total cost curve this total cost figure here, this is the lowest cost. And the duration of this project is eleven. So, you should not reduce this project beyond 11 days. So, if you want what is happening here? Let me draw total cost. So, this is the total duration initially 16 days. Now it is 11 days right. So, the total cost is now it has gone down like this right, but if you if you if you if you further reduce it is it will go up right. So, this is your eleventh day here the cost total cost is minimum. So, let me

summarize what we have done in this session. We have solved a question for finding optimum schedule of a project.

So, what we have done here? First of all we calculated slope for all the activities. Then we found critical path and for reduction in duration of the total project. We selected those critical activities along critical path for which slope were least right. And we then reduce it is duration by one day, in all these networks we have reduce the duration of critical activity every time by one day, though this is quit and inefficient method, but quite a simple method.

We will look at some efficient methods of reducing duration of the project in coming sessions. So, for the time being let me stop here.

Thank you very much.