

MCDM Techniques using R
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Lecture – 8
ELECTRE – Part I

Welcome to the course MCDM Techniques using R. So in previous few lectures, we talked about the AHP method that is Analytic Hierarchy Process. Now in this particular lecture, we are going to start our discussion on one another MCDM method that is ELECTRE. So let us start.

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ELECTRE

- ELimination Et Choix Traduisant la REalit´e
 - Meaning: elimination and choice expressing reality
 - Developed by Roy (1965, 1968)
 - Referred as ELECTRE
 - Belongs to the category of outranking methods
 - Based on pairwise comparisons of the alternatives
 - Considered to be a relatively complex method
 - Several technical parameters
 - Complex algorithm

So what we mean by ELECTRE. This in English actually means elimination and choice expressing reality method. So this was actually developed by Roy 65, 68 so that was the time the first version of ELECTRE method was actually proposed by him. So in brief, this is referred as ELECTRE and just like AHP this also belongs to the category of outranking school of thought, outranking methods. Just like AHP, this particular method also requires pairwise comparisons of the alternatives.

However, there are of course going to be certain advantages and disadvantages of ELECTRE that we are going to discuss in comparison to AHP also and other techniques. So this particular method ELECTRE is considered to be relatively complex method, the main reason being that a number of technical parameters are considered in this particular method and also the algorithm that is adopted is slightly complex in comparison to other MCDM methods.

So something that we have gone through AHP and once we discuss these steps through the procedure of ELECTRE method, then we will get to know that the complexity is a little on the higher side and the number of technical parameters which the decision makers need to specify they are on the higher side.

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ELECTRE

- ELECTRE
 - Advantages
 - Avoid compensation between criteria
 - Any normalization process, which distorts the original data

 - Different ELECTRE methods were developed
 - To solve different types of decision problems
 - Choice, ranking, sorting etc.

However, given those points, there are certain advantages of ELECTRE. So one is that avoid compensation between criteria. So any compensation effect something that we might have to do in other techniques, that is avoided here in ELECTRE, so that is one advantage. The second being if any normalization process which can actually distort the data, so something similar to what we discussed in AHP where we said that if introduction or removal of certain alternative, the denominator in the normalization process might change in the distributive mode that we talked about in previous lecture.

So therefore, there we talked about that ideal mode could be adopted where the best alternative could be used as the denominator in the normalization process, so this whole processes can actually be avoided in case of ELECTRE method and also the compensation between criteria. So typically in AHP also, one criteria is compared with others so that there is going to be a compensation effect, we will have to rate higher some criteria and others are rated lower. So that kind of thing is also avoided in this particular method.

Now different ELECTRE methods were developed depending on the type of problem that is being addressed; choice problem, ranking problem, sorting problems. So for all types of

problem, different ELECTRE methods are available, and depending on the problem that we are taking, we will have to pick the method appropriately. So let us move forward and discuss few important aspects when to use ELECTRE.

So given that we have already discussed AHP and given our discussion in the introductory part of this course where we talked about that it is slightly difficult for us to find out among all the MCDM techniques which one is going to be more suitable or more appropriate for a given decision problem, so this is a generic issue in the MCDM techniques, MCDM methods. However, we are going to discuss certain points specific to ELECTRE and that will give us pointer in terms of when ELECTRE as a MCDM method can be used.

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ELECTRE

- When to use ELECTRE?
 - More than two criteria
 - If criteria are expressed in different units
 - Compensation effect between criteria is not to be tolerated
 - If indifference and preference thresholds are required
 - To ignore insignificant small differences, but to allow sum of small differences
 - To handle imprecise or uncertain data
 - If alternatives are to be evaluated using ordinal or interval scale

So more than two criteria, then of course ELECTRE we can use. If the criteria are expressed in different units, so one particular criteria is being measured in duration, the another one is being measured in some other unit weight or something, the another criteria is being measured in meters or centimeter so that kind of scenario is there. If the criteria are being expressed in different units, then ELECTRE method can be suitable.

Because of the way the steps are adopted, the way underlying mathematics is performed, this particular problem can be overcome. The compensation effect between criteria is not to be tolerated. So in the previous slide we talked about that one of the advantages of this particular method that is ELECTRE is that we can avoid the compensation between criteria. So the same thing has been put in another way here that if we don't tolerate the compensation

effect between criteria, then also we can go for EELCTRE that being it's one of the main advantage.

Another situation could be when indifference and preference threshold are required. When we do comparisons between alternatives then if we have certain boundary limits, certain conditions, certain thresholds that we would like to use in making those comparisons, then that kind of scenario also we can go for ELECTRE because it specifically asks for these kind of parameters, indifference and preference thresholds. So sub-points are also mentioned here, for example to ignore insignificant small differences.

So if there are two alternatives and the difference in the performance of those alternatives is small something that as a decision maker we would like to ignore, then we can specify that in terms of indifference threshold. Similarly given the difference in performance of two alternatives if it is higher than a certain value, then we would like to make that as very clear preference, so that is something which can be specified using preference threshold. So the points are to ignore insignificant small differences but to allow sum of small differences or higher differences.

So if we would like to ignore small differences between alternatives and if we would like to allow higher differences, then these kinds of thresholds can be useful. Another situation could be to handle imprecise or uncertain data. So if there could be the data the numbers in terms of performance of the alternative if some impreciseness is there, some uncertainty is there, then because of these two parameters and other parameters as well indifference and preference threshold, you can easily get away from that kind of situation.

Because the way the methods and these parameters are being used as part of those steps, we can get away with some level of impreciseness because indifference threshold is there. So if there is little difference between two alternatives, then still we can make comparisons and so those kind of things are allowed and therefore we can handle imprecise or uncertain data in this particular technique. Another scenario could be if alternatives are to be evaluated using ordinal or interval scale so if comparisons, for example AHP we talked about that.

The scale has to be ratio scale so that is mainly because the way comparisons are done and the way underlying mathematics work it requires usage of ratio scale. So the alternatives or

criteria that are there should be compared using ratio scale. However in case of ELECTRE, we can use ordinal and interval scale. What we mean by ordinal or interval scale is something if you are not clear about what is ratio scale, ordinal scale or interval scale, you can refer to my previous course on NPTEL that is Business Analytics and Data Mining Modeling using R.

There I have clearly defined these scales ordinal, interval, ratio scales and I have also given certain examples over there which you can refer to get more clarity. Briefly, ordinal scale means where we are just looking at the order and the difference between two particular elements is actually not meaningful in that particular scale. Interval scale is something where the difference between two elements is meaningful; however, the concept of absolute zero is not there, so certain operations are allowed, certain operations are not allowed.

So that all depends on the kind of scales that we are using. So more details on difference types of scale, you can refer to my other course that is Business Analytics and Data Mining Modeling using R. So let us move forward. So now we are going to talk about different ELECTRE methods that are available and the method which have been developed for particular type of decision problem. So, we will start with choice problems.

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ELECTRE

- ELECTRE methods for choice problems
 - Select a smallest subset of best alternatives from a given set of alternatives
 - Accepting the possibility of incomparable alternatives
 - ELECTRE
 - ELECTRE I
 - ELECTRE Iv
 - Veto concept:
if an alternative performs badly on a single criterion compared to another alternative, the alternative will then be considered as outranked, irrespective of its performance on the other criteria
 - ELECTRE Is
 - Pseudo-criteria:
If a decision maker does not have a preference between two alternatives w.r.t a criterion, if the difference in their performance is smaller than the indifference threshold, or if the difference in their performance is higher than the preference threshold

So ELECTRE methods for choice problems. Though we have already talked about what we mean by choice problem in the introductory part of the course, however, let us again reiterate it in the context of ELECTRE. So by choice problem, we mean that we are supposed to select a smallest subset of best alternatives from a given set of alternatives. So why we are not

saying best alternative because under outranking school of thought something that we discussed in the introductory part of the course also that we might arrive at incomparable alternatives.

So sometimes the comparisons we might get the equivalent alternatives which cannot be further compared, so we can reduce to a smaller set of alternatives so that is what we are referring here selecting a smallest subset of best alternatives from a given set of alternatives. So that is what we are referring to here as a choice problem, specifically in this outranking school of thought.

So the sub-point also talks about this particular fact that accepting the possibility of incomparable alternatives so that is why smallest subset of best alternatives. So the ELECTRE methods which can be used to solve these kind of decision problems choice decision problems is the ELECTRE that is the first time this method was proposed, then ELECTRE I advanced version of it, then ELECTRE Iv where v actually stands for the veto concept which was introduced in this particular version of the method.

So what do we mean by veto concept, briefly few details are given here in the slide. If an alternative performs badly on a single criterion compared to another alternative, the alternative will then be considered as outranked irrespective of its performance on other criteria. So out of given set of criteria if even for one criterion a particular alternative performs badly in comparison to another alternative, then it would be considered outranked irrespective of its performance on other criteria.

So even though you might feel that some sort of weighted sum of the performance of alternative on other criteria might compensate, but we are not considering that compensation effect under ELECTRE method, so that is what we refer to in the previous slide of this particular lecture when we said the one of the advantage of ELECTRE method is that if you want to avoid the compensation effect between the criteria, then this is the method and the same thing is being indicated through the concept of veto.

So even if in a single criterion if alternative performs badly, then it would be considered outranked and therefore its performance on other criteria is not going to compensate for its bad performance on even one single criterion. So this is the ELECTRE Iv method. Then the

next one is ELECTRE Is version of this method, so here s is actually standing for pseudo-criteria. So what do we mean by pseudo-criteria, few pointers are indicated here.

If a decision maker does not have a preference between two alternatives with respect to a criterion, so that is one situation where we can use this particular version of the method; if the difference in the performance is smaller than the indifference threshold, so this is another scenario. If the difference in their performance is higher than the preference threshold, so in the given preference and indifference threshold if the performance is higher than the preference threshold, then also this method can be used.

The performance is smaller than the indifference threshold, then also this particular method would be suitable, and if does not have a preference that means incomparable alternatives, in that case also this concept of pseudo-criteria can be used and therefore this version of the ELECTRE method could be suitable. So let us move forward.

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ELECTRE

- ELECTRE methods for ranking problems
 - Produce a partial preference order on a set of alternatives
 - Accepting the possibility of incomparable alternatives
 - No scores are assigned
 - ELECTRE II
 - ELECTRE III
 - Pseudo-criteria
 - Outranking degrees:
Instead of binary outranking relations produced in ELECTRE II
 - ELECTRE IV
 - Relative importance of criteria is not needed

So now let us talk about the ELECTRE methods which are suitable for ranking problems to solve ranking decision problems. Now what do we mean by ranking problems, something that we have already discussed in the introductory part of the course. Now let us again understand it in the context of ELECTRE, so to produce a partial preference order on a set of alternatives. So because this is about ranking, therefore we need to have some sort of preference there, but the preference might not be complete.

So this is also a particular issue with the outranking school of thought, something that we have talked about in the introductory part of the course that we might not get the complete ranking and therefore the same thing is indicated here that produce a partial preference order on a set of alternatives. So again this particular way of defining this ranking problem in the context of ELECTRE is also accepting the possibility of incomparable alternatives.

So just like we discussed before, there could be incomparable alternatives and therefore the complete ranking might not be produced and therefore we are saying that partial preference order and no scores are assigned in this ELECTRE method. So what are the different ELECTRE methods which can be used for ranking problem. So first one is ELECTRE II. So if you remember you can see in the previous slide ELECTRE I, this was for the choice problem, ELECTRE II is for the ranking problem.

Then ELECTRE III and ELECTRE IV, they were also for ranking problems; however, certain new additions and certain new concepts were introduced in ELECTRE III and IV. So let us talk about them in brief. So in ELECTRE III which is considered to be the most popular most used ELECTRE method for ranking problems, two concepts were introduced. One is pseudo-criteria something that we talked about in the context of choice problem and the second one is outranking degrees.

So ELECTRE II, the previous version of ELECTRE method for ranking problems, it used to produce a binary outranking relation; however, in the ELECTRE III version, we produced outranking degrees. So we will talk about what we mean by outranking degrees, we will talk about later in this lecture or in the coming lecture. So when we discuss the method ELECTRE III in more detail, then we will talk about these particular concepts as well in more detail.

If we talk about the next version of ELECTRE method the ELECTRE IV for ranking problems, then relative importance criteria is not needed in this particular version. So in this particular version you can even avoid the relative importance of criteria right, so in those situations also, something that we discussed that imprecise or uncertain data, even if we have that kind of information that kind of data even then ELECTRE method can be used.

The similar thing is indicated here in the context of criteria where relative importance is not needed for ELECTRE IV. Now let us talk about ELECTRE methods for another type of problems that is sorting decision problems.

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ELECTRE

- ELECTRE methods for sorting problems
 - To independently assign a set of alternatives into one or several predefined categories
 - Similar to a supervised classification method:
A preference relation amongst the categories is produced
 - ELECTRE-Tri (ELECTRE-Tri-B)
 - ELECTRE-Tri-C

So what do we mean by sorting problems. So typically what we mean is that to independently assign a set of alternatives into one or several predefined categories. So independently given a set of alternative, we will like to assign each of those alternatives into a predefined set of categories, this seems to be something very similar to what we do in supervised classification method. If you want to understand what we mean by supervised classification method, then again you can refer back to my course on Business Analytics and Data Mining Modeling using R.

So there in the supervised classification method, typically we have dependent variables and independent variables, and for the dependent variable, it is going to be a categorical or nominal or ordinal variable and it will have number of categories or classes and we would like to classify the records into one of those categories. So the techniques which are available and which could be used to perform this kind of classification, they come under the category of supervised learning methods.

So when we say for sorting problems that we are going to independently assign a set of alternatives into predefined categories right, so that scenario is very similar to what we do in supervised classification; however, a preference relation amongst the categories is produced,

so that is slightly different, in terms of the overall sense it is similar to supervised classification, but here we get the preference relation.

The ELECTRE methods which could actually be used to solve sorting decision problems are ELECTRE-Tri also called ELECTRE-Tri-B and ELECTRE-Tri-C, these are the methods which can be used. So what are the inputs that we require from decision makers to implement our ELECTRE technique. So few things have been discussed here. So remember in this lecture in the first slide we talked about that the ELECTRE method is considered to be a slightly relatively complex method because of the number of technical parameters that are required to be specified by decision makers.

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ELECTRE

- ELECTRE requires decision makers
 - To specify various technical parameters
 - Therefore, process and outcomes of ELECTRE methods are difficult to explain and justify
 - Automatic elicitation
 - Decision makers provide a clear ranking of alternatives
 - Criteria weights and thresholds are inferred

Issue: decision maker's inconsistencies or contradictions may lead to reevaluation of the judgements

So now, we are going to talk about some of those aspects here. So in ELECTRE, we require decision makers to specify various technical parameters. Therefore, the process and outcomes of ELECTRE methods are slightly difficult to explain and justify because if the parameters are produced as an outcome of a series of steps or processes or algorithm, then people can easily understand okay how these parameters are being derived or being deduced.

However, because these technical parameters are being specified by decision makers, therefore we typically depend on the perceptions of decision makers while specifying these parameters, therefore it becomes difficult to explain the ELECTRE process and outcomes and justification also becomes slightly difficult. Now since there are a number of numerous technical parameters are to be specified, therefore certain researchers have developed automatic elicitation methods which could be used to actually specify these parameters.

So what happens in automatic elicitation is that decision makers are asked to provide a clear ranking of alternatives. So based on their perception of alternatives, they are asked to provide a clear ranking and based on the ranking that is provided the criteria weights and threshold. So about few threshold we have talked about while discussing different ELECTRE methods like indifference threshold, veto threshold and preference threshold.

Those are the main thresholds that are part of ELECTRE methods and of course the criteria weights. So these criteria weights and threshold then they are inferred from this ranking. So even before we can produce a ranking or choice using the ELECTRE methods, we asks decision makers to provide there a clear ranking among the alternatives and that is then used to actually infer the criteria weights and threshold. This is what we do in automatic elicitation process.

Now what is the problem with this approach is that the decision maker's inconsistencies or contradiction may actually lead to reevaluation of the judgments. Because as we have talked about one problem with the decision making in the general sense is that humans, often they are inconsistent just like we talked about, we sighted the example of IPL cricket where a team sitting at the top of the table might lose to a team sitting at the bottom of the table, so that kind of inconsistency can always be there in any effort, which involve basically mainly humans.

So similarly, decision makers they might have their own inconsistencies or contradiction. Therefore, they might go back on some of the preferences that they might have given, therefore it might lead to reevaluation of the judgments.

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ELECTRE

- ELECTRE III
 - Most used ELECTRE method for ranking
 - Two phases
 - Construction of outranking relations between the alternatives
 - Inputs from decision maker is taken in this phase
 - Preference order is produced using the outranking relations
 - In general, preference directions of all criteria are taken to be increasing
 - All the criteria have to be maximized

Now we are going to start our discussion on one of the most used ELECTRE method for ranking that is ELECTRE III. So let us start. So how ELECTRE III is actually conceptualized. So there are two phases which are used to implement ELECTRE III technique. So let us talk about these two phases. So first particular phase talks about the construction of outranking relation between the alternatives, what we mean by outranking relation we are going to discuss later on.

So first phase is about construction of these outranking relations between the alternatives. So for this construction of outranking relation, we take inputs from decision makers. So the inputs from decision maker is typically taken in the first phase itself and based on that, construction of these outranking relations is done. Then in the second phase, the preference order is produced using the outranking relations.

So whatever outranking relation that we have constructed in the first phase, they are later on exploited in the second phase and a preference order is produced. Another aspect of this ELECTRE III method is that in general, something that we have discussed in the previous slides as well, to avoid the compensation effect among criteria, so the related point is mentioned here that the preference directions of all criteria are taken to be increasing right.

So when we do pairwise comparisons of criteria in AHP so see there is going to be some sort of compensation, some of trade-off is going to take place over there. However, since preference direction in general they are taken to be increasing. So this is how we can actually avoid that compensation effect thing, trade-off thing and all the criteria have to be

maximized. So this is what we mean when we say that preference directions of all criteria are in general taken to be increasing, so actually we are looking to maximize all the criteria.

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ELECTRE

- Outranking relation
 - a outranks b
 - Denoted as 'a S b'
 - Means: Sufficient arguments support that 'a is at least as good as b' and There are no essential reasons to refute this
 - Outranking degree
 - Between a and b is computed
 - To 'measure' or to 'evaluate' the strength of the assertion on outranking relation
 - Denoted as $S(a, b)$
 - Value lies in $(0, 1)$: a value closer to 1 indicates stronger assertion

Now let us talk about the outranking relation. So this is in the first phase. As I said in the first phase, we typically construct the outranking relation. So let us talk about what we mean by it. So if we say a outranks b, so this is the typical assertion that we make in ELECTRE that a outranks b, this is denoted as a S b so as you can see in the slide. So what we mean is that there are sufficient arguments and sufficient justification are there in the place because of which we feel that alternative a is at least as good as alternative b.

There are no essential reasons to refute particular assert, then we can say that a outranks b. So to reiterate again when we say that a outranks b denoted by a S b, what we mean that we have arguments and logic to support that a is at least as good as b and there are no reasons to refute this. So therefore if that is the scenario, then we say that a outranks b. Then there is another aspect of outranking relation that is called outranking degree. So what do we mean by outranking degree.

So this is important aspect of our construction of outranking relations between alternatives. Given two alternatives a and b, so between a and b we are supposed to compute this outranking degree and this outranking degree is actually used to measure or evaluate the strength of the assertion on outranking relation. So if we say a outranks b and make that assertion as an outranking relation, how do we measure the strength of this assertion, so for that we need to compute outranking degree.

So this particular outranking degree is denoted as $S(a, b)$. Because this is a measurement, there is going to be a value, this value typically lies between 0 and 1. If the value is closer to 1, then it indicates stronger assertion. If the value is closer to 0, then of course it is going to be a weaker assertion. So with this much discussion, we would like to stop here and in the next lecture, we will continue our discussion on ELECTRE. Thank you.