Econometric Modelling
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Lecture No. 36
Simultaneous Equation System-II

This is Module 36 of the course on Econometric Modelling. We were discussing the model simultaneous equation system in module 35, and I further continue with the discussion on the problem of identification in this module and then further go to the discussion on estimation and inference.

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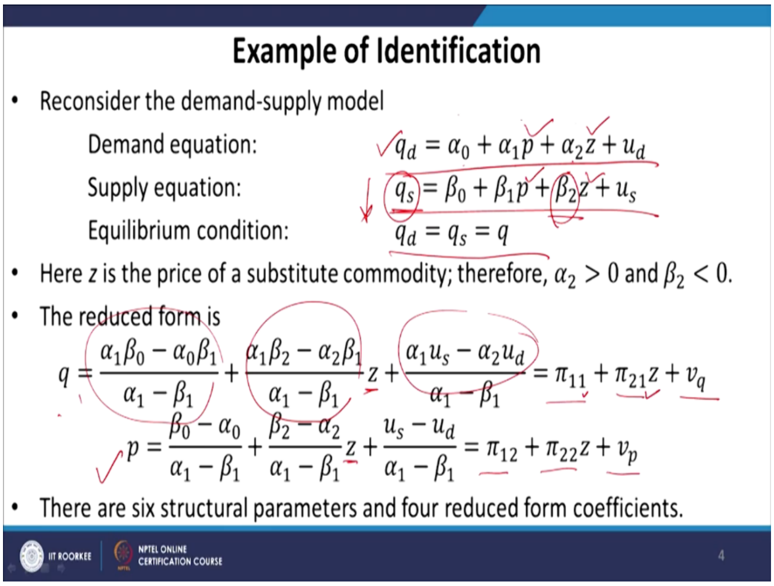

So, we first take an example of identification, we reconsider the demand-supply model that we had considered at the beginning of module 35 or when we started the discussion on a simultaneous equation system. So, this is a demand equation, this is a supply equation.

Now, you note that both these equations have one endogenous variable and the same exogenous variable. So (refer slide time: 1:12- 1:33). This implies that, if the price of the substitute commodity increases, then people will basically demand more of this commodity, and some of the demand for that particular commodity will be substituted by the demand for this commodity. On the other hand β_2 is expected to be less than 0, because if the prices of that substitute commodity is growing, then it is possible that producers or farmers would start producing this commodity more.

So, as a result of which, the supply of this commodity will go down, that is why we expect $\beta_2 < 0$. Now, the reduced form is, we actually obtain the reduced form when we express (refer slide time: 2:21- 3:00).

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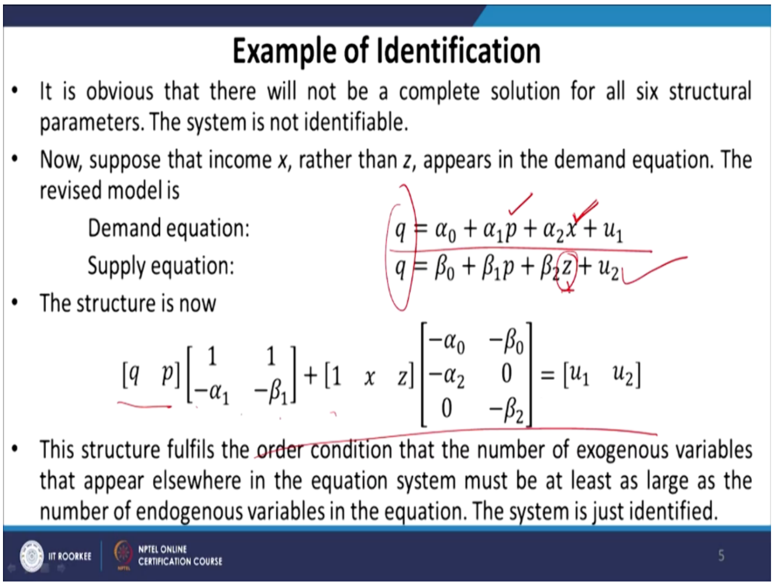

So, what happens? It is obvious that there will not be a complete solution for all 6 structural parameters the system is not identifiable. So, what do we need to do because the order condition is not fulfilled and that is why we are not able to identify the system?

Now, suppose the (refer slide time: 3:21- 3:55).

Essentially, I am writing this entire thing in terms of matrix form. Now, this structure fulfills the order condition that the number of exogenous variables that appear elsewhere in the equation system must be at least as large as the number of endogenous variables in the equation.

So, the system is just identified, because you can see that for both the equations what we have one exogenous variable which appears here but does not appear here and one endogenous variable. Similarly, one exogenous variable appears here, but not here, and one endogenous variable. So, that is why the system is just identifiable.

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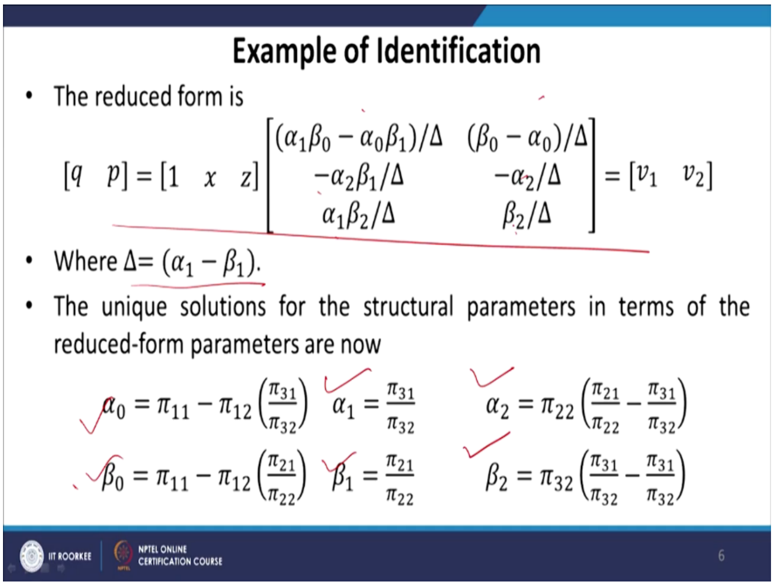

The reduced form is in terms of matrix form, we write it like this, (refer slide time: 4:42). So, this is how the system becomes just identified. Now, we talk about estimation and inference.

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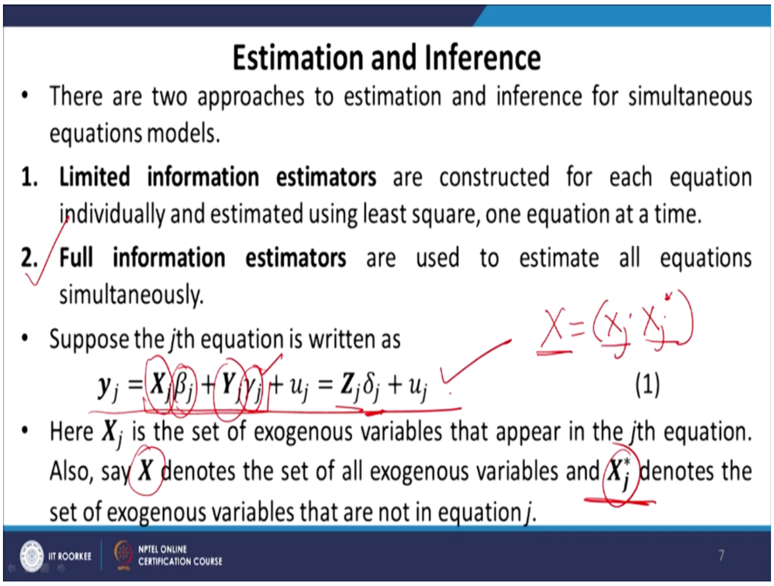

Now, there are two approaches to estimation and inference for the simultaneous equations model. The first one is limited information estimators, which are constructed for each equation individually and estimated using least squared one equation at a time. So, since we are not using all the information that is available to us, we call it limited information estimators. And the second is full information estimators that are used to estimate all the equations simultaneously. So, the complete set of information is being used, and that is why we call it full information estimators.

Now, suppose (refer slide time: 6:01- 7:34).

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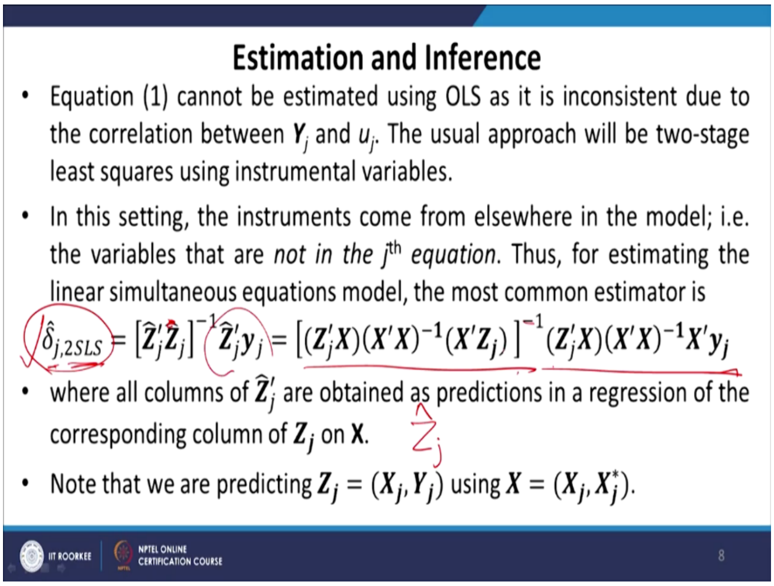

So, equation 1 cannot be estimated using OLS as it is inconsistent due to the correlation between Y_j and U_j the usual approach will be two-stage least squares using instrumental variables. So, we used instrumental variables in certain situations. So, once such a situation was when we expect that some of the exogenous variables are actually correlated with the error term.

So, this is a situation where some of the variables we do not expect. We know that they are correlated with the error term, so we need to use some instrumental variables. In this setting, the instruments come from elsewhere in the model that is the variables that are not in the jth equation can be utilized to come up with or to form the instrumental variables.

Thus, for estimating the linear simultaneous equations model, the most common estimator is denoted by (refer slide time: 8:30- 10:32).

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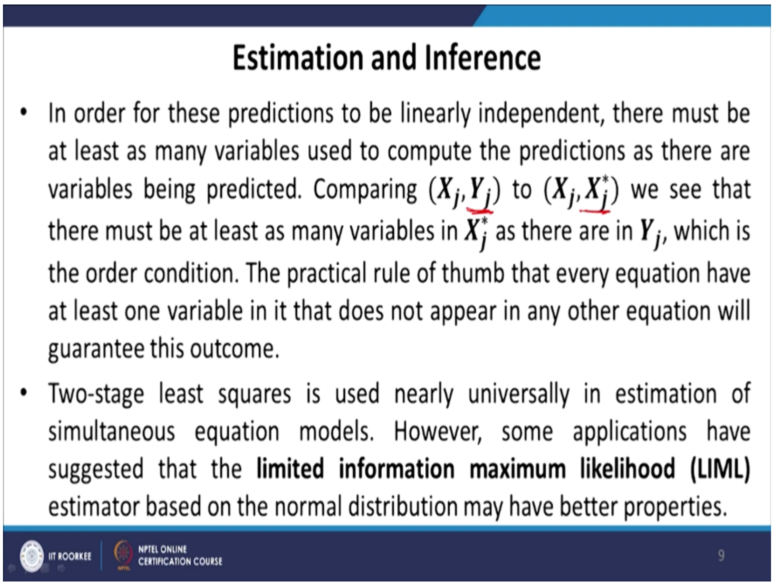

So, in order for the predictions to be linearly independent, there must be at least as many variables used to compute the predictions, as there are variables being predicted. So, when we compare X_j, Y_j to X_j, X_j^*, we see that there must be at least as many variables in X_j^* as they are in Y_j in order for them to be linearly independent. So, this is exactly the order condition. The practical rule of thumb that every equation has at least one variable in it that does not appear in other equations will guarantee this outcome.

So, two-stage least squares is used nearly universally in the estimation of the simultaneous equations model. However, some applications have suggested that the limited information maximum likelihood estimator based on the normal distribution may have better properties. Now, I am not going to discuss limited information maximum likelihood method or technique here in detail, because that is sort of outside the purview or scope of this course, this is a basic course. Instead, I will just discuss certain properties of a (LIML) that is limited information maximum likelihood method, what are the advantages it has over 2SLS if we can go for LIML.

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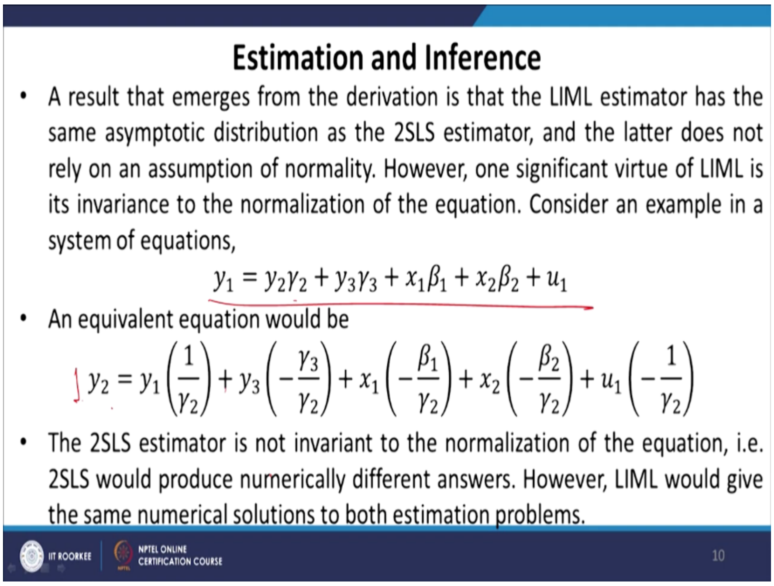

So, a result that emerges from the derivation is that the LIML estimator has the same asymptotic distribution as the 2SLS estimator, and the latter that is the 2SLS estimator does not rely on an assumption of normality.

However, one significant virtue of LIML is its invariance to the normalization of the equation. So, whenever we go for any kind of normalization, first of all, LIML results do not change with that. So, for example, we consider this equation, and the same equation is actually written while taking y to the left-hand side.

So, (refer slide time: 12:38). So, the 2SLS estimator is not invariant to the normalization of the equation, which implies that 2SLS would produce numerically different answers. However, LIML would give the same numerical solutions to both estimated problems. So, this is an advantage of LIML over 2SLS.

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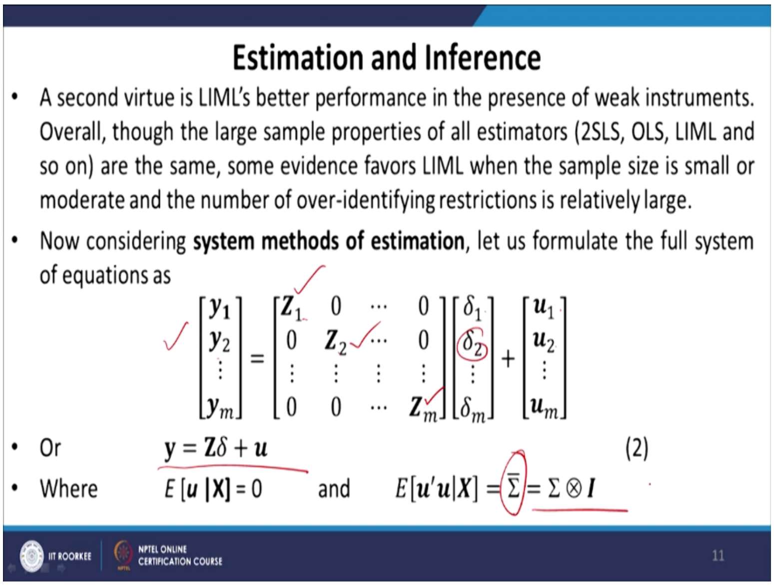

And the second virtue is that LIML's better performance in the presence of weak instruments. Overall, though, the large sample properties of all estimators that is 2SLS, OLS, LIML, and so on are the same, some evidence favors LIML when the sample size is small or moderate, and the number of over-identifying restrictions is relatively large. Then it is possibly observed that LIML gives us better results.

Now, we consider a system method of estimation that is the full information estimation technique. So, let us formulate the full system of equations, first of all, as (refer slide time: 13:52)

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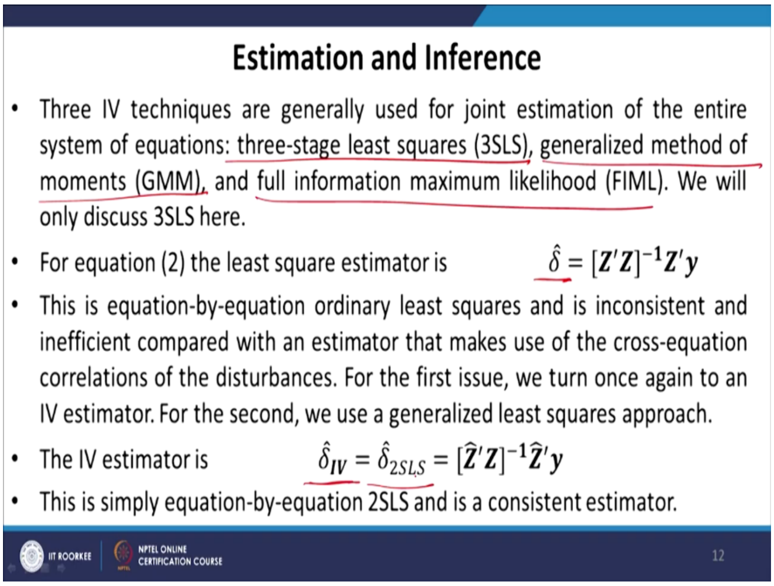

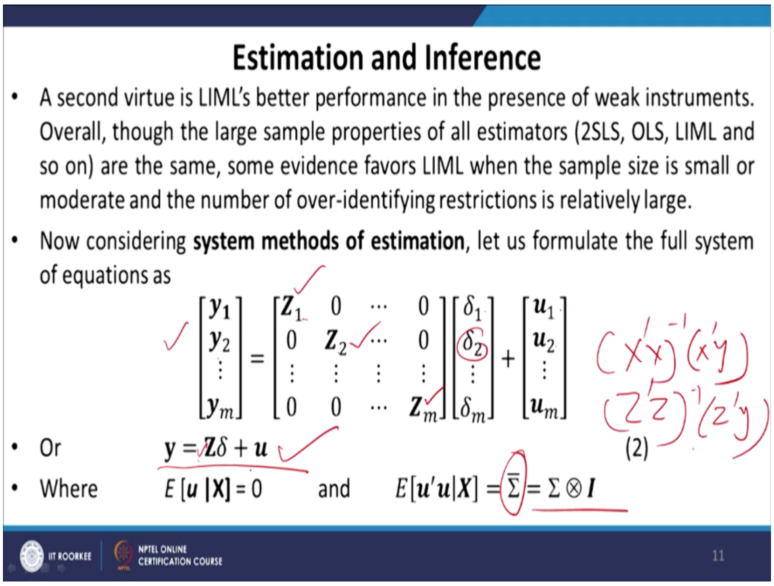

Now, the Three IV techniques that are generally used for joint estimation of the entire system of equations are three-stage least squares or 3SLS, generalized methods of moments (GMM), and full information maximum likelihood method or FIML. We will only discuss 3SLS here because the generalized method of moments and FIML not only there is time and space constraint, but also are much more complex estimation techniques and outside the purview of this course.

So, this is equation 2 and its least squares estimator. If you remember the OLS estimators (refer slide time: 15:55) this is equation by equation ordinary least squares and is inconsistent and inefficient. It is inconsistent because as you can see that Z consists of endogenous variables, and $Z\ and\ u$ are correlated, but still, we are going ahead with OLS, so that is why it is inconsistent. And previously we have also discussed that these, the system as a whole will produce inefficient estimators.

Now, this is inefficient compared with an estimator that makes use of the cross-equation correlations of the disturbances. For the first issue, we turn once again to an IV estimator. So, by considering the instrumental variable estimation technique, we will be first taking care of the inconsistency problem. And the problem of inefficiency will be taken care of by using a generalized least squares approach, which was also earlier discussed in the context of heteroscedasticity or serial correlation.

So, the IV estimator is (refer slide time: 17:19). This implies equation by equation 2SLS, and it is a consistent estimator.

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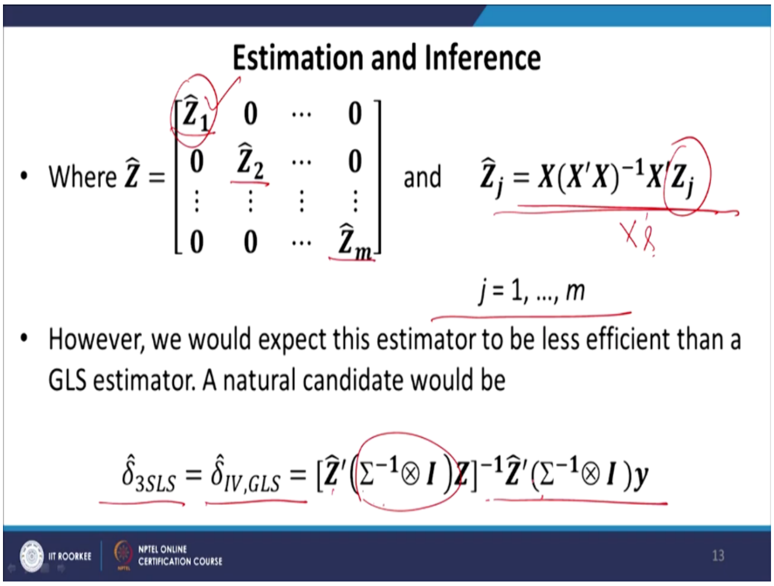

Now, where, you can see that (refer slide time: 17:45).

However, we would expect this estimated to be less efficient than a GLS estimator. So, our natural candidate would be to go for a third stage. So, that is how we have a 3SLS, that is the third one is denoted by $\hat{\delta}$ 3SLS, which is basically $\hat{\delta}$ IV, and we are applying GLS on it. If you remember when applying GLS what we used to do is that, in order to convert an error with constant variance, we would be dividing the entire equation or all the data by the non-constant component of the error term.

So, here, the error variance is (refer slide time: 19:16- 20:10).

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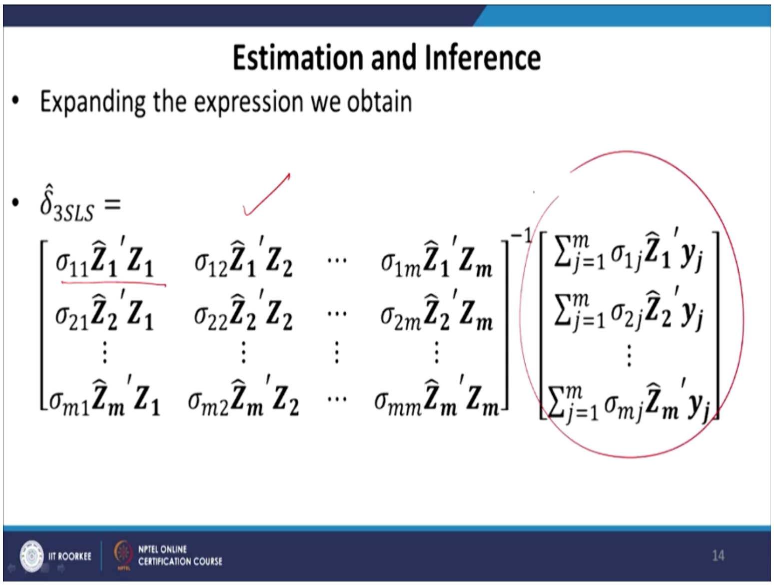

Now, expanding the expression, we will obtain (refer slide time: 20:13- 20:48).

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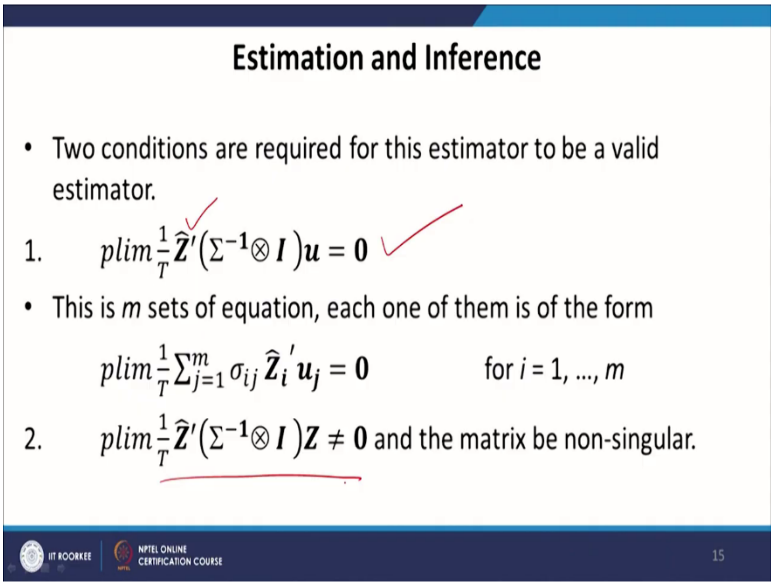

Now, two conditions are required for this estimator to be a valid estimator. Their first condition is that (refer slide time: 20:55- 21:24).

Now, ideally, this implies that the instruments that we obtain must be uncorrelated with the error term. And the second condition is that, this term should be invertible So, that this term should be not equal to 0 and the matrix be non-singular. If these two conditions are fulfilled then we can always obtain these three SLS estimators.

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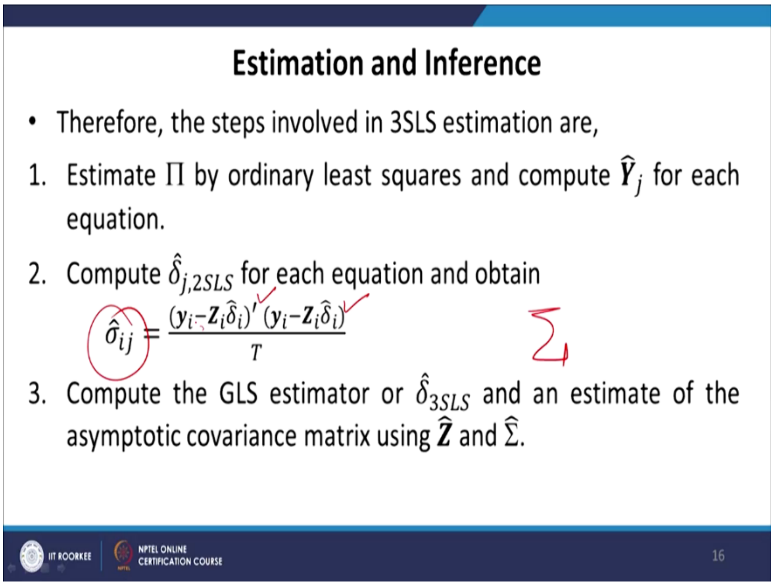

So, finally, the steps involved in the 3SLS estimators are first of all estimate π by ordinary least squares and compute $\hat{Y}_j$ for each equation. Then we compute (refer slide time: 22:14-23:14).

So, that is broadly about simultaneous equation systems. How do we tend to identify them? What are the methods of estimation and inferences that we generally go for? Now, most often these simultaneous equation systems, which are driven by structural assumptions require a lot of theoretical implications in order to make the system identifiable, and most often when the system is identifiable, then standard statistical packages can run this kind of regressions in order to produce the estimated values.

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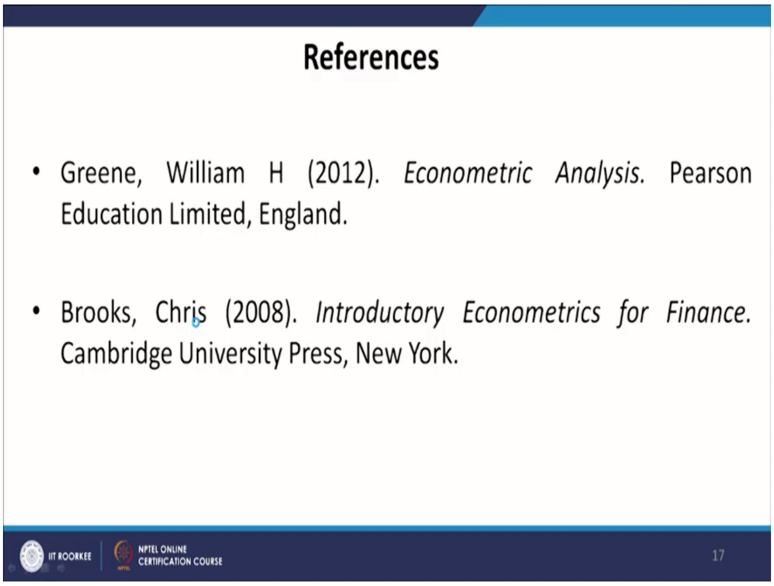

These are the references I have primarily used for coming up with a discussion on simultaneous equation systems. Thank you.