Econometric Modelling
Professor Sujata Kar
Department of Management Studies
Indian Institute of Technology Roorkee
Lecture 40
Basics of Cointegration

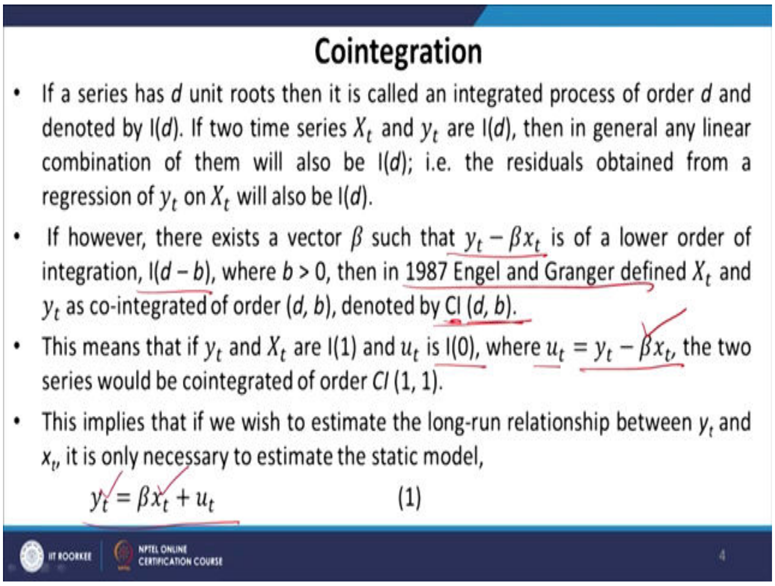
(Refer Slide Time: 0:31)

Hello and I welcome you to the last module of this course on Econometric Modelling. In this last part that is part 8, we are discussing modeling long-run relationships and I have already discussed stationarity and unit testing procedures. The last thing that I am going to discuss is the basics of cointegration.

Now, before that, I need to tell you that how cointegration is actually related to stationarity and unit testing. Once we find a series non-stationary then so far, I have discussed how we can difference the series and make it a stationary one, because we cannot directly work with non-stationary series.

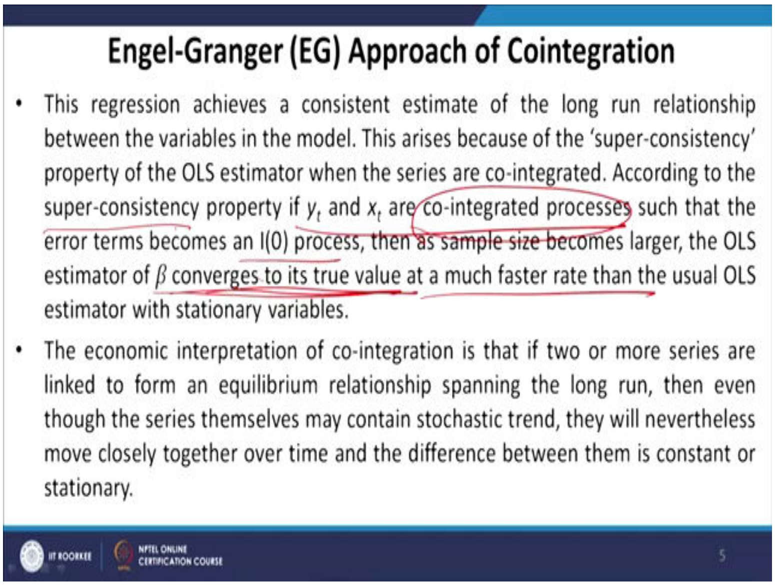
But then there is of course a possibility of working with non-stationary series as well because differencing leads to some kind of loss of information. So, in order to avoid that, what we can use is the concept of cointegration. Cointegration does not require us to difference the series but still, it can be used for estimation purposes. So, let us see what is cointegration all about.

(Refer Slide Time: 1:33)



If a series has d unit-roots, then it is called an integrated process of order d and it is denoted by $I(d)$ that has already been discussed. So, a series having d unit roots is actually a non-stationary series. The way we used to call the non-stationary process of 1 unit root as I (1) series and a stationary series can be denoted by I (0) series with d unit-roots would be called an integrated series of order d and denoted by $I(d)$. If (refer slide time: 2:04- 3:30).

(Refer Slide Time: 3:30)



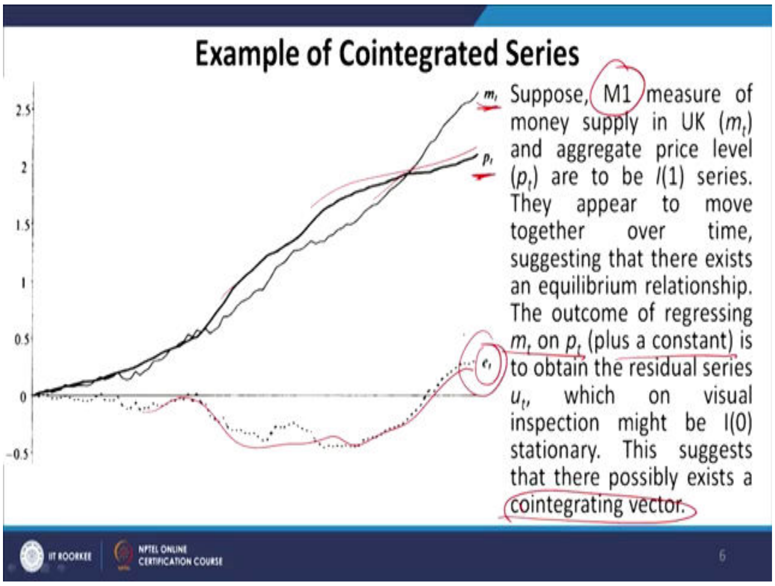
Now we are going to deal with primarily Engel-Granger's approach of cointegration. This regression achieves a consistent estimate of the long-run relationship between the variables in the model. This arises because of the super consistency property of the OLS estimator when the series are cointegrated.

So far, we discussed the concept of consistency, consistency was introduced in the very beginning itself, which is a property of the OLS estimators an asymptotic property or a large sample property. Now, there is something that comes is the concept of super consistency. So, according to the super consistency property, if y_t *and* x_t are cointegrated processes, such that the error term becomes an I (0) process. Then as the sample size becomes larger, the OLS estimator of βconverges to the true value at a much faster rate than the usual OLS estimator with stationary variables.

So, first of all, the concept of consistency actually told us that β converges to its true value. So, when βconverges to its true value with an increase in the sample size, then we call it a consistent estimator. But when β converges to its true value at a much faster rate than the usual OLS estimator, because of this cointegration properties, then we call it super consistency.

The economic interpretation of cointegration is that if two or more series are linked to form an equilibrium relationship spanning the long run, then even though the series themselves may contain a stochastic trends, they will nevertheless move closely together over time. And the difference between them is constant or stationary.

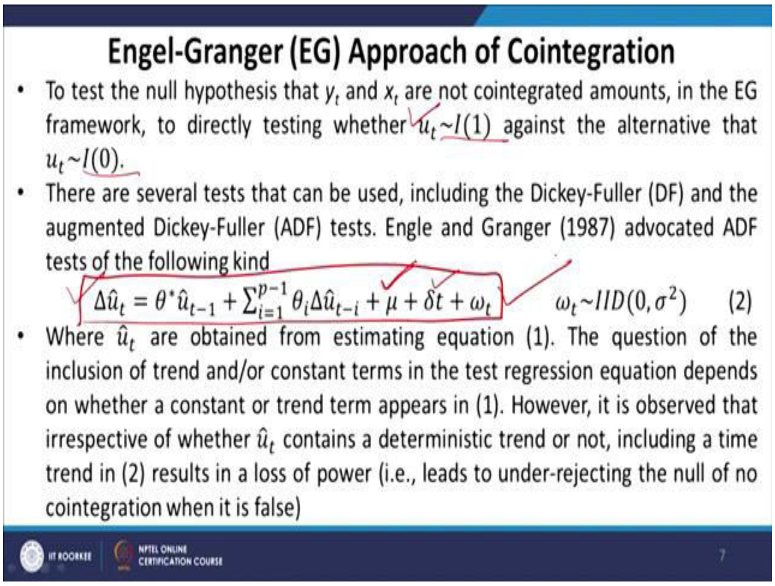
(Refer Slide Time: 5:21)

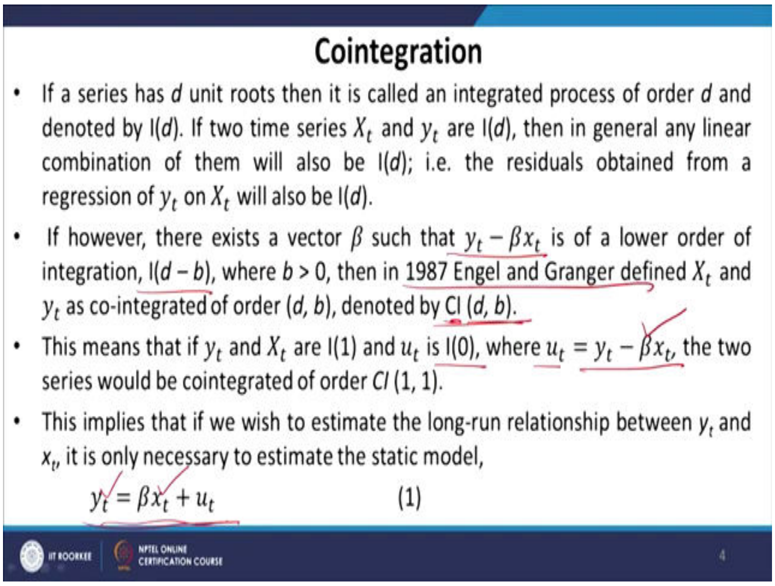


So, I will explain this concept with an example. Suppose the M1 measure of money supply, can be called the narrow definition of money in the UK which is denoted by m_t , and the aggregate price level denoted by p_t, are I (1) series. They appear to move together over time, you can see that they are moving together over time suggesting that there exists an equilibrium relationship between them, they may exist.

The outcome of regressing m_t on p_t plus constant is to obtain the residual series u_t, which on visual inspection might be an I (0) series. So, this is what is our, the estimated u_t, denoted by e_t, this is our estimated residuals. And this actually looks quite like a stationary series because it is hovering around 0 mean, or at least its variations are not increasing consistently over time. This suggests that there possibly exists a cointegrating vector between m_t and p_t which renders the series e_t, a stationary one.

(Refer Slide Time: 6:37)





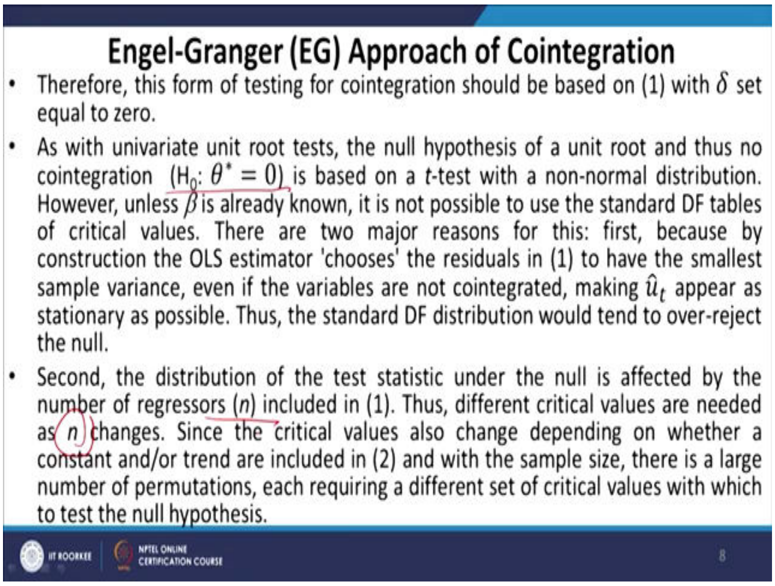
To test the null hypothesis that y_t and x_t and are cointegrated amounts, in the EG framework that is Engel Granger framework to directly testing for whether u_t is I (1) against the alternative that u_t follows an I (0) or u_t is an integrated series of orders 0, that is it is a stationary series.

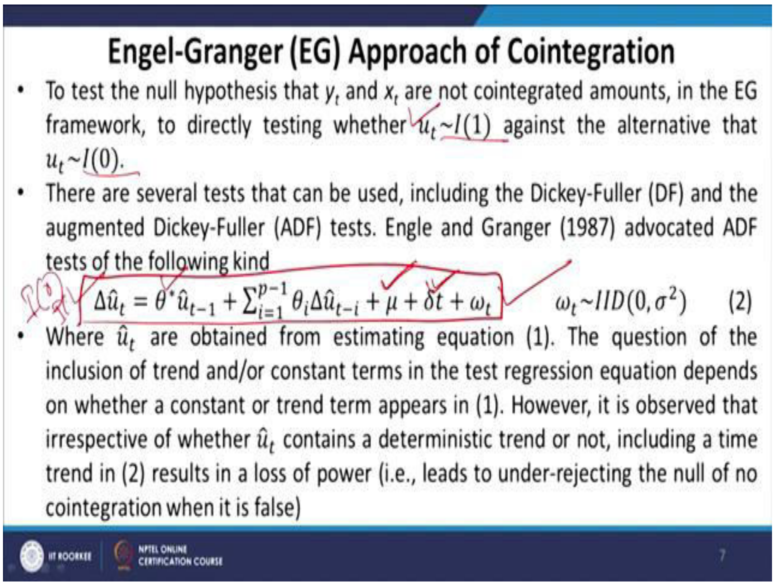
There are several tests that can be used, including the Dickey-Fuller and the augmented Dickey-Fuller tests. Engel and Granger advocated ADF tests of the following kind. So, this is an ADF test of the estimated residuals to test that for whether the residuals are I (1) or I (0) and how we are doing it.

So, it actually follows the usual ADF structure that I had just discussed in the previous module, $\widehat{u_t}$ are obtained from estimating equation 1. The question of the inclusion of trend and or constant term, whether they should be included or not in the test regression equation depends on whether a constant or trend term appears in equation 1.

So, if the original equation includes a trend and a constant term then that should also be included in this specification otherwise not. However, it is observed that irrespective of whether $\widehat{u_t}$ contains a deterministic trend or not, including a time trend in 2, results in a loss of power that is it leads to under rejecting the null of no cointegration when it is actually false.

(Refer Slide Time: 8:17)



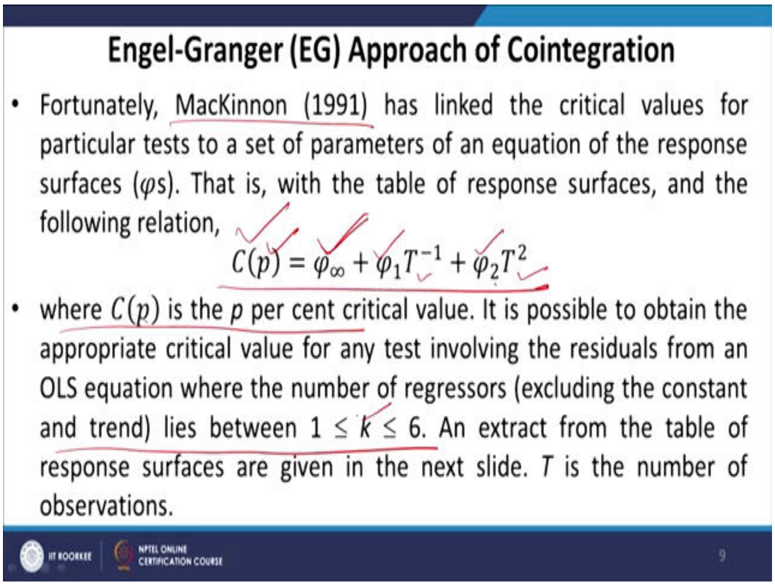


Therefore, this form of testing for cointegration should be based on (refer slide time: 8:18-8:59). However, unless β is already known, it is not possible to use the standard DF tables of critical values. There are two major reasons for this, first, because by construction the OLS estimator chooses the residuals in 1 to have the smallest sample variance, even if the variables are not cointegrated making $\widehat{u_t}$ appear as stationary as possible.

Thus, the standard DF distribution would tend to over reject the null that is rejecting the null which is actually should not be rejected. Second, the distribution of the test statistic under the null is affected by the number of regressors n included in the very initial specification, the model involving $y \; and \; x$.

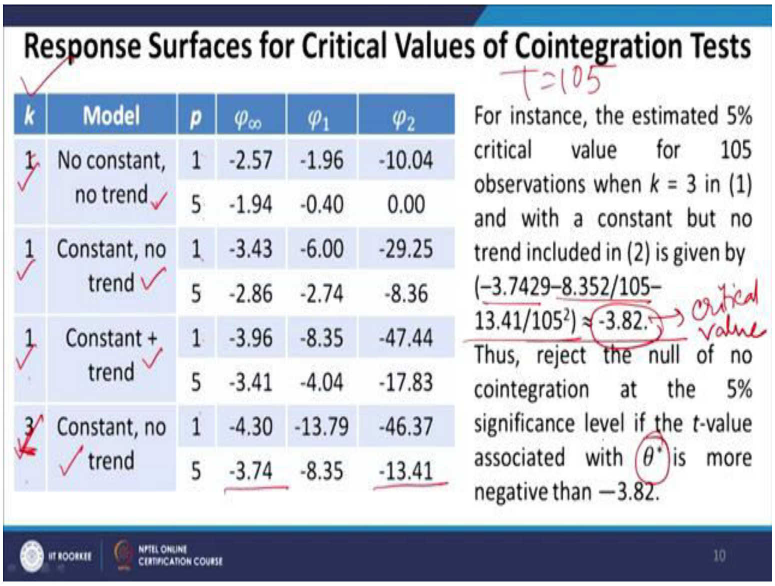
Thus, different critical values are needed as n changes. Now, here n refers to the number of regressors. Since the critical values also change depending on whether a constant and/or a trend is included in (2) and with a sample size, there is a large number of permutations, each requiring a different set of critical values with which to test the null hypothesis.

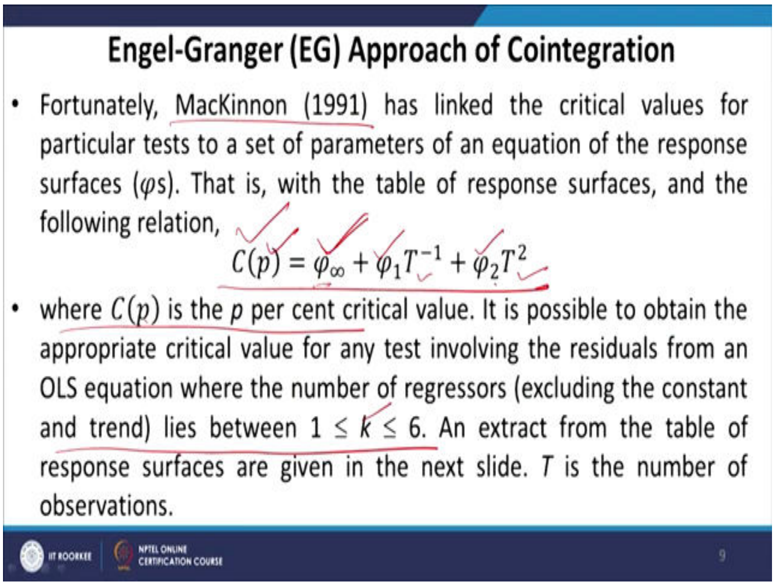
(Refer Slide Time: 10:16)



So, a solution was given by MacKinnon in 1991, who linked the critical values for particular tests to a set of parameters of an equation of the response surfaces. So, we call these as response surfaces, (refer slide time: 10:30), for different sample sizes for different values of t, we would be calculating the critical values and then examine the validity of the test statistic, it is possible to obtain the appropriate critical value for any test involving the residuals from an OLS equation where the number of regressors excluding the constant and trend lies between 1 and 6. Here k refers to the number of regressors. An extract from the table of response surfaces is given in the next slide and T is the number of observations.

(Refer Slide Time: 11:30)

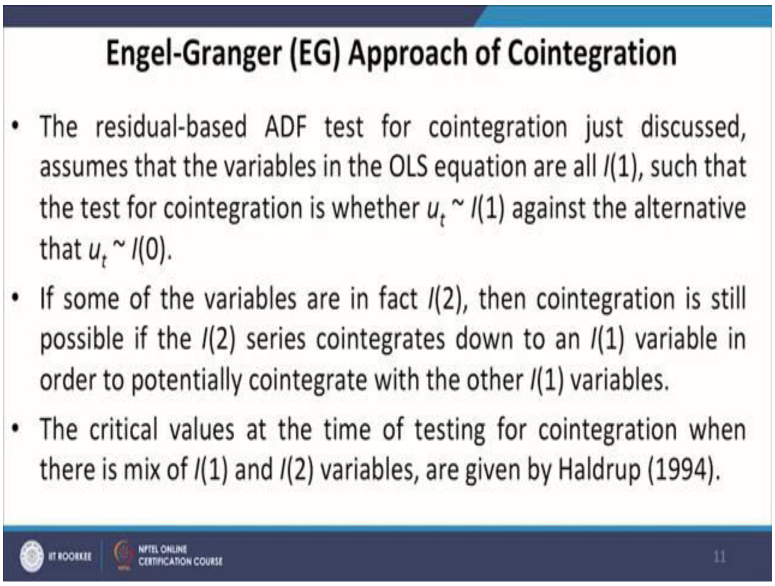




So, this is an extract from the table of response surfaces. So, first of all, we have considered several alternatives, like key values, are 1, 1, 1, and 3 and these are the models. The first model has no constant no trend, the second has one constant, but no trend. The third one has a constant plus trend. And the fourth one has 3 variables but no trend. So, constant and 2 other independent variables.

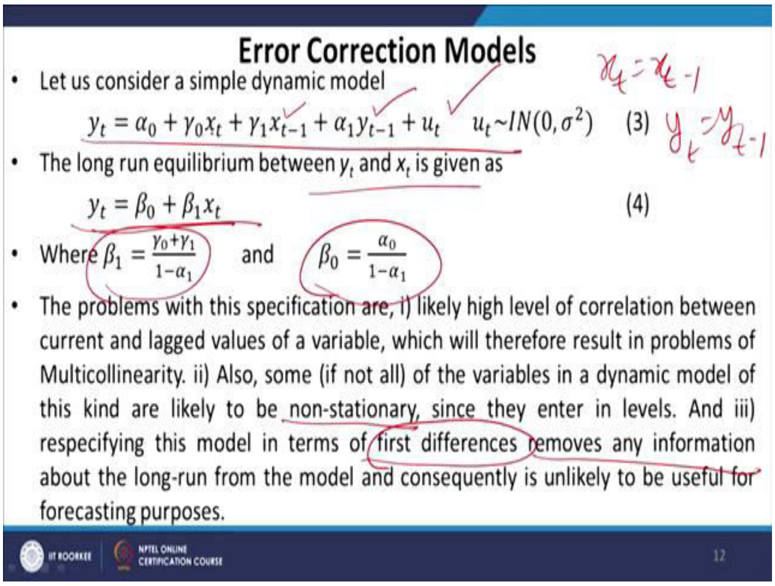
And then for different significance levels that is 1 percent and 5 percent. The $\varphi_\infty, \varphi_1, and\ \varphi_2$ values are given which were calculated by or given by MacKinnon. So, for instance, the estimated 5 percent critical values for 105 observations. So, here t is equal to 105 when k is equal to 3 in equation 1 with a constant but no trend included in equation 2 is given by this formula. So, this is actually obtained from simply like this (refer slide time: 12:30- 13:35).

(Refer Slide Time: 13:36)



So, the residual-based ADF test for cointegration just discussed assumes that the variables in the OLS equation are all I (1) such that the test for cointegration is (refer slide time: 13:48-14:17).

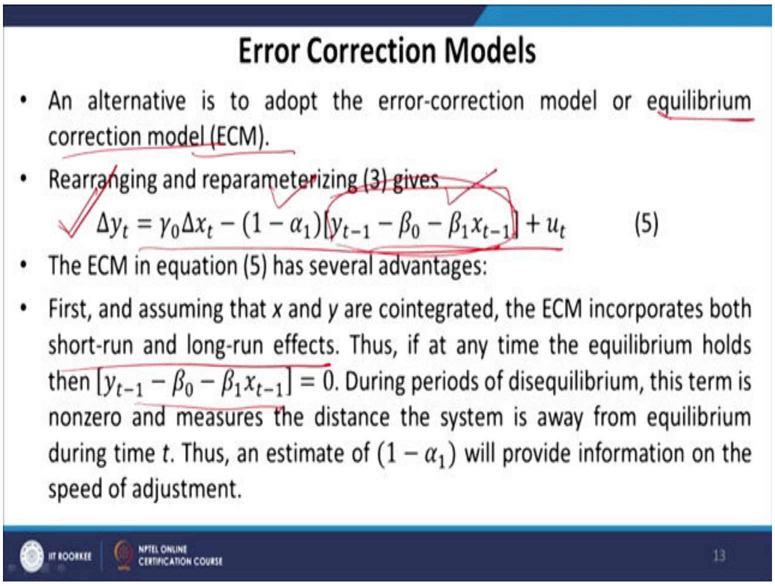
(Refer Slide Time: 14:20)

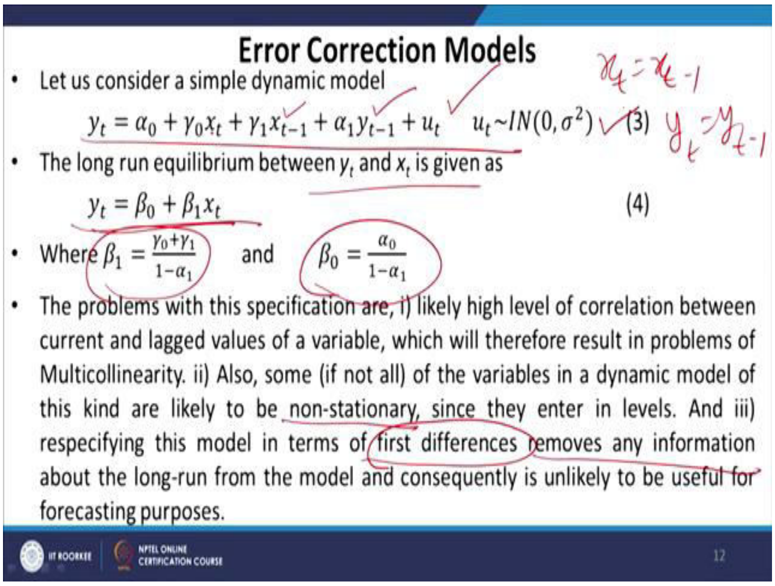


Now, we talk about a related concept that is error correction models. Let us consider a simple dynamic model. So, this is a dynamic model as you can see that we are also considering lag values of (refer slide time: 14:30- 15:10). The problems with this specification are first of all a likely high level of correlation between current and lagged values of a variable which will therefore result in problems of multicollinearity. So, we are, of course, talking about the first short-term dynamic model.

Second, also some if not all of the variables in a dynamic model of this kind are likely to be non-stationary. Say, since they enter in levels. And third, respecifying this model in terms of first differences, because they could be non-stationary series and removes any information about a long-run relationship from the model and consequently is unlikely to be useful for forecasting purposes.

(Refer Slide Time: 15:54)



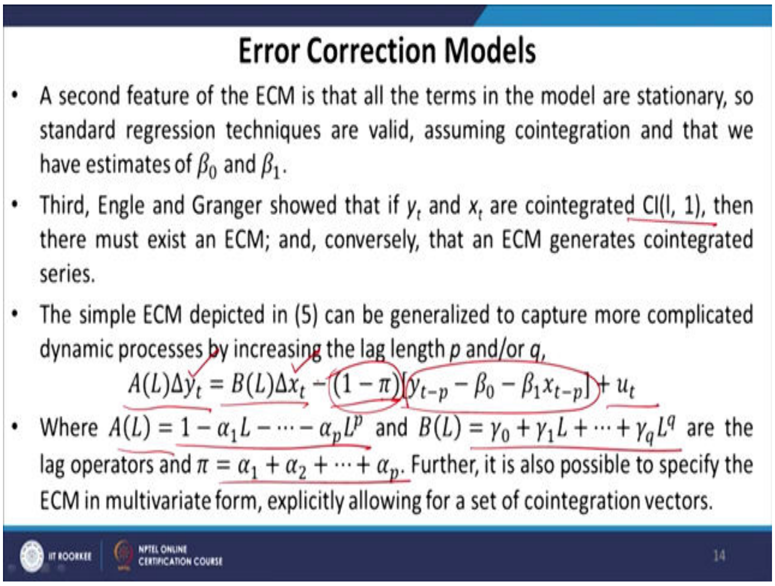


So, an alternative is to adopt the error correction model or equilibrium correction model denoted by ECM. Rearranging and re-parameterizing 3, this was my initial model the short-term dynamic model 3, we rearrange the terms and write it like this. So, the ECM in equation 5 has several advantages, this is called an error correction model or equilibrium correction model or ECM in short.

So, the advantage with this kind of specification is that first of all assuming that x and y are cointegrated. The ECM incorporates both short-run and long-run effects. So, this is the short run specification in that you can also see the long run specification. Thus, if at any time the equilibrium holds then (refer slide time: 16:42).

During periods of disequilibrium, this term is nonzero and measures the distance, the system is away from the equilibrium during time t. Thus, an estimate of $1 - \alpha$ will provide information on the speed of adjustment of the system from being disequilibrium to the equilibrium level in the long run.

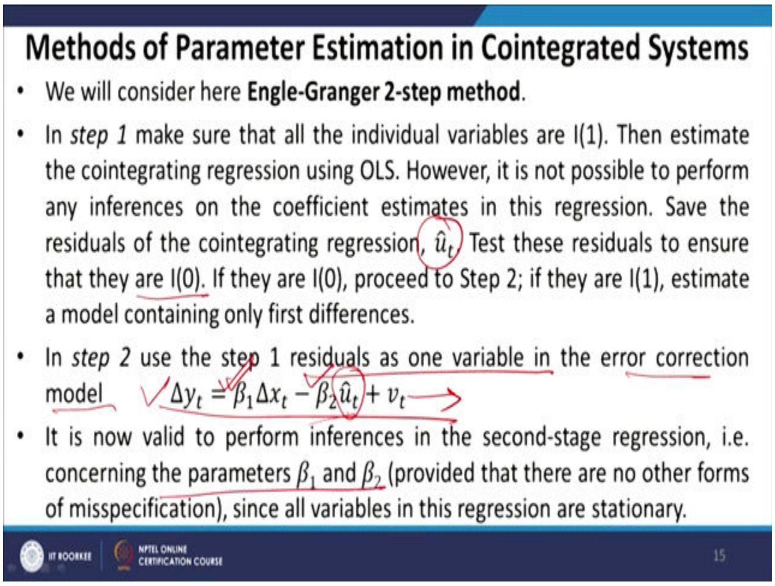
(Refer Slide Time: 17:14)

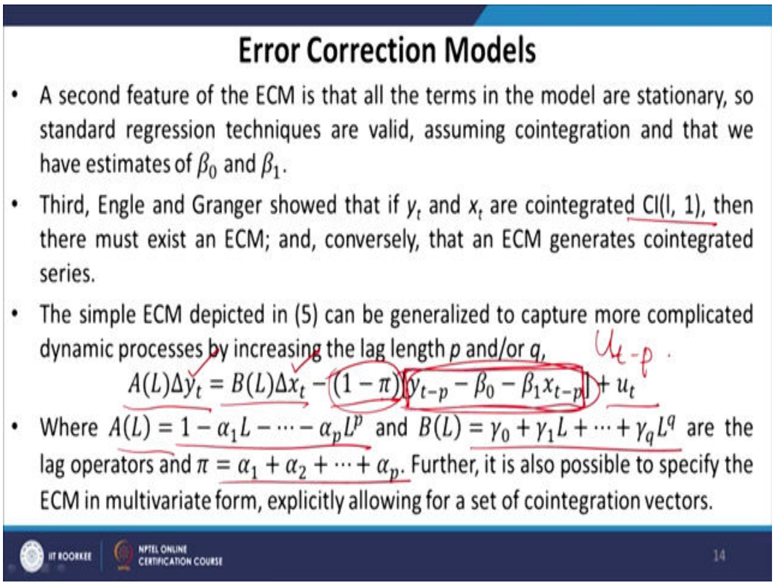


A second feature of the ECM is that all the terms in the model are stationary. So, standard regression techniques are valid, assuming cointegration and that we have estimated β_0 and β_1. Third Engel and Granger showed that if y_t and x_t are cointegrated that is they are CI (1, 1) then there must exist an ECM, and conversely that an ECM generates cointegrated series.

The simple ECM depicted in 5 can be generalized to capture more complicated dynamic processes by increasing the lag length p and/or q. For example, we are having (refer slide time: 17:55- 18:38). This measures the speed of adjustment. Further, it is also possible to specify the ECM in multivariate form explicitly allowing for a set of cointegration vectors.

(Refer Slide Time: 18:51)





Finally, we talk about the methods of parameter estimation in cointegrated systems. So, we will consider here Engel-Granger 2 step method. In step 1, make sure that all individual variables are I (1), then estimate the cointegrating regression using OLS. However, it is not possible to perform any inferences on the coefficient estimates in the regression because the error terms are not well behaved.

Then save the residuals of the cointegrating regression that is $\widehat{u_t}$ test these residuals to ensure that they are I (0) if they are I (0) then you proceed to step 2. If they are I (1) then estimate a model containing only first differences, then we cannot do much. So, in step 2 use the step 1 residuals as one variable in the error correction model.

So, in the error correction model, this expression is actually equivalent to (refer slide time: 19:55- 20:26).

It is now valid to perform inferences in the second stage regression, that is concerning the parameters β_1 *and* β_2 provided that there are no other forms of misspecification since all variables in this regression are stationary. So, this is how once we have that parameter estimates of β_1 *and* β_2, then, of course, they would give us the usual interpretations. And they are reliable estimates. So, that is all about the discussion on cointegration.

(Refer Slide Time: 20:58)

These are the references I have followed; some are mentioned here which are actually there in the discussion. And that is all about cointegration. Thank you.