Econometric Modelling

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Lecture 39

Stationarity & Unit Root Testing - II

Hello everyone, this is the 39th module of the course on econometric modelling. So, in the previous module, we started discussing the problem of stationarity, we defined it, and we also discussed what kind of problem it poses to the data if the data is non-stationary. So, stationarity has been defined and its various properties are discussed.

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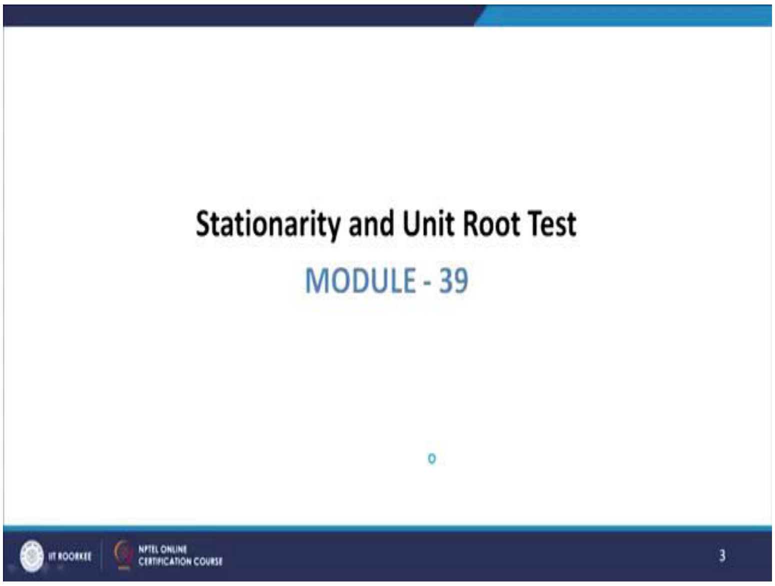

And in this current module, module 39, continuing with the discussion on stationarity and unit root test, we primarily are going to discuss the testing procedure.

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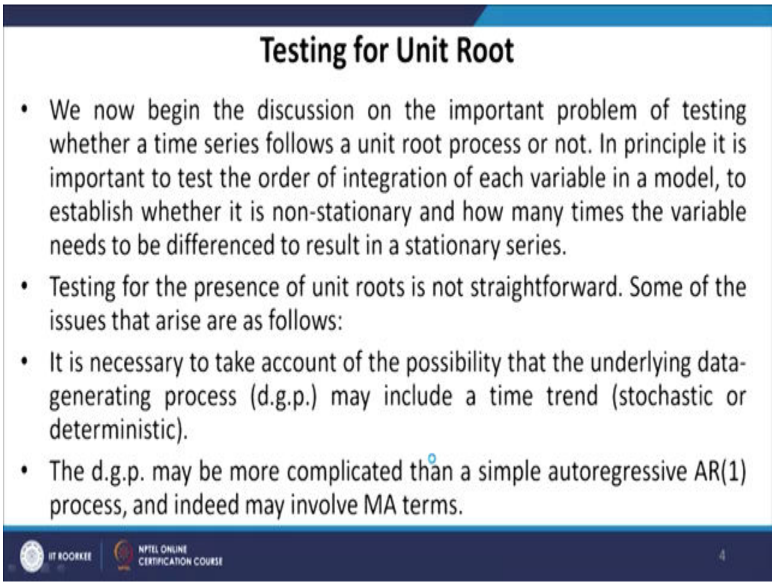

So, first of all, we begin with a discussion on the important problem of testing, whether a time series follows a unit root process or not. In principle, it is important to test the order of integration of each variable in a model to establish whether it is non-stationary and how many times a variable needs to be differenced to result in our stationary series.

Testing for the presence of unit roots is not straightforward. Some of the issues that arise are as follows: First of all, it is necessary to take into account the possibility that the underlying data generating process (d.g.p). may include a time trend that is stochastic, or deterministic.

The problem arises when it has a deterministic trend possibly because by testing for stationarity, we are going to test whether this stochastic trend is present or not. The second thing is that the d.g.p. may be more complicated than a simple autoregressive AR (1) process, and indeed may involve some moving average terms.

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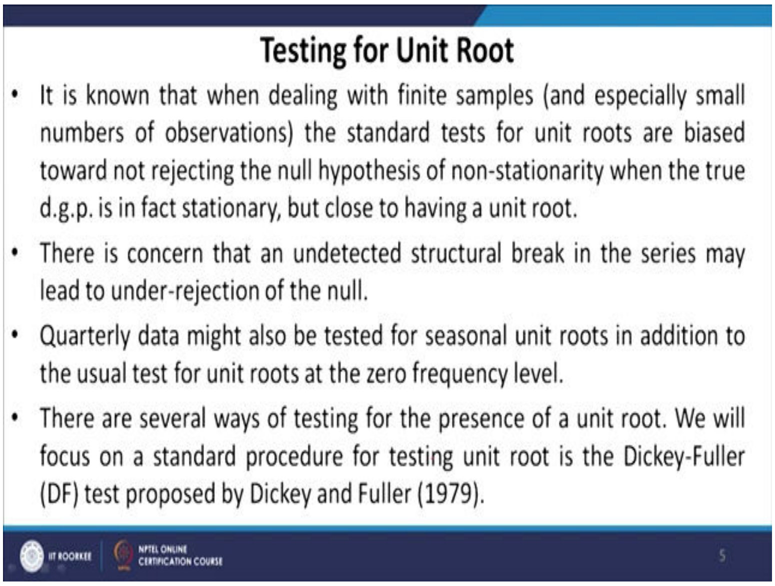

Third, it is known that when dealing with finite samples and especially a small number of observations, that standard tests for unit roots are biased towards not rejecting the null hypothesis of non-stationarity when the true d.g.p.is in fact stationary but close to having a unit root.

There is concern that an undetected structure break in the series may lead to under rejection of the null. That is, the null is not rejected when it should be ideally rejected. Quarterly data might also be tested for seasonal unit roots in addition to the usual test for unit roots at the 0 frequency level. There are several ways of testing for the presence of unit root; we will focus on a standard procedure for testing unit root is the Dickey-Fuller test proposed by Dickey and Fuller in 1979.

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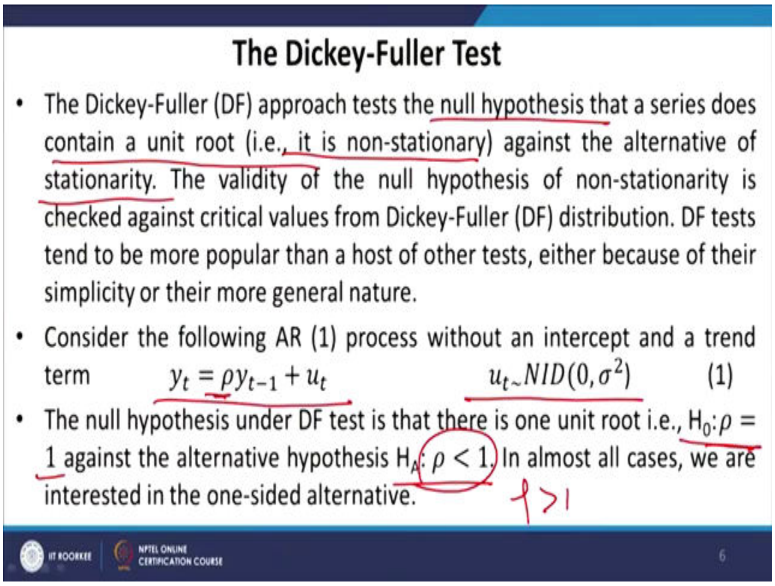

So, the broad discussion is around the Dickey-Fuller test. The Dickey- fuller approach tests the null hypothesis that a series does contain a unit root, that is it is non-stationary. So, the most important thing is that the null hypothesis is of non-stationarity, against the alternative of stationarity.

The validity of the null hypothesis of non-stationarity is checked against critical values from the Dickey-Fuller (DF) distribution. So, the critical values are given by Dickey-Fuller. DF tests tend to be more popular than a host of other tests, either because of their simplicity or their more general nature.

Now, consider the following AR(1) process, without an intercept and a trend term. So, this is an AR(1) process, which does not have either a constant term, that is intercept term, or a deterministic trend term, you also assume that the u_t, that is the errors are normally and independently distributed with a mean 0 and constant variance are sigma squared. The null hypothesis under the DF test is that, there is 1 unit root as already stated above that the null hypothesis is of non-stationarity and there is only 1 unit root for the time being.

So, this implies that we are testing $\rho = 1, H_0: \rho = 1$ against the $H_A: \rho < 1$. In almost all cases, we are interested in the 1 sided alternatives. The reason is that we know if $\rho = 1$, then it is an explosive series and it actually does not help us. So, the stability condition requires us to have $\rho < 1$ and that is why we test for $\rho < 1$.

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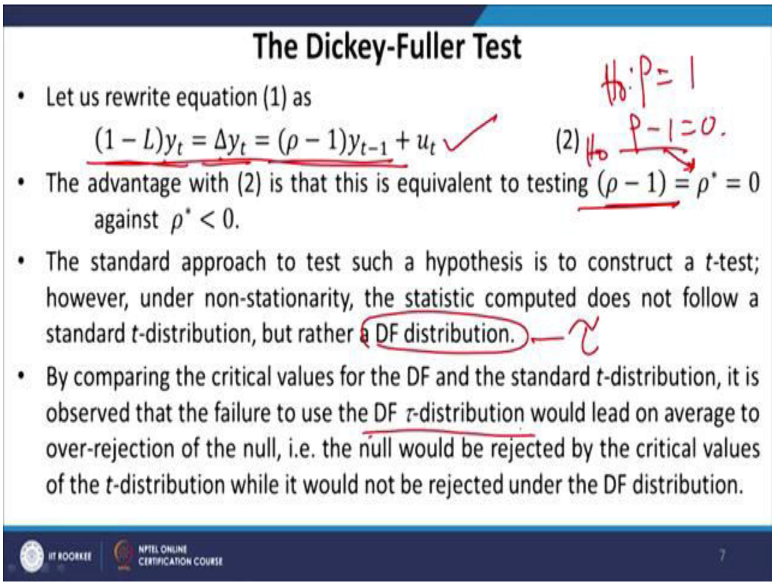

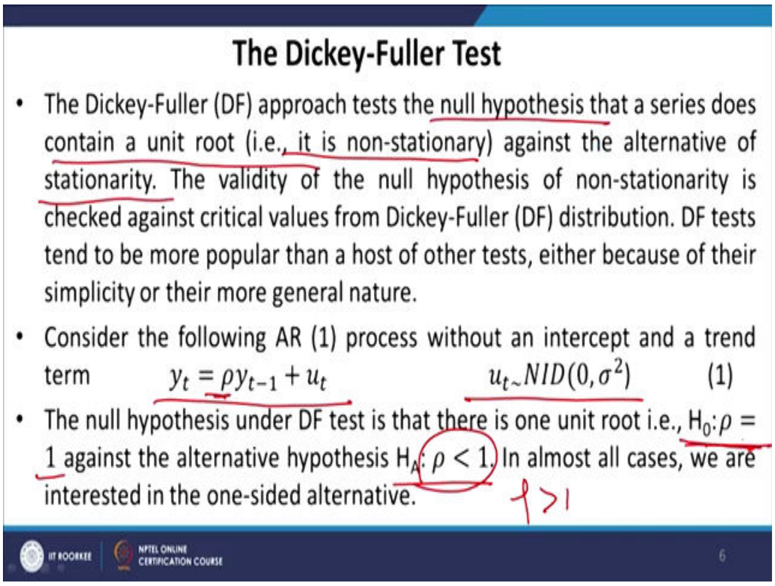

So, let us rewrite equation 1, in terms of the lag operator. So, we have (refer slide time 4:55-5:45). The standard approach to testing such a hypothesis is to construct a t-test. However, under non-stationarity, the statistic computed does not follow a standard t-distribution rather it follows a Dickey-Fuller distribution.

So, by comparing the critical values for the DF and the standard t-distribution, it is observed that the failure to use the DF τ-distribution (DF distribution for this kind of model specification is denoted by τ), would lead on average to over-rejection of the null, that is the null would be rejected by the critical values of the t-distribution while it would not be rejected under the DF distribution. So, if we follow the DF distribution, then we are going to reject the null hypothesis, while it should not be ideally rejected.

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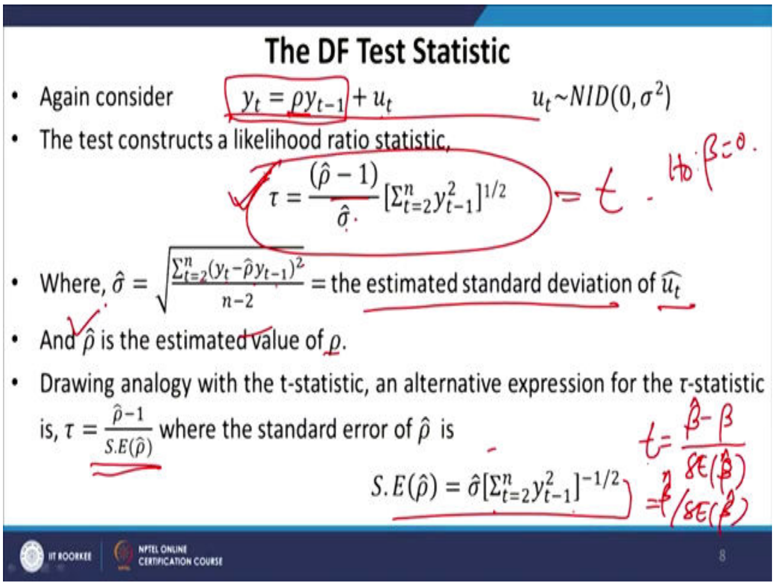

Now, again we consider, this specification AR(1) process, the test constructs a likelihood ratio statistic, the τ statistics look something like that. But, then ideally this is exactly equal to the t-statistic because you can see that, this (refer slide time: 7:03- 8:39). But yes, they do not follow the same distribution, this does not follow the DF distribution, if the series does not follow the t-distribution, if this series is actually non-stationary, it follows the DF distribution.

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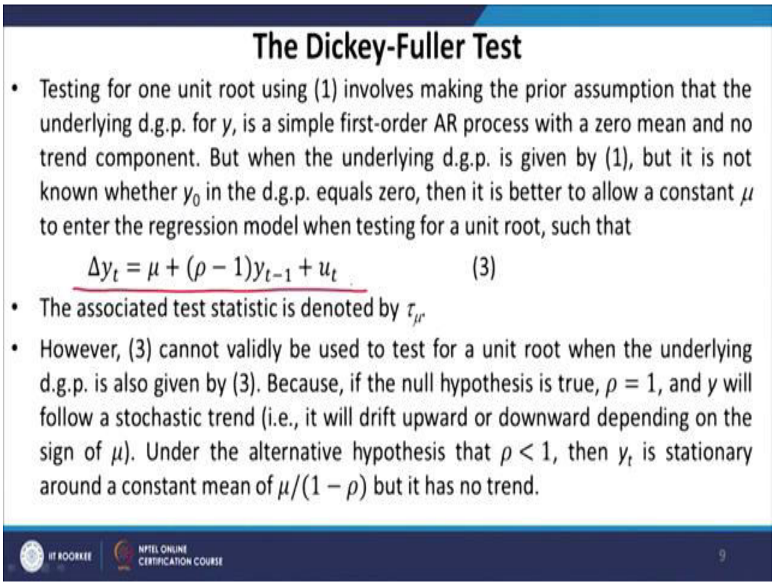

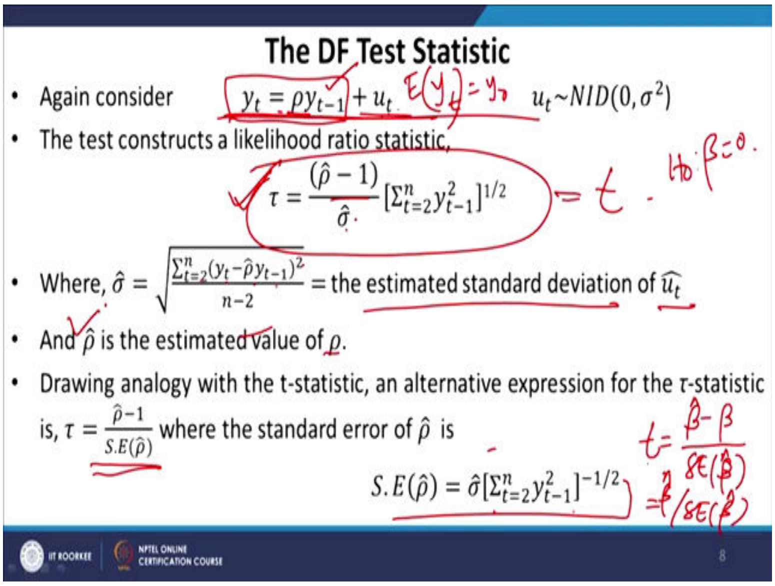

Testing for a 1 unit root using (1), involves making the prior assumption that the underlying d.g.p. for y, is a simple first-order AR process with a zero mean and no trend component. But, when the underlying d.g.p. is given by (1), but it is not known whether y_0 in the d.g.p. equals zero, then it is better to allow a constant μ to enter the regression model when use testing for a unit root, such that (Refer slide time: 9:30- 10:30).

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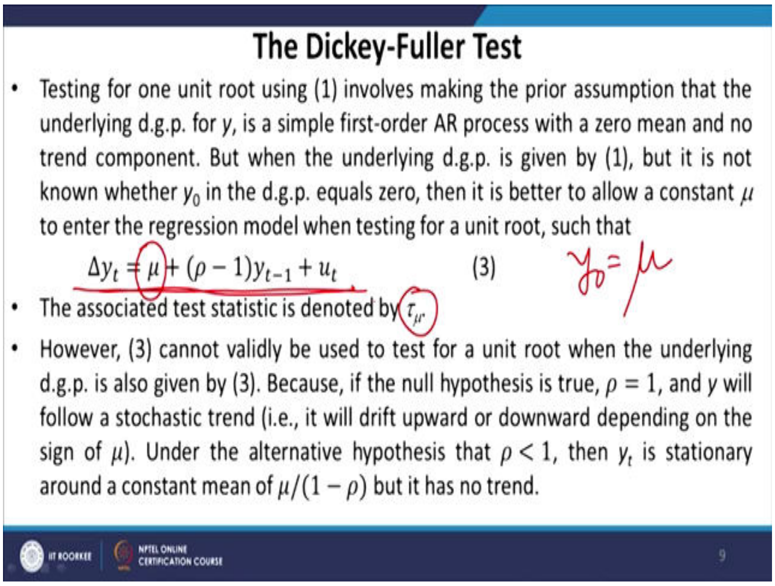

But, suppose if you are not sure about the fact that whether it is 0, or not, then we should ideally include a constant term here, and that any y_0 is denoted by μ and included in this expression. The associated t-statistic is denoted by τ_μ. So, if we are using a model like this, then the statistic is known as τ_μ.

However, 3 cannot validly be used to test for unit root, when the underlying d.g.p. is also given by 3, because if the null hypothesis is true, and $\rho = 1$ and y will follow a stochastic trend, that is it will drift upward or downward depending on the sign of μ. Under the alternative hypothesis, that $\rho < 1$, then y_t is actually a stationary series around a constant mean of μ, divided by $1 - \rho$, and it has no trend.

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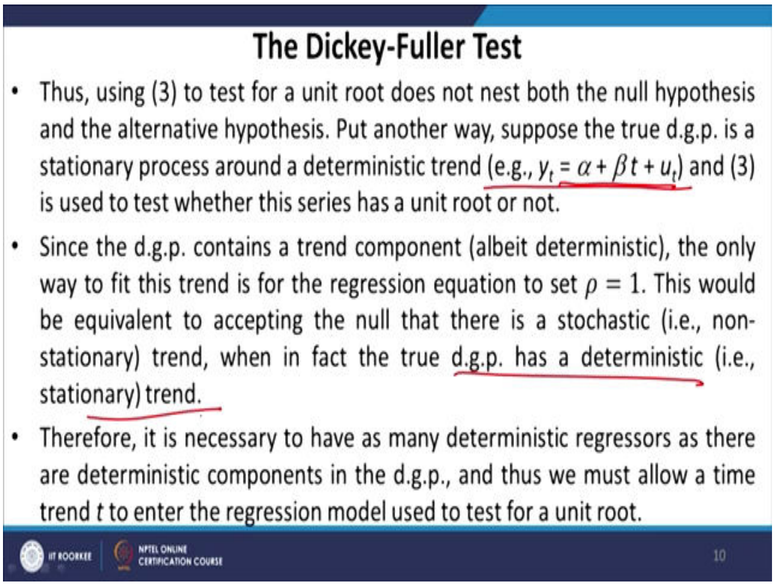

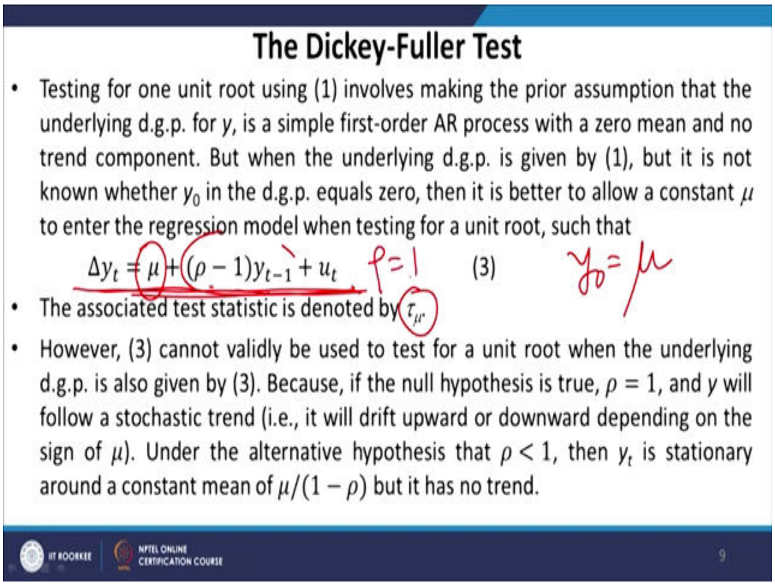

So, using (3), to test for unit root does not nest both the null hypothesis and the alternative hypothesis. This implies that the implications are both in, both the null hypothesis and the alternative hypothesis are not actually nested by the d.g.p. or the data generating process given in 3. If the null hypothesis is true, the d.g.p. would take a form, which would be different from the d.g.p. which would be taken up by the series, if the null hypothesis is not true.

Put another way suppose, that true d.g.p. is a stationary process around a deterministic trend. So, this is a stationary process with a deterministic trend and we are not even trying to find out whether the series is stationary or not, we know that this is the d.g.p. And 3, expression 3 is used to test whether this series has a unit root or not.

So, expression 3 does not have any deterministic trend component in it. Since the d.g.p. contains a trend component, despite it being a deterministic trend, the only way to fit this trend is for the regression equation to set $\rho = 0$, because this d.g.p. does not include a deterministic trend term. What the analysis would do is that, convert this stochastic trend term, or treat the stochastic trend term as a deterministic trend term, or a replacement for the deterministic trend term.

And this is what it would do by assigning a value 1 to ρ, and that is how we would assume that this series 3 is actually has a stochastic trend, but it is having a deterministic trend. This would be equivalent to accepting the null that, there is a stochastic that is a non-stationary trend, when in fact the true d.g.p. has a deterministic, that is a stationary trend.

Therefore, it is necessary to have as many deterministic regressors as there are deterministic components in the d.g.p. And thus we must allow a time trend t to enter the regression model used to test for unit root.

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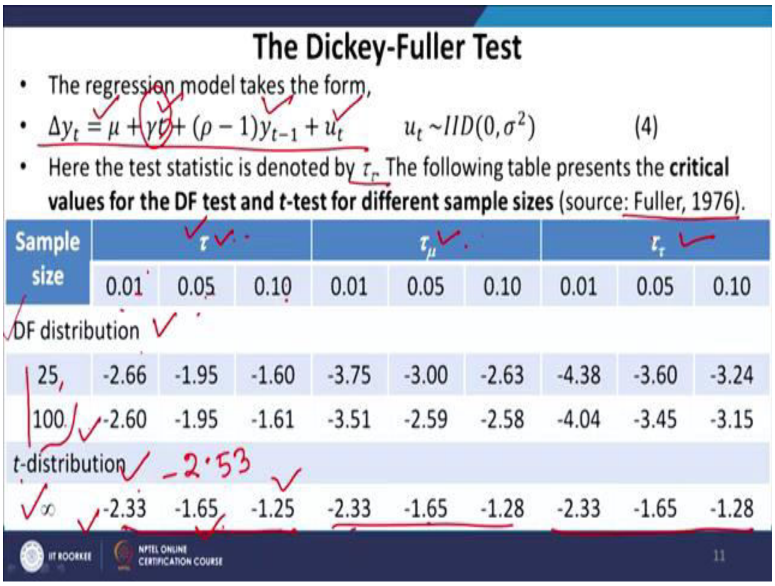

The regression model takes this form. So, you can see that it is a very general specification, when we have a constant term, we have a deterministic trend term, here we are testing for stationarity and of course a random error term.

Here the test statistic is denoted by τ_τ, that is when the model includes a deterministic trend component, then the corresponding test statistic is denoted by τ_τ. The following table presents the critical values for the DF test and t-tests for different sample sizes. This table has been extracted from the original distribution or critical values given by Fuller in 1976.

So, I have just considered two alternative sample sizes 25 and 100, under DF distribution, under t-distribution only a very large sample represented by infinity is considered, and for each type of you know the statistic, that is τ, τ_μ, and τ_τ, three significance levels are also considered 1 percent, 5 percent, and 10 percent. For τ distribution, of course, there are not different statistics that is why for all of them, these values 1 percent, 5 percent, 10 percent, figures are basically the same.

What you observe is that you can see the value of t-statistic is actually lower than all the values, all possible values under the τ-statistic, this itself implies that, if we reject the null hypothesis under that t-statistic, it is not necessary, that we will also be rejecting it under the DF distribution.

Because, suppose the statistic takes up a value of -2.53, then and it is the τ-statistic, when you can see that at 1 percent level of significance for a large sample like 100 units, the t-distribution rejects the null hypothesis, where DF distribution does not reject the null hypothesis. So, this is the problem with using the t-distribution.

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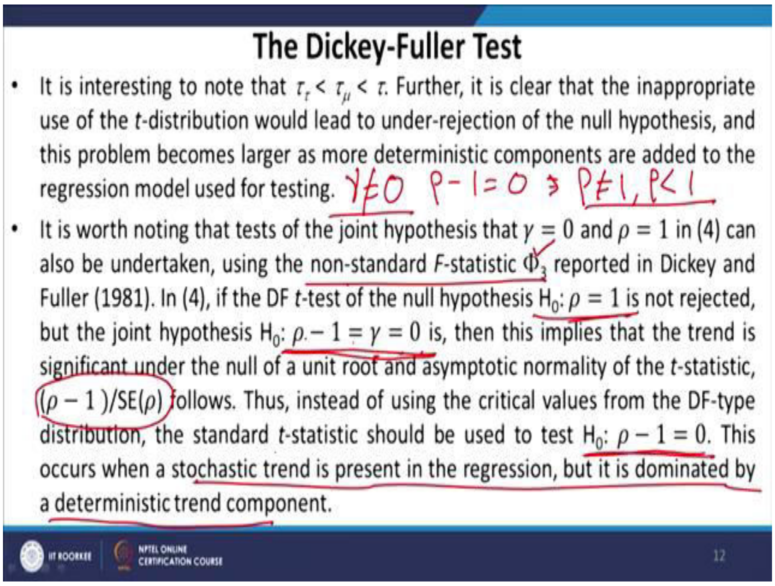

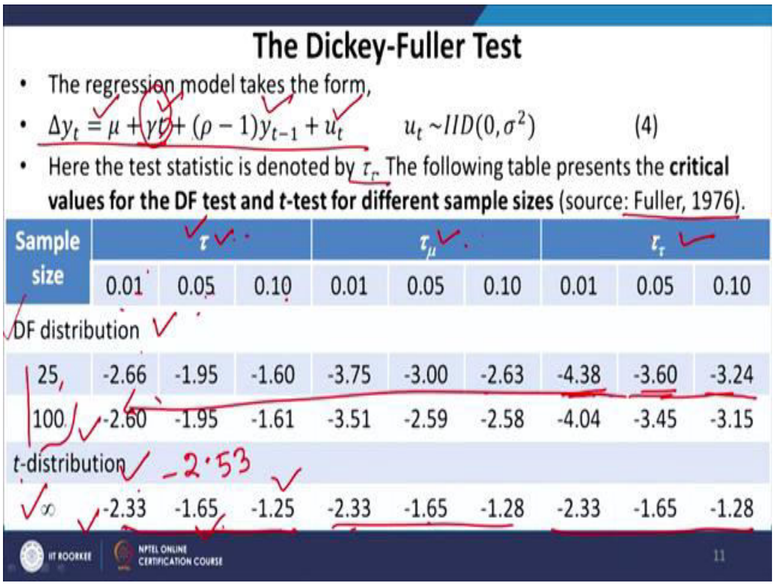

Sample size	τ			τ_μ			τ_τ		
	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
DF distribution									
25	-2.66	-1.95	-1.60	-3.75	-3.00	-2.63	-4.38	-3.60	-3.24
100	-2.60	-1.95	-1.61	-3.51	-2.59	-2.58	-4.04	-3.45	-3.15
t-distribution									
∞	-2.33	-1.65	-1.25	-2.33	-1.65	-1.28	-2.33	-1.65	-1.28

It is interesting to note that $\tau_\tau < \tau_\mu < \tau$, these are actually smaller numbers. So, you can see that gradually we are going up. Further, it is clear that the inappropriate use of the t-distribution would lead to under rejection of the null hypothesis, and this problem becomes larger as more deterministic components are added to the regression model used for testing.

It is worth noting that, tests of the joint hypothesis that, (refer slide time: 16:36). So, this is a non-standard F-statistic, the statistic is calculated using the usual formula of F-statistic, but it since it has a different distribution and it does not follow the F-distribution, rather it follows the φ_3, distribution given by Dickey-Fuller, that is why it is called a non-standard F-distribution φ_3.

So, in 4, if that DF t-test of the null hypothesis, (refer slide time: 17:23- 17:57). This occurs when a stochastic trend is present in the regression, but it is dominated by a deterministic component.

So, what I am trying to tell here is that, if I go for a joint test of (refer slide time: 18:09- 18:44).

So, this implies that there is no stochastic, non-stationarity trend and this implies that, there is 1 deterministic trend. So, that is why we say that the stochastic trend is present in the regression, this is possible, but it is dominated by a deterministic trend component. And in that case, we actually test for the presence of unit root using the t-statistic.

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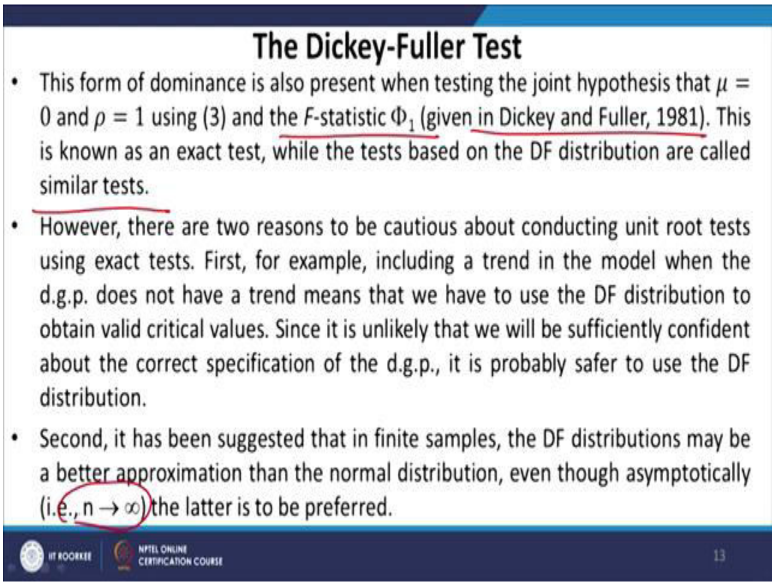

This form of dominance is also present or possible when testing the joint hypothesis that (refer slide time: 19:16- 19:34).

And in that case, we use F-statistic φ_1, this is also a non-standard F-statistic denoted by φ_1, given by again Dickey and Fuller in 1981. This is known as an exact test, while the test based on DF distribution are called similar tests.

However, there are two reasons to be cautious about conducting unit root tests using exact tests. First, for example, including a trend in the model, when the d.g.p. does not have a trend means, that we have to use the DF distribution to obtain valid critical values. Since it is unlikely that we will be sufficiently confident about the correct specification of the d.g.p., it is probably safer to use the DF distribution.

And second thing is that it has been suggested that in finite samples, the DF distributions may be a better approximation than the normal distribution, even though asymptotically, that is when n tends to infinity, or for a large sample, the latter is to be preferred.

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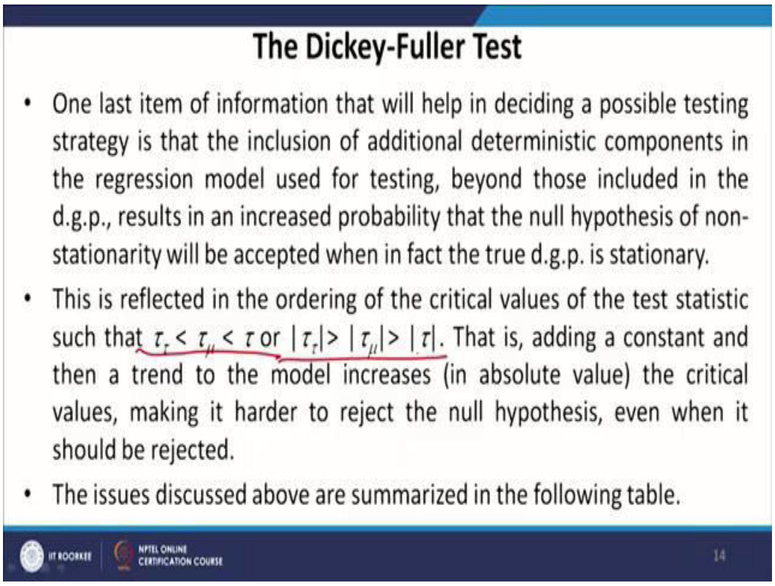

One last item of information that will help in deciding a possible testing strategy is that the inclusion of additional deterministic components in the regression model used for testing, beyond those included in the d.g.p. results in an increased probability that the null hypothesis of non-stationarity will be accepted, when in fact, the true d.g.p. is stationary.

This is reflected in the ordering of the critical values of the test statistic such that, as I have already discussed or shown that $\tau_\tau < \tau_\mu < \tau$. Alternatively, $|\tau_\tau| > |\tau_\mu| > |\tau|$, that is adding a constant and then adding a trend to the model increases in the absolute value of the critical values making it harder to reject the null hypothesis, even when it should be rejected.

So, that is why we should be careful in adding deterministic components, or additional variables into the regression because that reduces the power of the test. The issues discussed above are summarized in the following table.

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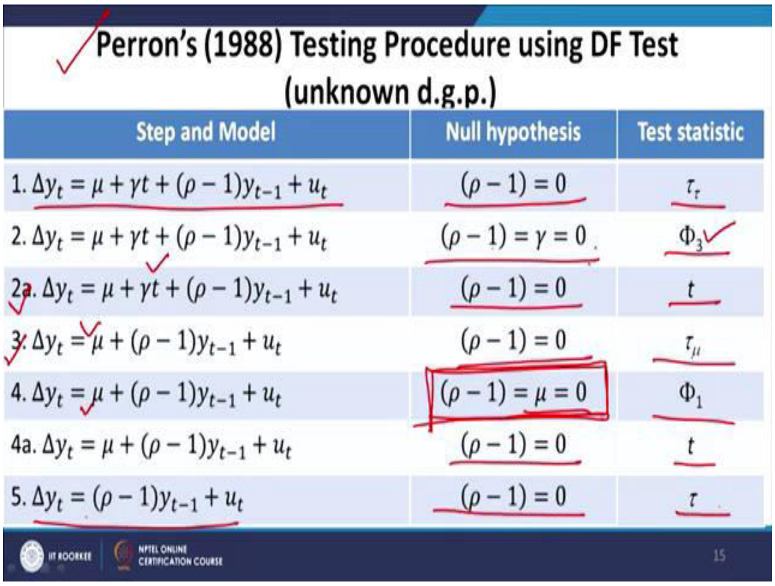

Perron's (1988) Testing Procedure using DF Test (unknown d.g.p.)

Step and Model	Null hypothesis	Test statistic
1. $\Delta y_t = \mu + \gamma t + (\rho - 1)y_{t-1} + u_t$	$(\rho - 1) = 0$	τ_τ
2. $\Delta y_t = \mu + \gamma t + (\rho - 1)y_{t-1} + u_t$	$(\rho - 1) = \gamma = 0$	Φ_3
2a. $\Delta y_t = \mu + \gamma t + (\rho - 1)y_{t-1} + u_t$	$(\rho - 1) = 0$	t
3. $\Delta y_t = \mu + (\rho - 1)y_{t-1} + u_t$	$(\rho - 1) = 0$	τ_μ
4. $\Delta y_t = \mu + (\rho - 1)y_{t-1} + u_t$	$(\rho - 1) = \mu = 0$	Φ_1
4a. $\Delta y_t = \mu + (\rho - 1)y_{t-1} + u_t$	$(\rho - 1) = 0$	t
5. $\Delta y_t = (\rho - 1)y_{t-1} + u_t$	$(\rho - 1) = 0$	τ

So, this stepwise discussion is given by Perron in 1988. This is a testing procedure using a DF test for unknown data generating processes or d.g.p. So, first of all, the steps are we go for a very general specification of the model and test for the null hypothesis using the test statistic τ_τ.

After that, we go for a joint test (refer slide time: 22:11). Then we basically go to model 2a and simply test for non-stationarity using the asymptotic t-distribution. If we reject the null hypothesis here, then of course this implies that the series has a stochastic trend, if we do not reject the null hypothesis, then this implies that the series is actually a non-stationary series, which implies that $(\rho - 1) = 0$ is not rejected.

Then the third model specification is to include only a constant term and in no deterministic trend component. Then we use the null hypothesis or test the null hypothesis $(\rho - 1) = 0$ using τ_μ statistics. And a joint hypothesis of (refer slide time: 23:27).

So, again if this null hypothesis of joint significance of both of them is not rejected, then it implies that probably the d.g.p. does not have a stationary trend. Then we go for a statistic testing of $(\rho - 1) = 0$ using the asymptotic t distribution. If this null hypothesis is not rejected, or the null hypothesis is accepted, in that case, this implies that the series is actually a non-stationary series with a constant term equal to 0.

And then we finally go for testing of this series that is, which does not have either a constant term or deterministic trend component. And then we test for the presence of non-stationarity using the τ statistic.

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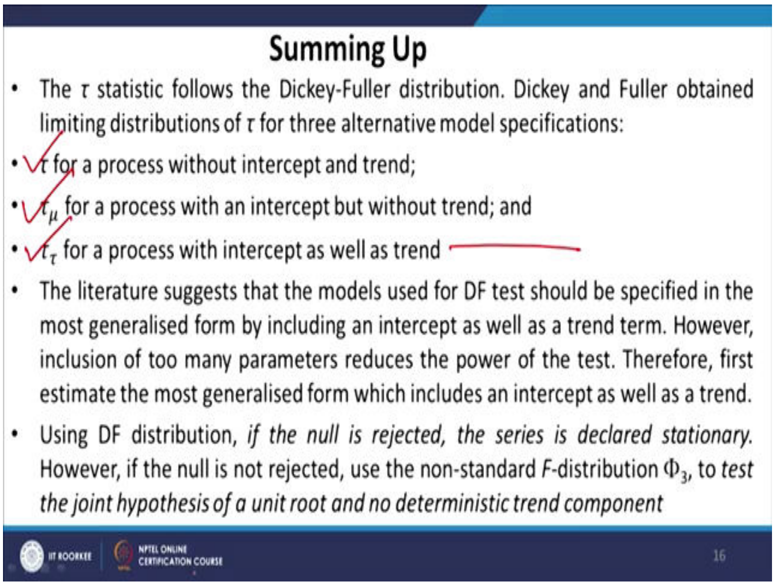

So, I just summed this up here, that τ-statistic follows the Dickey-Fuller distribution. Dickey and Fuller obtained limiting distributions of τ for three alternative model specifications, τ for a process without intercept and trend, τ_μ for a process with an intercept but without trend, and τ_τ for a process with intercept as well as a trend term, the most generic specification.

The literature suggests that the models used for the DF test should be specified in the most generalized form by including an intercept as well as a trend term. However, the inclusion of too many parameters reduces the power of the test.

Therefore, first examine the most generalized form, which includes an intercept as well as a trend. Using DF distribution, if the null is rejected, the series is declared stationary. However, if the null is not rejected, use the non-standard F-distribution φ_3 , to test the joint hypothesis of a unit root and no deterministic trend component.

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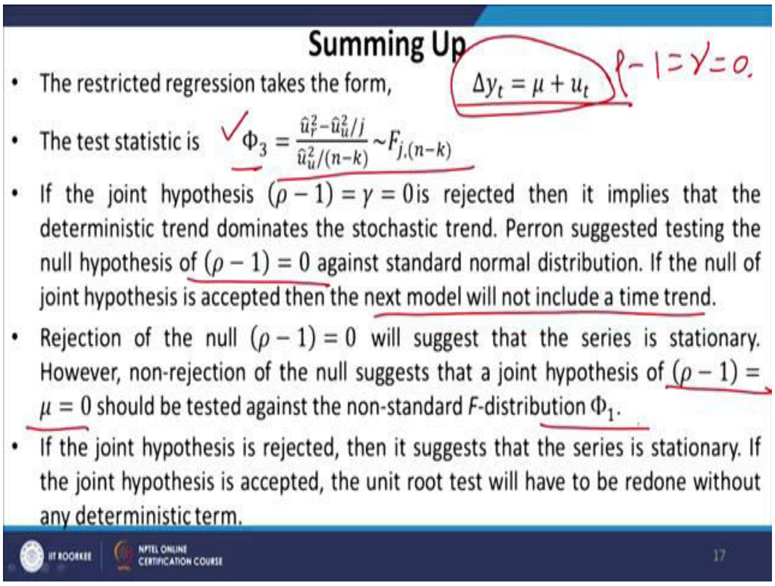

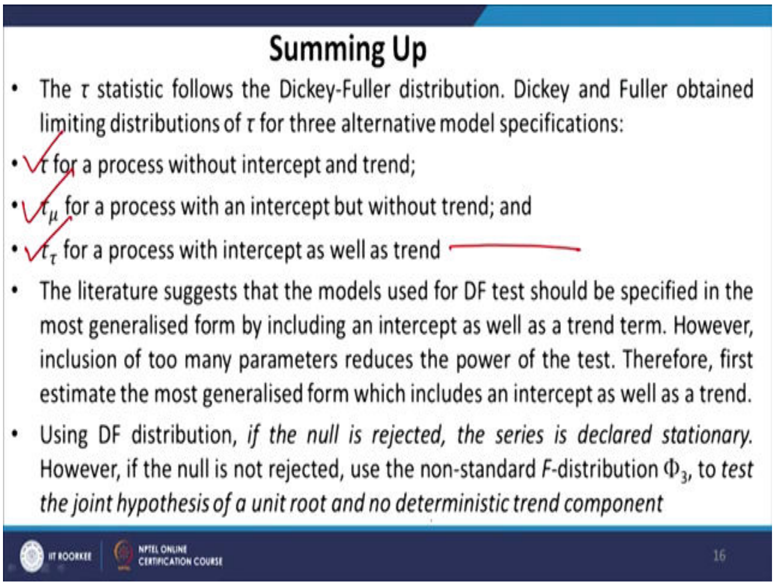

The restricted regression takes this form because you remember, when we put the restrictions, these are the restrictions, that (refer slide time: 25:43- 26:27). However, non-rejection of the null suggests that a joint hypothesis of $(\rho - 1) = \mu = 0$, should be tested against the non-standard F-distribution φ_1. If the joint hypothesis is rejected, then it suggests that the series is stationary. If the joint hypothesis is accepted, the unit root test will have to be redone without any deterministic term.

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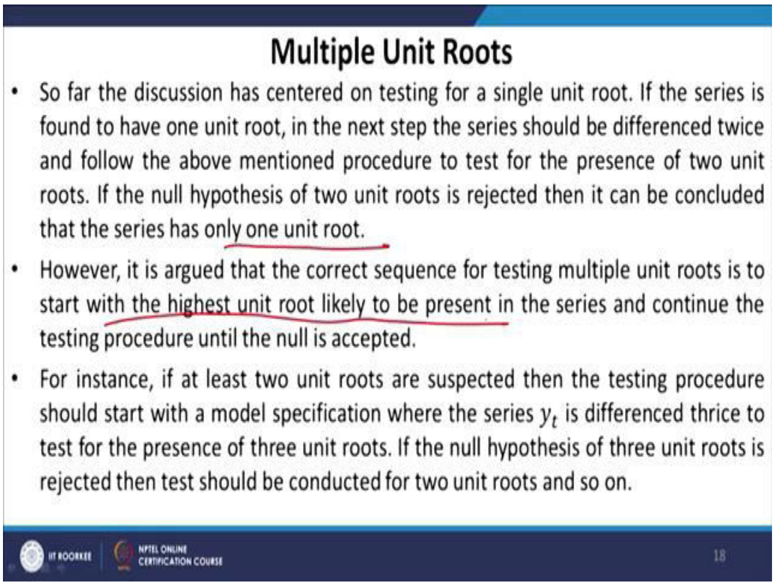

Now, we talk about testing for multiple unit-roots. So far, the discussion has centered on testing for a single unit root. If the series is found to have one unit root, in the next step the series should be differenced twice and follow the above-mentioned procedure to test for the presence of two unit-roots. If the null hypothesis of two unit roots is rejected, then it can be concluded, that the series has only one unit root.

However, it is argued that the correct sequence for testing multiple unit roots is to start with the highest unit root likely to be present in this series and continue the testing procedure until the null is accepted.

For instance, if at least two unit-roots are suspected, then the testing procedure should start with a model specification when the series is series y_t is differenced thrice to test for the presence of three unit-roots. If the null hypothesis of three-unit roots is rejected, then a test should be conducted for two-unit roots and so on.

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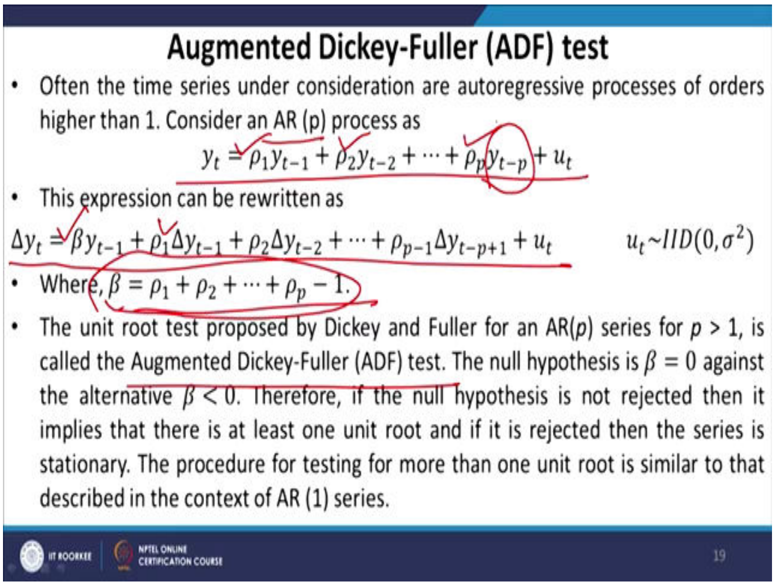

And finally, talk about the Augmented Dickey-Fuller test. Often the time series under consideration are autoregressive processes of order higher than 1. So, consider an AR (p) process, so this is an AR(p) process as we can, you can see that, we have considered y_t up to lag length p. This expression can be rewritten as (refer slide time: 28:15).

The unit root test proposed by Dickey and Fuller for an AR (p) series for p greater than 1, is called the Augmented Dickey-Fuller test. The null hypothesis is (refer slide time: 28:34).

Therefore, if the null hypothesis is not rejected, then the series is stationary. It implies that there is at least one root and if it is rejected, then this series is stationary. The procedure for testing for more than one unit root is similar to that described in the context of the AR (1) series.

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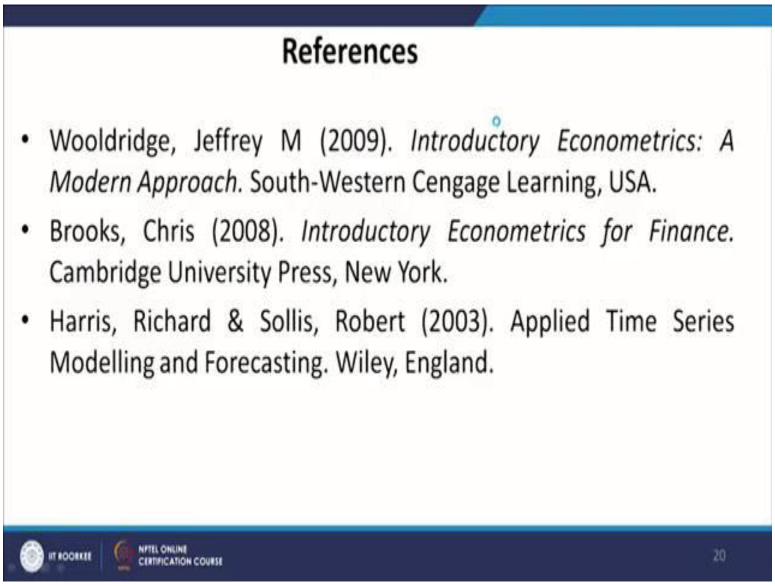

And these are the references I have followed, or some of them are also mentioned in the text, or in the discussion there. And this completes a discussion on the testing procedures of stationarity of unit root or presence of unit root.

Of course, there are alternative testing procedures, or methods available. One another very popular test is given by the Phillips Perron test. But again, that is probably outside the purview of the course. So, people can further build on their interest by following the text, or the reference books mentioned here. Thank you.