MCDM Techniques Using R

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Lecture - 18
Fuzzy AHP – Part I

Welcome to the course MCDM Techniques Using R. So in previous lecture we started our discussion on fuzzy sets. So let us do a quick recap of what we have discussed so far. So we talked about fuzzy set theory and how it has been used in combination with MCDM techniques quite frequently. And what is the main idea behind usage of fuzzy set theory and development of fuzzy set theory. So this was mainly to deal with uncertainty in the decision problem.

So this particular aspect we talk about that there could be subjective preferences which might lead to impreciseness or uncertainty in the human decision process and the way the preferences or responses are given by decision makers. So fuzzy set theory has certain advantages using which we can handle some of these problems.

So we talked about that how it has been proposed by Zadeh in 65 and then another development along with Bellman in 70'. We talked about the role of fuzzy sets and fuzzy logic in the modeling sense. So that part also was covered. We talked about how fuzzy set is an extension of the conventional set.

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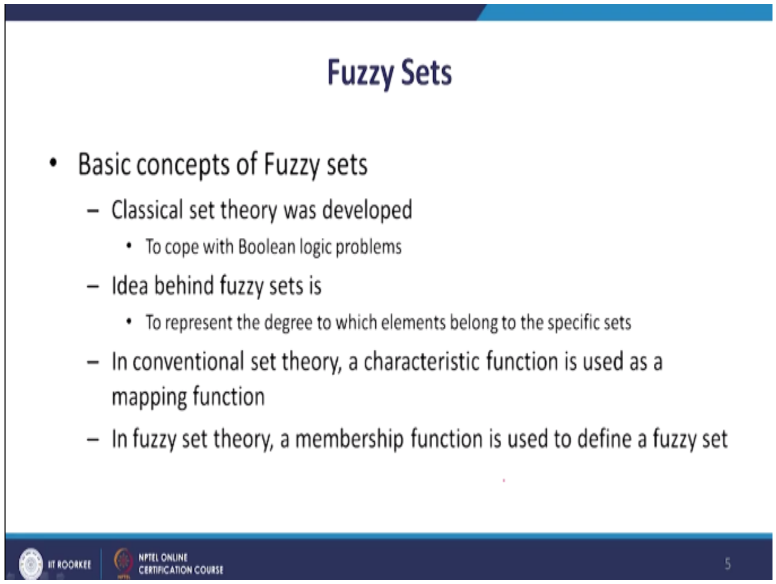

And how they are different there in the conventional set theory be require a characteristic function for mapping. Here in fuzzy sets we require a membership function, right, so all these of aspects were talked about. We talked crisp set and how fuzzy set is different, because it allows the partial membership.

So there is something of you know, continuum of grades or degrees of membership. So these aspects related to fuzzy set were discussed. Then we also talked about how to define a fuzzy set. So we talked about that A tilde can be defined in curly braces as x, Mu A in parenthesis x where Mu is the membership function which takes values from capital X and maps them into a 0 to 1 range.

So this is the membership function and where the value the value of this membership function Mu Ax comes out to be 0. Then x does not belong to A tilde, if it is 1 then x completely belongs to A tilde. If the value is, a value lies between 0 and 1 then possibly belongs to A tilde. So we also discussed how the fuzzy set is defined. Then further we talked about the linguistic variable.

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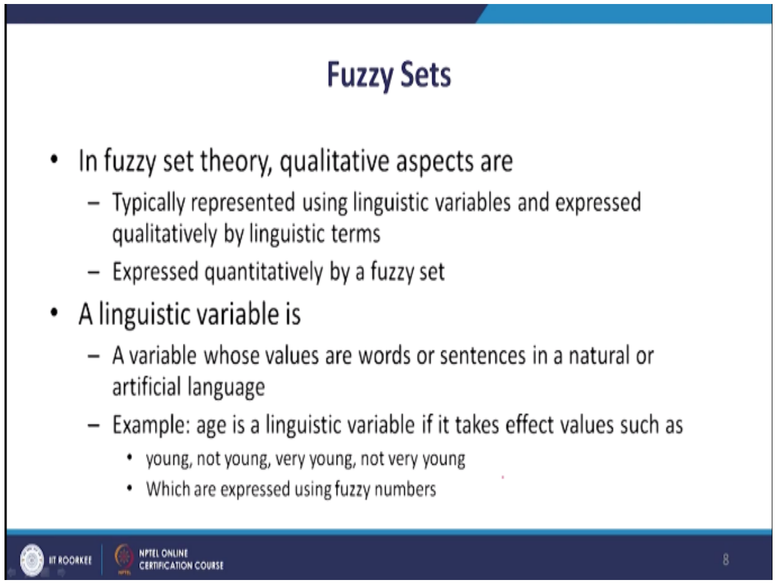

For example, we discussed age as an example and set that linguistic terms like young, not young, very young and not very young you know, some of these kinds of terms are actually used to express the preferences by decision makers and which can be then expressed using fuzzy numbers. We talked about the concept of linguistic variables and you know its importance. And the fields where it has been applied, so artificial intelligent, information retrieval, human decision processes so all these fields it has been applied.

Then we move to another concept that is fuzzy numbers. So we talked about how fuzzy numbers are defined.

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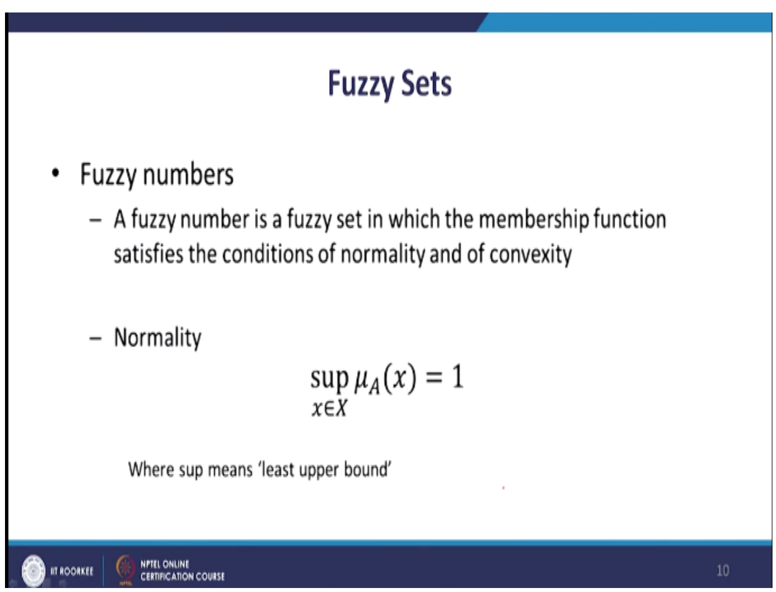

A fuzzy number is a fuzzy set having satisfying two properties normality and convexity. So you can see that this, so a normality about having a least upper bound value as 1. So at least in a sense you know one element belonging to the set. Similarly, convexity is where the all the elements there, you know they can be expressed in the fuzzy numbers that is the case for all the element of x the decision matrix can be expressed in that form and that is the convexity property that is satisfied.

Then how you know, different fuzzy numbers different forms of fuzzy numbers can be used. So depending on the decision problem and the overall context trapezoidal fuzzy numbers, triangular fuzzy numbers have been proposed. However, more popular in the triangular fuzzy number. All these aspects we talked about. We talked about TFNs as triplet l, m, u.

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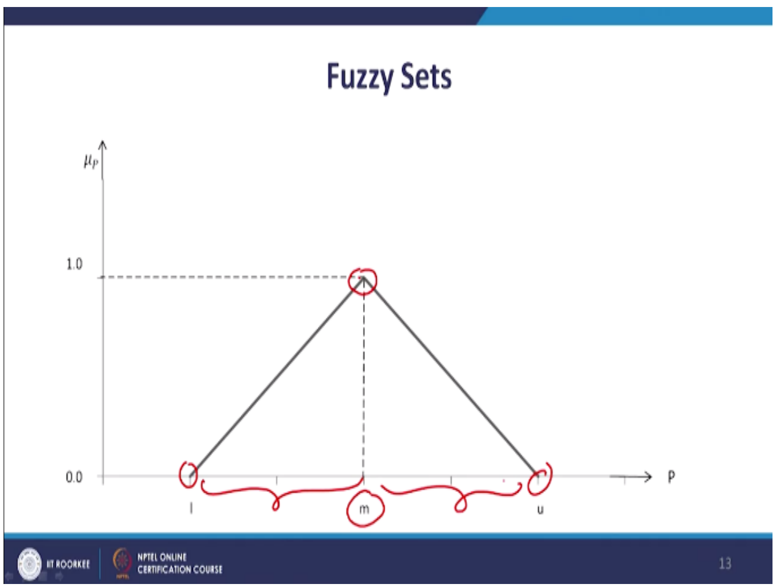

We also discussed this particular graphic to explain a TFN. And we also talked about this membership function Mu x as you can see in the slide and how different values are been taken by the membership function.

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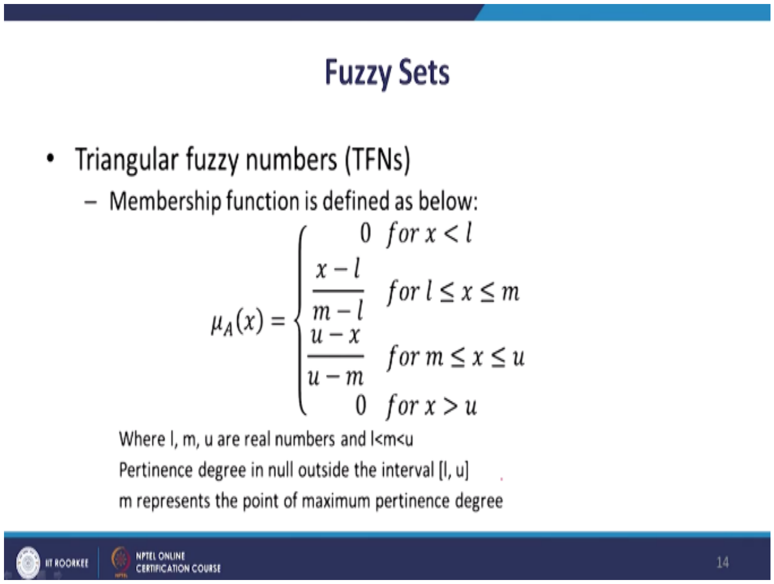

As per the membership functions, so all these aspects we talked about. Then we stopped at this point where we were about to discuss the algebraic operations with fuzzy numbers. Now, having understood the how to define fuzzy numbers let us discuss the algebraic operations.

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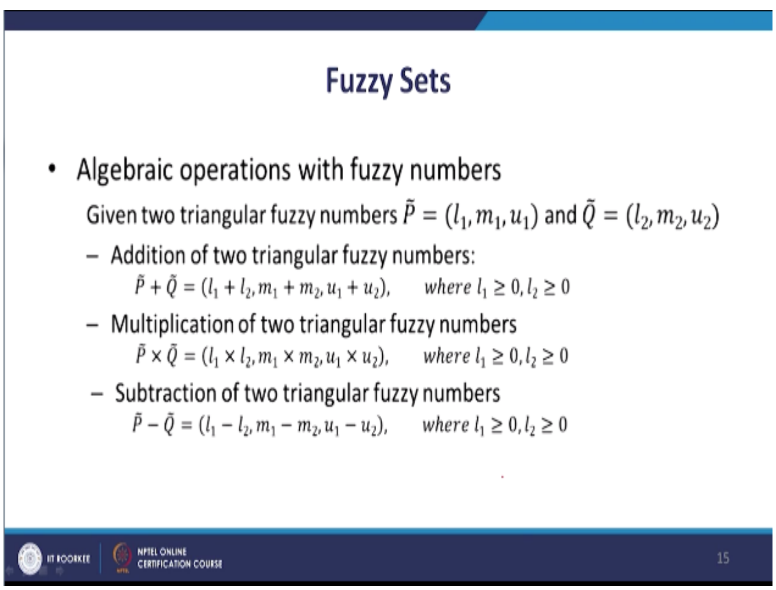

So let us assume we have been given these two triangular fuzzy numbers P tilde which is defined using l1, m1 and u1 and Q tilde which is defined using 1, m2 and u2. So given a these two triangular fuzzy numbers how the fuzzy addition can performed, so that is the first algebraic operation that we are going to talk about.

So addition of two triangular fuzzy numbers can be given like this. So P tilde + Q tilde can be given as l1+l2 as the first parameter; m1+m2 as the first parameter and u1+u2 as the third parameter. Given that, l1 and l2 both are greater than or equal to 0. Then this fuzzy addition is justified. Similarly, we can move to another algebraic operation with fuzzy number that is about multiplication of two triangular fuzzy numbers.

So P tilde multiplied with Q tilde can be given as you know, multiplication of l1 and 12, second parameter being multiplication of m1 and m2 and then the third parameter being multiplication of u1 and u2 where l1 and 12 both are given as greater than or equal to 0. So this is how multiplication, fuzzy multiplication can be defined.

Similarly, subtraction of two triangular fuzzy numbers can also be defined. So here P tilde – Q tilde can be defined as first parameter being 11-l2, second one will become m1-m2, third one will become u1-u2 where l1 and l2 both are greater than 0, so these three algebraic operation, addition, multiplication and subtraction rather fuzzy addition, fuzzy multiplication and fuzzy subtraction.

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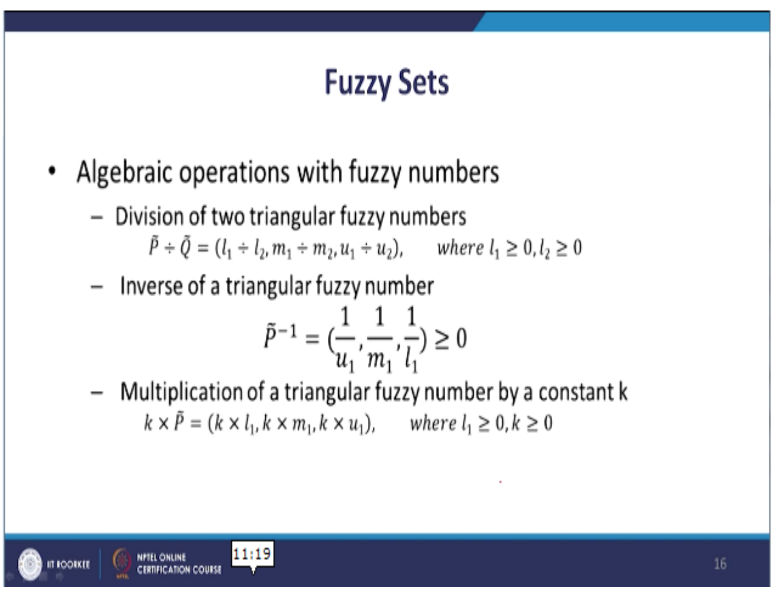

So this is how they can be performed, quite similar to the way we do it in the conventional theories. Now, division of two triangular fuzzy numbers similarly, P tilde divided by Q tilde can be given by you know, first parameter can be given by l1/12; m1/m2 and u1/u2 where l1 and l2 all three are greater than 0. So in this fashion division can also be defined. Now let us talk about inverse of a triangular fuzzy numbers. So if we are considering P tilde as the triangular fuzzy number for inverse operation then P tilde – 1 for inverse is going to be equal to first parameter will become 1/u1.

You see instead of l1 now u1 is being used because we are taking inverse 1/u1 then 1/m1 then 1/l1. You can also understand from the point of view that l1 is less than m1 and m1 is less than ul so therefore, once you do inverse it is the you know, 1/u1 which is going to will having the which is going to be the smallest value followed by 1/m1, followed by 1/l1 so therefore, the parameters will be like this, right.

Because for P tilde inverse 1/u1 is suppose to the smallest possible value, so therefore, it has to be 1/u1 because that is going to be also the smallest possible value. So from that point of view also you can understand why this. So all this has to be greater than or equal to be 0, so you know, this is the inverse. And then let us talk about multiplication of triangular fuzzy number by constant k.

So any constant k it can be multiplied by multiplied with triangular fuzzy number P tilde. So k multiplied with p tiled can be given as you know, like this first parameter k*l1, second k*m1 and third one k*u1 where l1 is greater than r=0 and k1 also is greater than r=0. So these are some of the algebraic operations which can be performed for fuzzy numbers.

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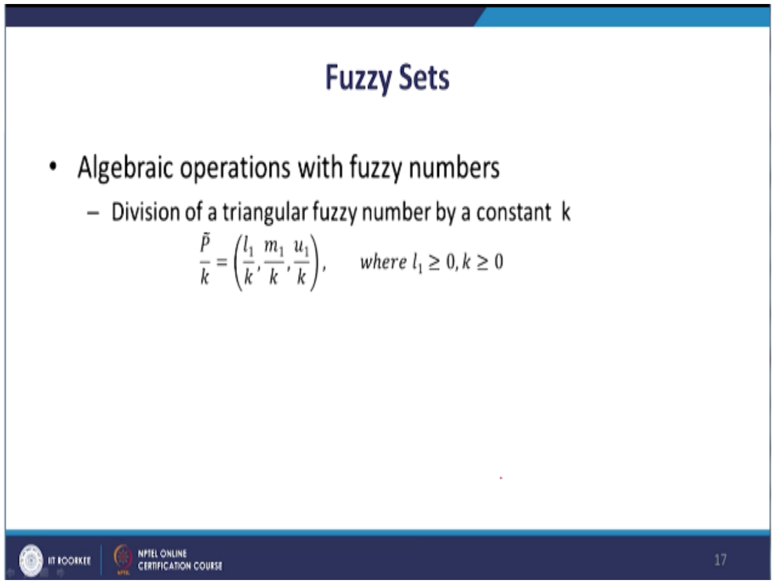

So you know, another operation which is about of division of a triangular fuzzy number by a constant k. So here P tilde is divided by this constant k can be expressed as l1/k, m1/k and u1/k. So these are the important algebraic operations that can be performed with fuzzy numbers. So with this we have you now, we have covered the basic background that is required for fuzzy sets and with this understanding we can see how fuzzy sets can be used in combination with other MCDM techniques and decision making problems, multi criterion decision making problems can be solved.

So now we will move to the application of you know, this method fuzzy sets method in combination with AHP that is Analytic Hierarchy Process. So we will move to our discussion on that part, that is you know Fuzzy AHP. So let us move forward. So let us talk about Fuzzy AHP.

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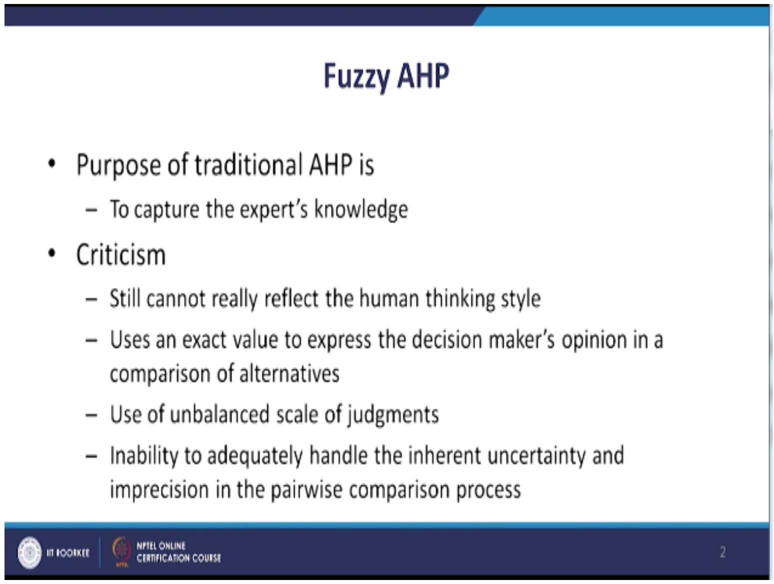

So fuzzy sets they have been used in combination with AHP by quite; this combination is quite popular and have been used by a number of research studies and other executive reports. So the purpose of traditional AHP as we already understand is to capture the expert's knowledge in the overall decision making process. So the AHP provides us a method you know, that can be used to capture you know the expert's knowledge.

However, there are certain criticisms of AHP which are in line with while we are going to use fuzzy sets in combination with AHP. So we look at some of this criticism of fuzzy AHP. This AHP, traditional AHP some of the criticism of traditional AHP still cannot really reflect the human thinking style, but use an exact value to express the decision maker's opinion in a comparison of alternatives. So these are some of the criticism of traditional AHP and that is why we have this, we are going to discuss the fuzzy AHP.

Then use of unbalanced scale of judgments. Inability to adequately handle the inherent uncertainty and impreciseness in the pair wise comparison process, so these are some of the criticism which you look at the kind of criticism that they are; fuzzy AHP can be used to actually you know, handle most of them.

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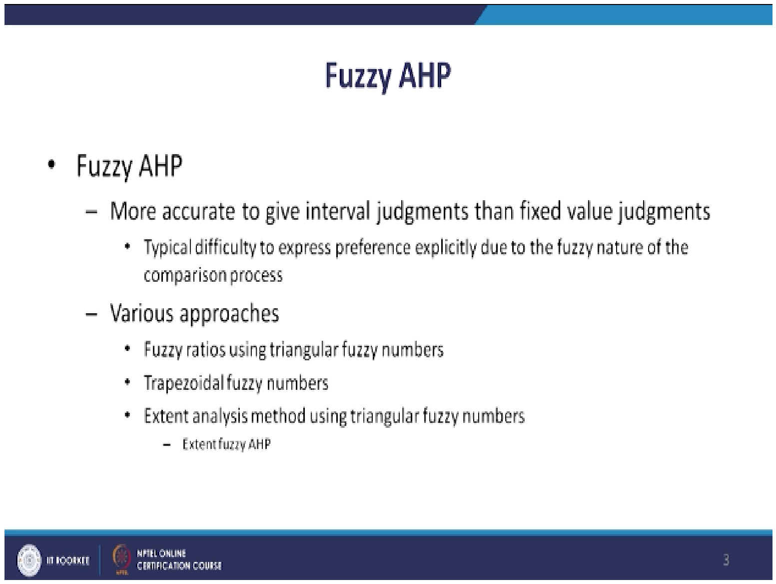

So now let us start our discussion on Fuzzy AHP. So as we have talked about while our discussion on fuzzy sets is that, sometimes it is very difficult for decision makers to give exact comparison, exact preferences in a quantitative fashion, right. So to overcome that scenario fuzzy AHP it takes into account that it is sometimes it might be more accurate to give interval judgments rather than the fixed value judgment or exact judgment.

Sometimes it might be more easier and more accurate as well to give interval judgment. So as mentioned in the slide typical difficulty to express preference, explicitly due to the fuzzy nature of the comparison process. So the way comparison might be being performed between two objects or elements you know, that might be you know, difficult so fixed value might be difficult to specify. So therefore, interval judgments can actually be used.

Various approaches with respect to AHP fuzzy AHP have been used. For example, fuzzy ratios in triangular fuzzy numbers, trapezoidal fuzzy numbers and the extent analysis method using triangular fuzzy numbers also referred as extent fuzzy AHP. So this extent fuzzy AHP, the extent analysis method using triangular fuzzy numbers is the most popular one. So in our discussion now onwards we are going to talk about External Fuzzy AHP.

So if we look at the we try to understand what happens in you know, extent you know, fuzzy AHP. So we will also try to understand and compare it with what we have already this discussed in some of the starting lectures in traditional AHP. So we will, while we are discussing extent fuzzy AHP we will also talk about the steps that are required to execute the traditional AHP. So let us start our discussion on extent fuzzy AHP.

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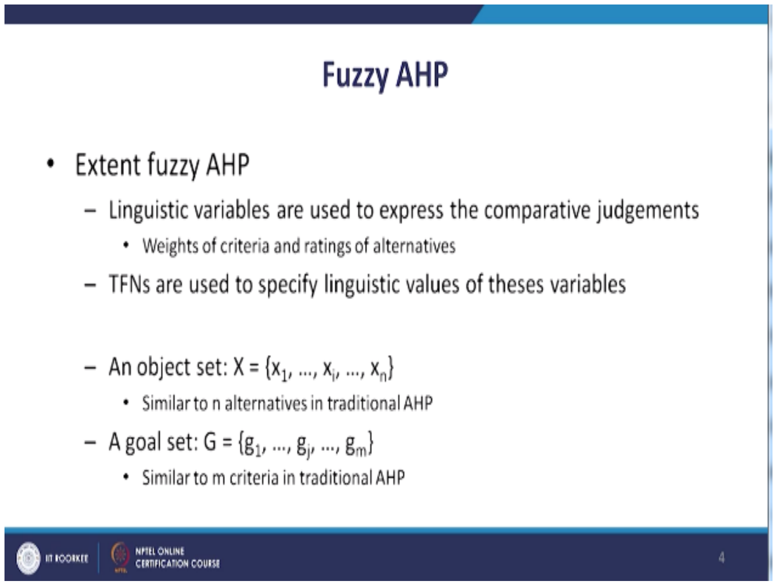

So linguistic variables as we have talked about in the previous lecture on fuzzy sets, that linguistic variables are used to express the comparative judgments in fuzzy AHP and extent to our fuzzy AHP as well. So now these linguistic variables, so just like in the previous lecture we talked about we had given the example of age where we set young, not very young right, no young so those are the kind of linguistic variables, linguistic values that can be used by decision makers to express their preferences.

So and to; in case of AHP to perform the pair wise comparison to indicate their pair wise comparisons, so weights of criteria and rating of alternatives they can actually be you know, indicated in that sense using these linguistic values, linguistic variables. So there in the traditional AHP you know, when we take preferences from decision maker whether for the pair wise comparisons of criteria with respect to the given goal.

Or you know, pair wise comparisons among alternatives with respect to a particular criterion so that was the decision makers in this AHP but suppose to specify that values. Suppose to give fixed value judgments. So we use to get a number of comparison metrics there in traditional AHP so one comparison matrix was about the you know, a pair wise comparison among criteria and then the other comparison metrics were about the pair wise comparisons among the between the alternatives with respect to different criteria that could be there.

So number of comparison metrics were generated there in traditional AHP. Now that process under extent fuzzy AHP is to be performed using linguistic variables. So therefore, the decision makers would be making those pair wise comparisons using linguistic term, just like the example that we have given like age, we talked about that you know, not young, young, not very young, correct terms can actually be used to you know, perform the pair wise comparisons.

Why this is being done? Because we are assuming that we are operating in a fuzzy environment where it is difficult for decision makers to give you know, exact comparisons you know, to give so rather we would like them to specify like them to give the interval judgments rather than fixed value judgments.

Now once these pair wise comparisons have been taken for extent fuzzy AHP form the decision makers then we can use TFNs to specify these linguistic values, right. So this TFNs once we have taken then, once we have converted the linguistic values to TFNs then we can talk about then we can move ahead with other steps. So steps are quite similar to traditional AHP. However, the mathematics, the underlined mathematics is going to be different.

Now we are going to be using fuzzy algebraic operations, fuzzy addition, subtraction, multiplication, inverse and other operations. So therefore, the kind of steps and the kind of mathematics that is going to be required, we are going to talk about some important steps that we will have to perform. So focus on that, we are supposed to compute a number comparison metrics.

So we have a number of comparison metrics, pair wise comparisons are there using linguistic variables and then TFNs so that is there, with that assumption now you know, once that is their so one comparison matrix would be for criteria then others one would be for alternatives. So we are taking whether it is for the criteria comparison matrix or for alternative comparison metrics, we are taking them as you know, object.

So this is we are defining as an object set capital x; the elements being x1, xi up to xn. So this is similar to an alternatives and traditional AHP for that comparison matrix for alternatives. And then we are taking a goal set that is capital G, which is g1, gj to to gm similar to m criterion and traditional AHP, right. However, we are talking about these object set, we are talking about this object set and goal set in more general sense because one comparison matrix is about criteria and the other comparison metrics are about alternatives.

So to understand the underlying mathematics and these steps that are required to be implemented in extent fuzzy AHP, we are using these notations which are different from what we have used in the lecture of AHP and in other lectures as well. So we have object set and elements are there, right. And these object sets are had been compared with respect to you know, goal set so there could be number elements in the goal set. So these comparison values are available with us. So with this we will move forward and talk about these steps.

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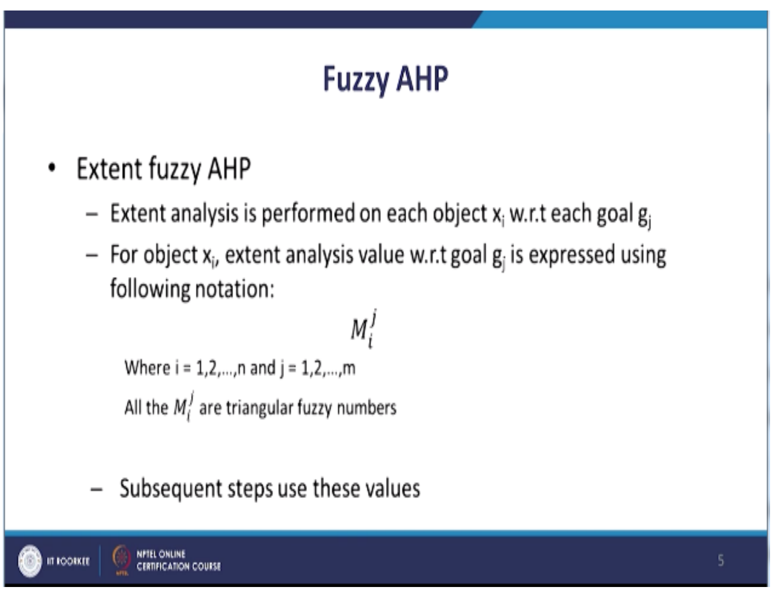

So one thing is that extent analysis is performed on each object xi with respect to goal gj. So this is there. So for object xi, extent analysis value would be obtained with respect to goal gj. And it is expressed using following notations, capital Mi j. i is indicating the object and j is indicating the goal here. So i can be from 1 to m, you know objects and j can be from 1 to m there are m goals. So these extent analysis values have been obtained.

So what this extent analysis values are? So there are triangular fuzzy numbers. So these fuzzy numbers we have. So what we talked about, the comparison metrics and having completed the having taken the responses from decision makers using linguistic terms. And then you know, converting them into a triangular fuzzy numbers. So the values that we obtained is what we are referring as the extent analysis values here.

And this capital m, this i j, i for object and j for the goal, this is what how we are denoting this, these values. Now subsequent steps we are going to use is they are going to use this values. So let us move forward.

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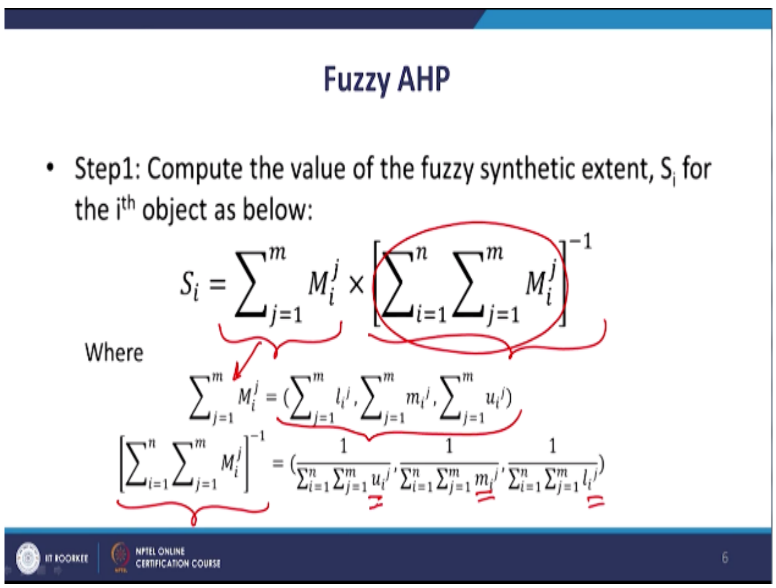

So in the step number 1, we are supposed to computer the value of fuzzy synthetic extent. So we have already computed extent analysis values that is capital M. Now we are going to computer this fuzzy synthetic extent values that is capital S for each of the object, that is why you are seeing Si, so i is for object there are n objects from 1 to m. So the values of you know, this extent analysis where will the capital Ms are going to be used to computer these synthetic values S values.

And these S values are going to be later on used in the subsequent steps. So let us see what expression is about. So capital Si this can be computed using this summation, j=1 to m where m is the number of goal elements, so the summation is over that. Then we have these values, Mi j and this is multiplied by another expression here. So this is one expression that we have is this one and then other one is this one.

So you see this expression we are supposed to take the inverse of it. So this is again summation of our 1 to n that is for n objects, right. And then summation over j=1 that is for m goals and there is extent analysis values Mi j. So after completing this inverse and then multiplication with this expression, so this will give us the synthetic extent values. So now that we are already familiar with the fuzzy algebraic operations so how this is going to be computed because all these numbers are going to be TFNs.

So we look at this first this expression, so it can be computed like this. So three parameter first parameter corresponding to L is going to become summation of j=1 to m li j and then the second parameter is going to become summation of j=1 to m mi j. So this summation is over m goals. So this S values that we are computing is you know, for each object. And then a third parameter summation j=1 to m ui j. So with this the first part of the expression would be computed and then second part the inverse part of the expression can be computed like this.

So fuzzy inverse of algebraic operation we have already talked about, right. So this is going to be, we use the same formula that we have discussed in this lecture. So we use this, then this would be the expression first by now 1/u related component here. So this summation would be would come as is, so first summation in the denominator is going to be summation over i=1 to m then summation over j=1 to m and then ui j. In the second parameter is on m and then the third parameter is on l.

So the algebraic part how this is being expressed, so we have already discussed. So with all these summation also it is going to be expressed in the similar fashion because as you remember the fuzzy addition and fuzzy multiplication also we are familiar with. So therefore, the you know, this expression can be extended in this fashion, given our understanding of fuzzy addition and fuzzy multiplication and also the inverse; fuzzy inverse that is being taken here.

So after the computation that are been done here, we will get the fuzzy synthetic extent values here, right. So let us move forward.

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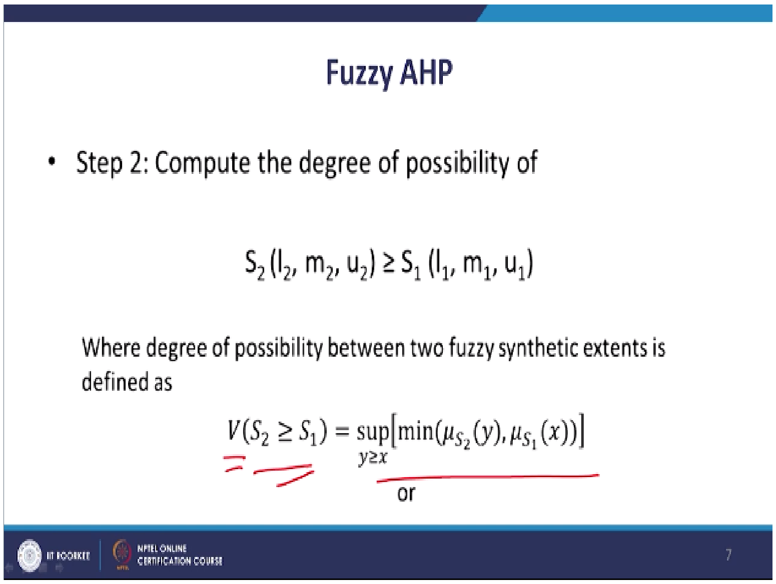

So once this is done then we move to step number 2. Compute the degree of possibility of S2 being greater than or equal to S1. So now again let us consider that we have two synthetic values fuzzy synthetic extent values S1 and S2 with respect to two objects, object 1 and object 2. Now we were to compare them and what is going to be in that comparison, what is going to be degree of possibility that S2 is going to be greater than or equal to S1?

So how do we computer that? So considering S2 is l2, m2 and u2 and S1 is l1, m1 and u1. So the degree of possibility between these two fuzzy synthetic extent is defined as this V S2 >= S1. So this value, so this possibility value is going to be given by this expression as you can see. So we are taking a minimum of this membership function value for Y and X. Mu s2 y and Mu s1 x. And this values then taken and then we take sup of it given that given for all elements of y where it is greater than or equal to x, because you see it is corresponding to our computation of an S2 to be greater than S1. So with this we will get this value.

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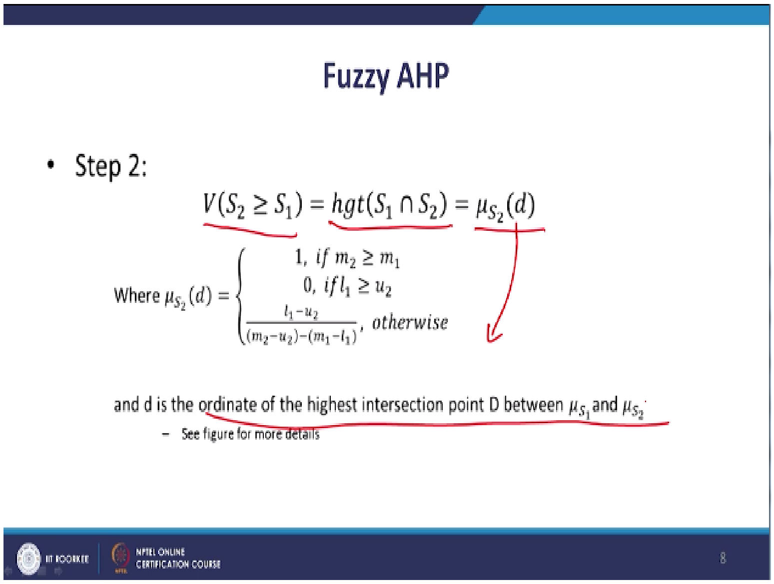

The same computation can also be performed in another fashion. So you can see here, same is been written here another form is this. So height of S1 intersection S2, so that will also give the same value, and Mu of Mu s2 of this d, this will give the same value, where Mu s2 of d is defined like this 1, if m2 is or equal to m1, 0 if l1 is u2 and otherwise l1-u2/this expression m2-u2 in parentheses then – then in parenthesis m1-l1.

So this is the membership function for S2, so it will convert into this and it will change into this and the values can be computed from here. So what we will get is the degree of you know, possibility between fuzzy extent S2 being greater than S1 that value will get. Where, this D that we are using here, this is the alternate of the highest intersection point D with the Mu s1 and Mu s2. So what do we mean by this?

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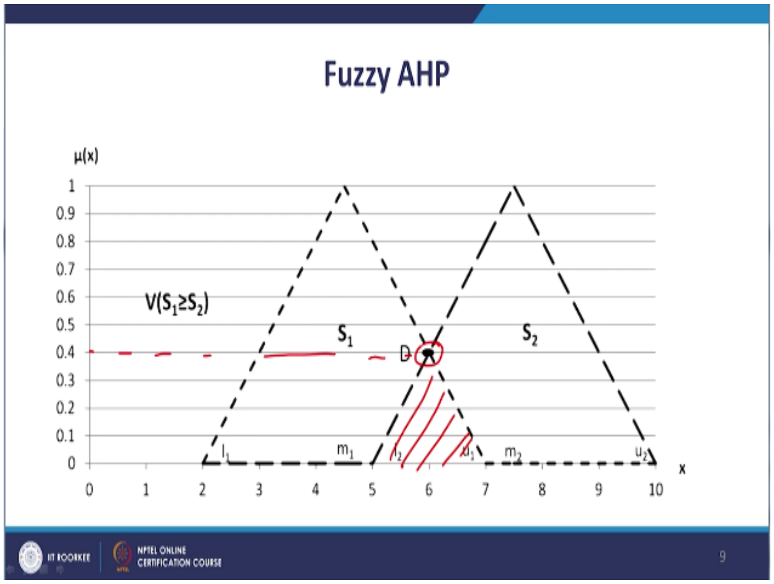

Whatever the mathematics that we are talking about, we look at in the graphic scenes so we can look at this diagram here. So you can see l1, m1 and u1 and because we are talking about fuzzy numbers so right, there is going to be overlapping. So this is the overlapped region here. And you can see l1, m1 and u1 and then l2, m2 and u2. Because you know, m1 being the most promising value but l1, u1 being you know, the small possible value and large possible value similarly for the second, second fuzzy number that is S2 as well.

So there is going to be this overlap so this intersection is going to be there. And D is this point of intersection and the; in the highest point of this intersection and when we are referring to Mu s2 d, this was actually this alternate value. So corresponding to this will get some value here. So that will give us the possibility, that will tell us about the possibility of this capital V, you know S1 being greater than or equal to S2. So this is what we will get. So the mathematics that we talked about here can also be understood through this graphic.

The steps that we are talking right now discussing right now they will be more clear, once we do an exercise in or where we will see through an example, each of these steps, the computation that we would be performing and what they mean, so we will elaborate more on this computations what they actually mean, more in the, more when we do exercise in our studio in our environment. Right now, focus is to understand the kind of computations that are, that are going to be performed under extent fuzzy AHP. So the step number 3 that is to be performed is.

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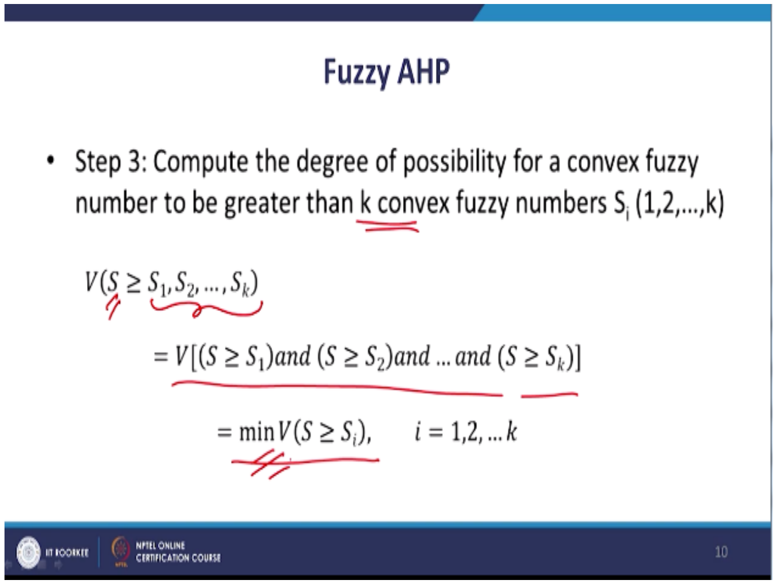

To computer the degree of possibility for a convex fuzzy number to be greater than k convex fuzzy number. If you see that, you know if you see that all the steps that we have talked till now, there are about certain computations that are to be performed and therefore the discussion is more in a generic sense.

The decision is on the mathematics part of it; how the mathematics, how the certain computations that are to be performed under extent fuzzy AHP, how these mathematics, how these underlying mathematics is expressed. And later on we will do in more; perform these steps more in a application sense, more in a through an example. So if we have to find out that a convex fuzzy number that is S and you know, whether what is the degree of possibility for it being greater than k convex fuzzy number.

So we are supposed to compute this then it can be written as this and this it can be written as this. So capital V of S greater than or equal to S1 and S greater than or equal to S2, so if it is individually if this is greater than with all other synthetic values then overall we can say it is greater than all convex fuzzy numbers. So if we take the minimum of this V being greater than or equal to Si where i could be 1 to k then that would actually indicate that this degree of possibility.

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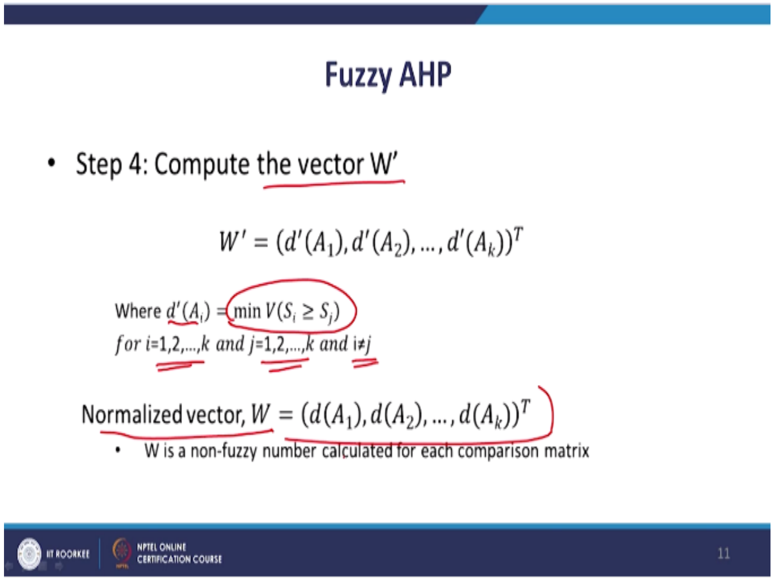

So having understood these kind of mathematics that would be required in this extent fuzzy AHP, now let us understand the last part which is the computer the vector, the weight vector. So we talked about different comparison matrix and later on the weights are to be computed. So here this W dash can be defined like this d dash A1, d dash A2 and d dash Ak and transpose of this.

So what is this d dash, small d dash Ai? So this is nothing but we are referring to previous step computation. A minimum of V Si greater than Sj, so one synthetic value being greater than the other one, so that is what we are referring to and that is being use to define the W dash. You can see here 1i is from 1 to k and j is from 1 to k and i should not equal to j. So this is like comparison being performed.

So we are talking about the underlying mathematics behind a comparison between two elements. So we were write to normalized vector form of this, it would become something like this. So this is W and this is normalized form will become a non-fuzzy number calculated for each comparison matrix. So we will get non-fuzzy numbers after doing this normalization. So with this we have covered the mathematics part of the extent fuzzy AHP.

So at every steps we have talked about certain mathematics that would be performed. So if we look at this you know, we take the linguistic variables to gather the you know, comparisons from the decision makers and those linguistic values then are converted into you know TFNs

Triangular Fuzzy Numbers. Now, once we have those metrics then we are supposed to perform certain number of computations to find out the weights and then the global and all those things.

Just like we did in traditional AHP the priority calculation that we are suppose to do, priority derivation. So similarly, here for that to happen given that we have the, you know comparison metrics what are the certain mathematics that are in computation that would be performed, so we talked about that. We referred synthetic, we talked about synthetic values; how they are going to be used to tell us about comparison between two, because there is going to be.

Because we are operating fuzzy environment and there is going to be overlap and how we can still indicate the whether a particular synthetic values and that fuzzy number is going to be greater than the other one and we you know, talked about the degree of possibility that capital V you know, of you know, one synthetic value being greater than the other one. Then we talked about this weight which is directly coming from that based on that degree of possibility.

So from that these weights are going to be computed. Then once we have these weights we can always go ahead and compute the; do our further computations. So all these we will understand more clearly through exercise and/or environment, so that we will do in the coming lecture. So we conclude here. Thank you.