Econometric Modelling
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Lecture No. 04
Review of Basic Concepts - 2

Hello, and this is module four of econometric modelling. In module 3, I discussed certain basic concepts, which are useful in the field of econometrics as well, they are primarily concepts of statistics. Now, I continue with that review of basic concepts in module 4 as well.

So, in the previous module I had introduced expected values, which are equivalent to tpopulation mean, then variance, population variance, and then we also started talking about their sample counterparts. So, I had introduced estimators, the concept of estimate and then examples of sample estimates or population mean, and sample estimates of the population variance.

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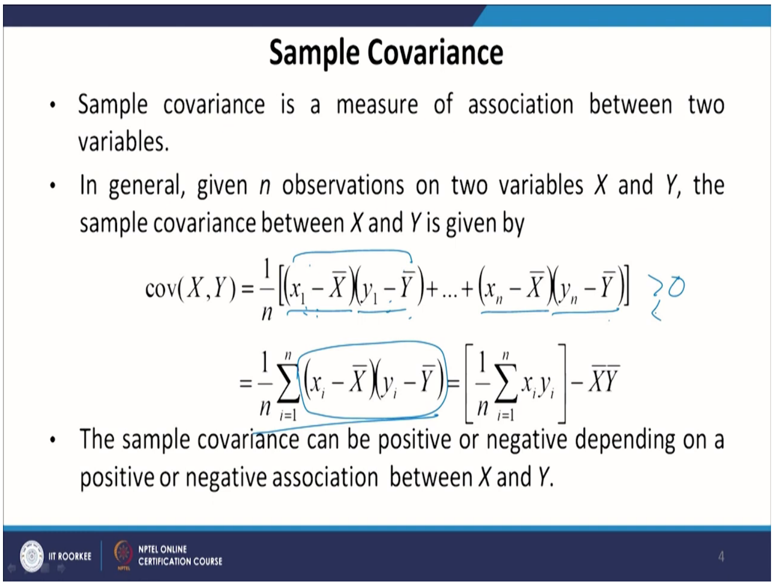

Now, we will talk about sample covariance defined as a measure of association between two variables. So far, we were dealing with only one variable. Now, another variable is brought into the picture. In general, given n observations on two variables X and Y, the sample covariance between X and Y is given by this expression or formula below:

$$\text{cov}(X,Y)=\frac{1}{n}\left[(x_1-\bar{X})(y_1-\bar{Y})+\ldots+(x_n-\bar{X})(y_n-\bar{Y})\right]$$

It shows how much X has deviated from its mean value and then correspondingly for Y i.e. how much its first observation has deviated from its mean value. Then they are multiplied and finally summed up.

This is finally written as:

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{X}\right)\left(y_i - \overline{Y}\right) = \left[\frac{1}{n}\sum_{i=1}^{n} x_i y_i\right] - \overline{XY}$$

Here, you can see that, if there are co-movements between the two variables, that is, if one is increasing the other is also increasing, which means that they are deviating from their mean in the same direction, then we will have these multiplications positive. On the other hand, if one is decreasing, the other is also decreasing, which implies that if one is drifting away from its mean, the other is also drifting away from its mean. In that case, both these are negative, and their multiplication would be negative. So, whenever there is co-movement between two variables and the multiplications are most often positive, we will finally have positive covariance. If, this multiplication is most often negative, then finally, we will have a negative covariance. And if it turns out to be 0, then there are no discernible co-movements between the two variables. So, sometimes one is positive, the other is negative. And this positive, negative almost cancel out each other. So, that is how we calculate covariance. And, that is how we can talk about the co-movements between the two variables. And what kind of values a covariance might take. So, the sample covariance can be positive or negative, depending on a positive or negative association between X and Y.

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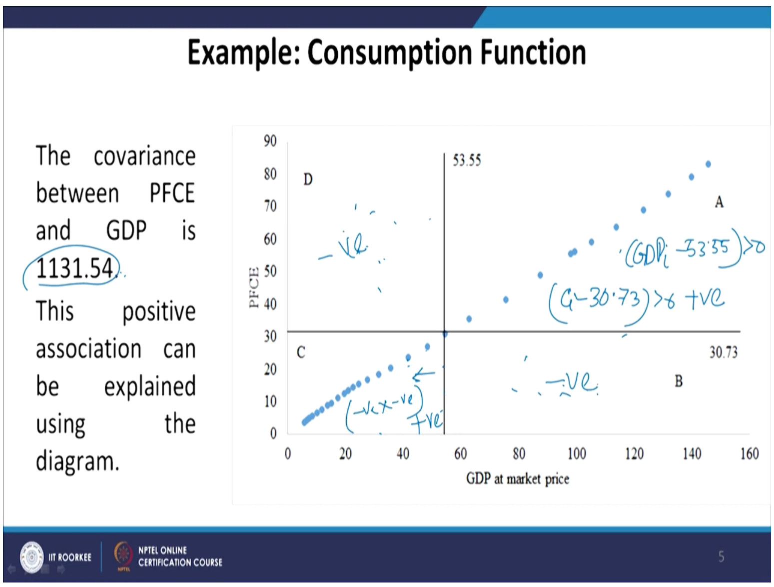

We take the example of the consumption function which had been taken in the first two modules. So, GDP and private final consumption expenditure in the Indian context from 1990 to 2020, that data we had picked up from RBI, and if we calculate the covariance between these two variables, it turns out to be a very large number 1131.54 *(Refer to slide above: Example consumption function)*.

So, this positive association can be explained using a diagram like this *(Refer to slide above: Example consumption function)*. You can see that it is almost an upward sloping straight line. So, there is so perfect co-movement between the two variables, that this turns out to be a very large number 1131.54. Now, you can see that the consumption is averaged at 30.73, and GDP at market price is averaged at 53.55.

So, all these values, which are basically below this 30.73 line, when the average is subtracted from individual consumption values, then will be negative. At the same time, all of them are also to the left of this average vertical line of 53.55. So, when the mean is subtracted from the individual values, that will also be negative. So, two negatives are multiplied here, which makes it positive.

On the other hand, if we look at this *(Refer to slide above: Example consumption function)*, quadrant, then we will find that all the values are to the of this vertical line, and above this horizontal line. So, Ci minus it's mean that is 30.73, will all be positive, and GDP, that is GDP i minus 53.55, will also be positive. So, when these two positives are multiplied again, we will be having positive numbers. So, here also, we have a very large positive covariance. If we had a lot of observations here, then you can see that for consumption, this would have been positive, but for GDP, this would have been negative.

Similarly, in this quadrant also *(Refer to slide above: Example consumption function)*, for consumption, they would have been negative, for GDP, they would have been positive. So, negative-positive, so these two quadrants would give us negative multiplication values. Now, here in this example, we do not have any negative values as such. And that is why I repeat, this number is very large.

If there are negative values that can outweigh the positive values, then there can be covariance, which is negative. Negative and positive values canceling out each other would result in near 0 covariances, which would indicate that there is not much co-movements between the two variables. Now, I will talk about certain covariance rules, which are also going to be used repeatedly in the forthcoming modules.

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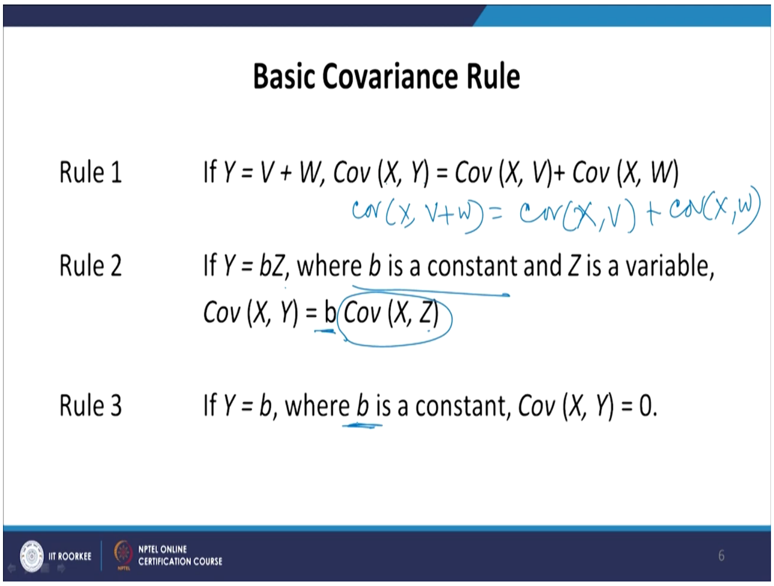

So rule 1; if Y is equal to V plus W multiplied by covariance X, Y, then it will be equal to covariance X, V plus covariance X, W. So, simply, a covariance between X and Y will be written as the covariance between X and V plus W. And then this can further be expanded as the covariance between X and V plus covariance between X and W *(see above slide).*

Rule 2, if Y equals bZ, where b is a constant, and Z is a variable, then covariance between X and Y is equal to b covariance X, Z. So, b is constant, it can always be taken outside this covariance expression or operation. And, that is how it comes out what remains are the two variables.

Rule 3, if Y equals b, where b is a constant covariance between X and Y is equal to 0 because y is a constant. So, the possibility of any co-movements is not possible. b constant implies that Y is a number and a constant number. For example, 300, 900, 450, etc. and there is no co-movement and no randomness associated with it, that is how covariance between X and Y is 0.

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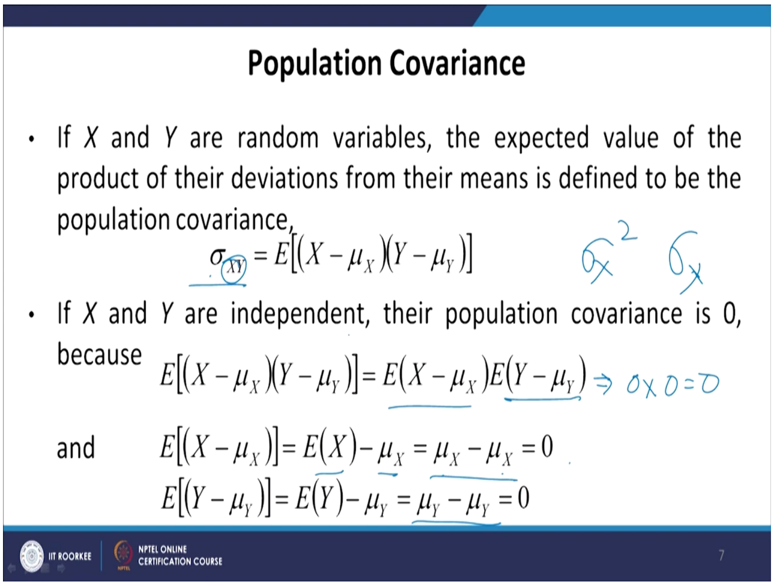

Now, we will talk about population covariance, if X and Y are random variables, the expected value of the product of their deviations from their means is defined to be the population covariance. Very similar to sample covariance, the difference is that when you talk about population, we bring in the concept of expectation, since the population is not observed, it hinges or depends a lot on probability values. And that is why we term it as the expected values, whereas sample counterparts are calculated. So, there is less expectation associated with it, they can be calculated with a certain amount of certainty, and that is why expected values are associated with population.

So, population covariance is,

$$\sigma_{XY} = E[(X-\mu_X)(Y-\mu_Y)]$$

Where population covariance is denoted by sigma (σ) X Y, not to be confused with variance, variances are denoted with sigma square (σ^2), standard deviations are denoted with sigma (σ). Then, we have two variables X and Y, calculated as expected value of X minus μ_X minus multiplied by Y minus μ_Y. If X and Y are independent, their population covariance is 0. Now, you got to recall how independence was defined. So, if X and Y are independent, then the expected value of X minus μ_X into Y minus μ_Y can be broken into two separate multiplications like the expected value of X minus μ_X into expected value of Y minus μ_Y.

$$E[(X-\mu_X)(Y-\mu_Y)] = E(X-\mu_X)E(Y-\mu_Y)$$

And

$$E[(X-\mu_X)] = E(X) - \mu_X = \mu_X - \mu_X = 0$$

Here expected value of X minus μ_X is equal to the expected value of X minus μ_X equal to μ_X minus μ_X is equal to zero, the expected value of μ_X is μ_X, as I had earlier also taught that μ_X is a number, a constant. So, the expected value of μ_X is μ_X itself. So, μ_X minus μ_X is 0.

Similarly, we will also have the expected value of Y minus μ_Y is 0.

$$E[(Y-\mu_Y)] = E(Y) - \mu_Y = \mu_Y - \mu_Y = 0$$

That is how this becomes multiplication of two 0s and this becomes 0. So, if X and Y are independent, their population covariance is 0.

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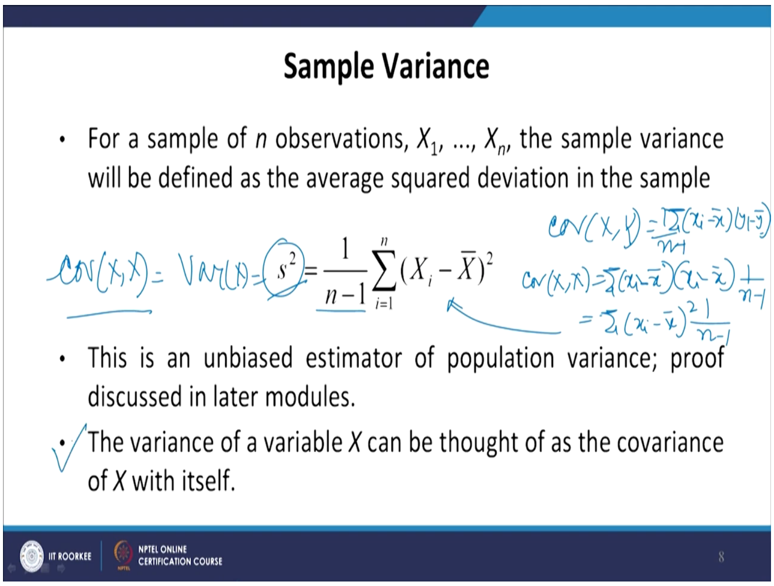

Now, we will talk about sample variance. For a sample of n observations X1 to Xn, the sample variance will be defined as the average squared deviation in the sample. So, I had already introduced this sample variance in the context of the unobserved component, here it is repeated. We have sample variance denoted by a square, which is 1 upon n minus 1 multiplied by summation Xi minus X bar whole square. And, as I have previously said that, why this is n minus 1, that would be taken up in a later module *(refer to slide above on sample variance).* This is an unbiased estimator of the population variance *(proof discussed in later modules*), the variance of a variable X can be thought of as the covariance of X with itself.

Referring slide above, we see, the variance of X can be written as the covariance of X with X itself, because when we have a covariance of X and Y, then this is written as summation xi minus x bar into yi minus y bar. So, here if we have covariance X and X, then this is summation xi minus x bar into summation xi minus x bar, which is summation xi minus x bar square. And that is exactly, is the various formula, we can have divided by 1 minus n or n minus 1. So, that is how this concept is also used in the future. And that is why they are being mentioned here, that the variance of a variable X can be thought of as the covariance of X with itself.

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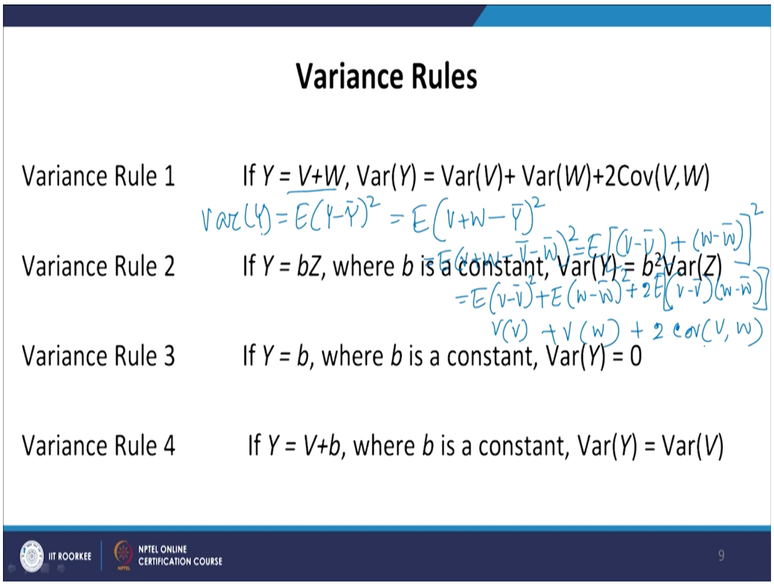

Now, we talk about certain variance rules:

Variance rule 1: If $Y = V+W$, $\mathrm{Var}(Y) = \mathrm{Var}(V)+ \mathrm{Var}(W)+2\mathrm{Cov}(V,W)$

If Y equals V plus W, then variance of Y is the variance of V plus variance of W plus 2 covariance of V, W. Now, this follows from the fact that, how do we write variance of Y, variance of Y is an expression where we have expected value of Y minus Y bar whole square. So, if we have Y equals V plus W, then that becomes the expected value of V plus W minus Y bar square. (*Refer to slide on Variance Rules)*

Now, if Y equals V plus W, then the Y bar equals V bar plus W bar. So, this can be further written as the expected value of V plus W minus V bar minus W bar whole square, which is equal to the expected value of V minus V bar, plus W minus W bar, the entire thing whole square.

Now, if we apply a plus b whole square formula, then we will find that this is equal to E, expected value of V minus V bar whole square, plus expected value of W minus W bar, whole square plus 2 expected value of V minus V bar into W minus W bar, entire thing under the bracket, so this is variance of V. This is variance of W. And this is 2 covariance between V and W. So, that is how, we have this variance rule one proof that, if Y equals V plus W, then variance of Y is equals variance of V plus variance of W plus 2 covariance V W (*Refer to slide on Variance Rules)*.

Variance rule 2: If $Y = bZ$, where b is a constant, $\mathrm{Var}(Y) = b^2\mathrm{Var}(Z)$

If Y equals bZ, where b is a constant variance of Y equals b square variance of Z, it also follows from the same rule that Z is the variable or Z is the variable here, b is constant, so b needs to come outside. So, when it comes out of the expected expression, then it becomes a square because, in the case of variance, we are having squares, ahead of, or inside the expected expression.

Variance rule 3: If $Y = b$, where b is a constant, $\mathrm{Var}(Y) = 0$

If Y equals b, where b is a constant, then the variance is equal to 0 variance of Y equals 0, which implies that when b is a constant, there is no variations. Variance stands for measuring the variations in the observed values of a variable with respects to its mean value. So, if we are considering a constant, a number, then it cannot, it does not have any variation. So, its variance will definitely be 0.

Variance Rule 4: If $Y = V+b$, where b is a constant, $\mathrm{Var}(Y) = \mathrm{Var}(V)$

The last variance rule, if Y equals V plus b, where b is constant and V is a variable, then variance of Y equals the variance of V, V is actually removed. The reason is that again, very simple, b here is constant it does not have any variations in it and as a result of it variance of Y becomes variance of V, b will not have any covariance also with either Y or V, the reason is that b is constant.

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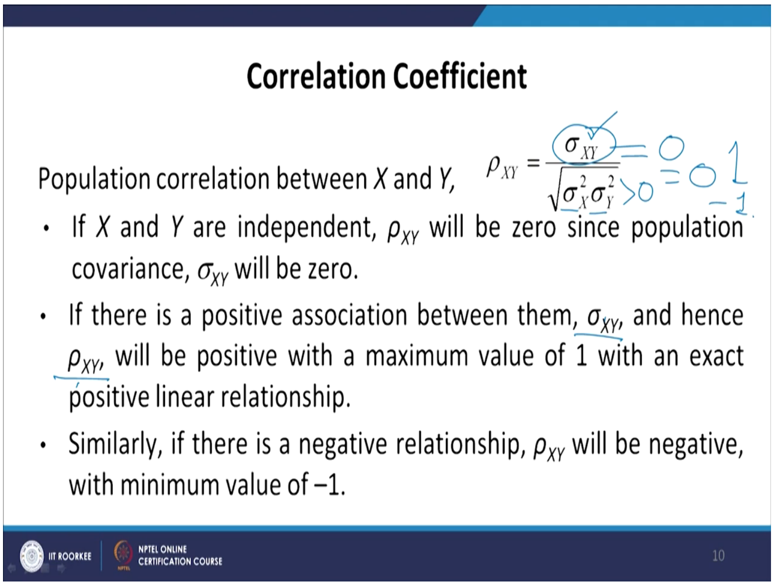

Now, we will talk about correlation coefficient. So far, we have talked about mean, variance, standard deviation and covariance. A very related concept is the correlation coefficient, which is a very popular concept and known to most. So, what it talks about is correlation between X and Y.

It is measured as in the population, it is measured as the covariance between X and Y. Remember, sigma XY refers to the covariance between X and Y divided by root over sigma X square, sigma Y square. So, this is in the denominator, we have variance of X multiplied by variance of Y, and then root over is taken, so it can alternatively be replaced with standard deviation of X and standard deviation of Y. So, this is the correlation between X and Y in the population *(Refer to slide above on correlation coefficient*).

If X and Y are independent, then you remember that sigma XY will be 0, in case X and Y are independent then in that case, rho XY will also be 0 because you have a numerator, which is 0. So, if X and Y are independent, rho XY will be 0, since population covariance, sigma XY will be 0. If, there is a positive association between them, sigma XY, and hence, rho *(denoted* *by ρ)* XY will be positive with the maximum value of one with an exact positive linear relationship.

Now, you notice one thing that in the denominator we have variance or standard deviations. Variance and standard deviations can never be negative numbers because they are obtained from squared and then, the squared numbers are summed up, so certain numbers are squared, and then they are summed up and that is why it can never be a negative number. So, in the denominator, we always have some positive numbers.

So, the sign of the correlation depends exclusively on the sign of the covariance, if the covariance is positive, then the correlation will also be positive. If the covariance is negative, then the correlation will be negative. Then, we talk about the values it can take. It can take a maximum value of 1 and a minimum value of minus 1. So, if there is a negative relationship rho (ρ) x y will be negative and the minimum value is equal to minus 1.

Now, what happens is that if there is a perfect correlation, then we will be having a value which is perfect positive correlation, then we will be having a variable value very close to 1, if we have a perfect negative correlation, then we will be having a value very close to minus 1. So, that is the range of values correlation coefficient takes i.e 1 to -1.

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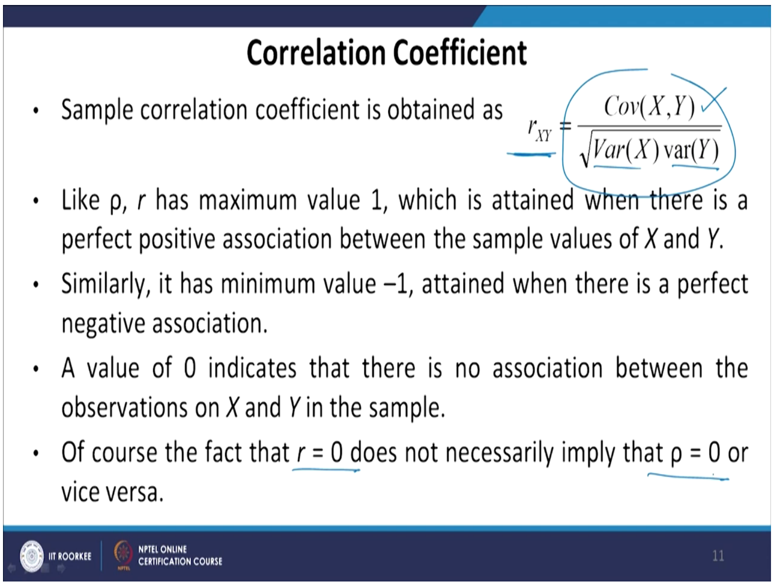

The sample correlation coefficient is obtained like this:

$$r_{XY} = \frac{Cov(X,Y)}{\sqrt{Var(X)\,\mathrm{var}(Y)}}$$

It is denoted by r XY, it has sample covariance calculation in the numerator, which is the covariance between X and Y and sample variance calculations in the denominator, this is the variance of X and variance of Y, then the square root is taken. Like rho (ρ) has maximum value of 1 which is attained when there is a perfect positive association between the sample values of X and Y. Similarly, it has a minimum value of minus 1 attained when there is a perfect negative association between X and Y. A value 0 indicates that there is no association between the observed observations on X and Y in the sample. Of course, the fact that r equals to 0 does not necessarily imply that rho(ρ) equals 0 or vice versa.

So here, it implies that if you are observing a 0 value for the sample r, this means for the sample that I have picked up from the population that is not showing any correlation between the two variables X and Y, but that does not necessarily mean that in the population, which could be much larger than the sample, there is no correlation between X and Y.

Similarly, it is possible that the sample randomly picked up is showing some correlation between X and Y. But, if we could consider the entire population, then this impact would have been diffused. And in that case, for the population, we probably would not find any substantial or significant, positive or negative correlation between the two variables. So that is why it is said that r equals 0 does not necessarily imply that rho equals 0, so sample correlation does not necessarily imply that population correlation is also 0 or vice versa. So, a non-zero sample correlation does not necessarily imply a non-zero population correlation.

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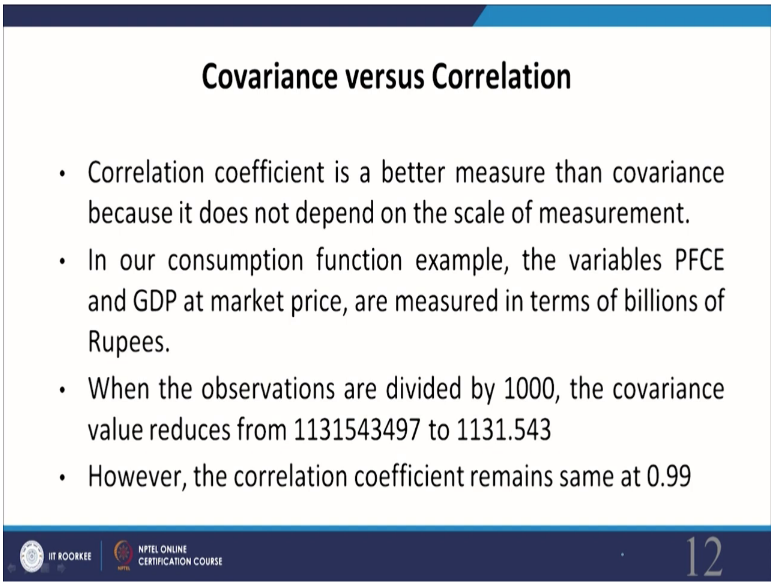

Now, we compare covariance with correlation, correlation coefficient is considered to be a better measure than covariance because it does not depend on the scale of measurement, it is a ratio, so it is a scale free measurement. In our consumption function example, the variables PFCE and GDP at market price are measured in terms of billions of rupees. When the observations are divided by say 1000, the covariance value reduces from a very large number. So, when we initially measured in terms of billions of rupees, then the covariance was as large as 1131543497. But, when we divided the observations with 1000, then the covariances values also came down, so 1131.543.

However, the correlation coefficient remains same at 0.99, it does not depend on the scale of measurement of the variables. So, it is a scale free measure and as a result of which, it is more comparable across a large number of variables where the scale measurements are not comparable. And that is why it is a preferred measure compared to covariance.

This takes me to the end of the review of basic concepts we have been discussing for the current module as well as in the previous module. Below are the references that I have followed for this module.

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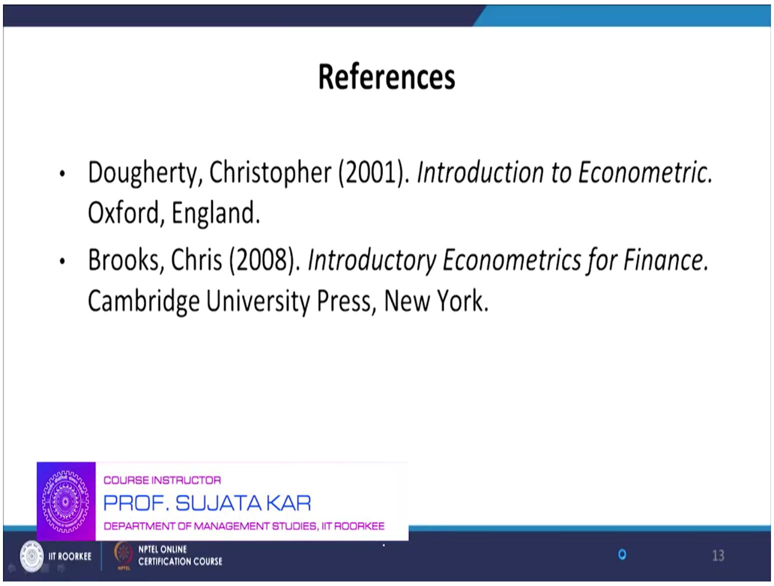

And in the next module, we will begin with the basic econometry tools like simple regression analysis. Thank you.