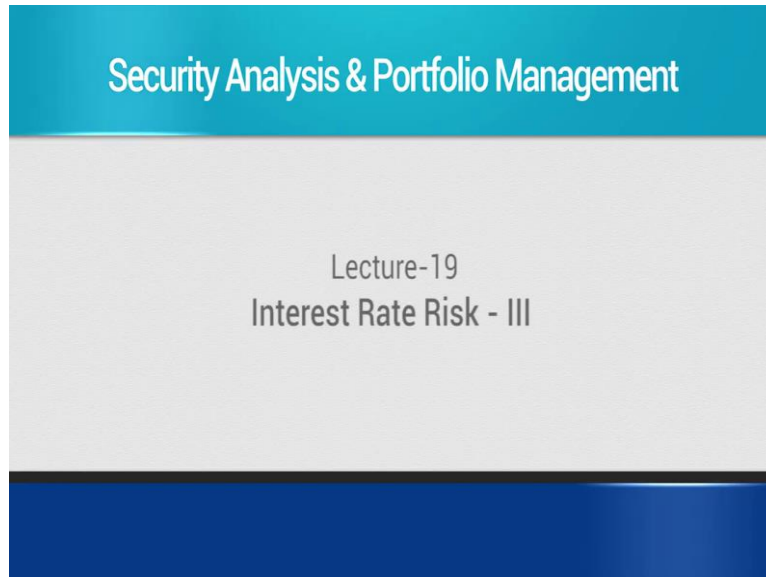


**Security Analysis and Portfolio Management**  
**Professor J.P Singh**  
**Department of Management Studies**  
**Indian Institute of Technology, Roorkee**  
**Lecture 19**  
**Interest Rate Risk - III**

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Welcome back. So, before we continue a quick recap of where we stand at the moment. I introduced the concept of interest rate risk. Whenever a bond holder or a person who is invested in a bond holds the bond for a period, which is different from its maturity, which is less than its maturity, then he gets exposed to two types of risks on account of changes in market interest rates.

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A slide titled "INTEREST RATE RISK" with a blue header. The main content is a list of three bullet points in red text. The first bullet point states that the price of a bond fluctuates with market interest rates, and the risk is that an investor faces when investing in a bond portfolio. The second bullet point (i) states that the price of the bond at the end of the holding period will decline if market interest rates rise UNANTICIPATEDLY, and this is referred to as interest rate risk. The third bullet point (ii) states that the income from reinvestment of coupons may not yield the desired return if interest rates fall, and this is called reinvestment rate risk. At the bottom of the slide, there are logos for IIT Roorkee and NPTEL Online Certification Course.

**INTEREST RATE RISK**

- Since the price of a bond fluctuates with market interest rates, the risk that an investor faces when investing in a bond portfolio is that:
- (i) the price of the bond at end of holding period will decline if market interest rates rise **UNANTICIPATEDLY**. This risk is referred to as interest rate risk.
- (ii) the income from reinvestment of coupons may not yield the desired return if interest rate fall. This is called reinvestment rate risk.

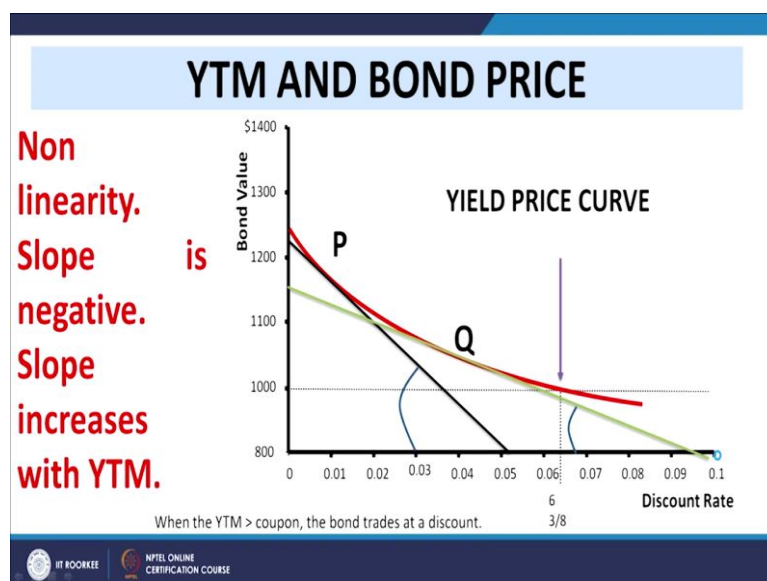
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Number one, if there is a fall in market interest rates, the income that he would derive from the reinvestment of his coupon that he receives on his bond investment would decrease, would decline. And number two, if interest rates increase, the price that the bond would fetch in the market, when he liquidates the investment at the end of the holding period would decrease.

So, two things are obvious from here number one, that increase in interest rates, decrease the market price on the liquidation of the bond. However, at the same, on the same token, they increase the reinvestment income and on the other hand, if there is a decrease in interest rates the converse happens.

So, these are two types of risks that the investor faces on account of his investment in a fixed income security, if he holds the investment for a period, which is less than its maturity. The risk arising from a change in his anticipated reinvestment income is called reinvestment rate risk, and the risk arising from the change in price at which he would liquidate the investment due to change in market interest rates is called interest rate risk.

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Then, we discuss the structure or the relationship between the yields and YTM and the bond prices and we saw that number one, the two are inversely related which is quite obvious the slope of the yield price curve was negative. And number two, the slope of the slope that is the second derivative of the yield curve, yield price curve at any point in time is positive.

The two of the, both of these combined together imply that the yield price curve is convex towards the origin that why is it is bulging towards the origin. So, that is an important feature of the yield price curve that it is a convex curve it is not a straight line in any case.

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### CONVEXITY OF PRICE-YIELD CURVE


$$P_0 = \sum_{t=1}^T \frac{C_t}{(1+y)^t} \text{ so that}$$


$$\frac{dP_0}{dy} = -\frac{1}{(1+y)} \sum_{t=1}^T \frac{tC_t}{(1+y)^t} < 0;$$

$$\frac{d^2P_0}{dy^2} = +\frac{1}{(1+y)^2} \sum_{t=1}^T \frac{t(t+1)C_t}{(1+y)^t} > 0$$

thereby establishing the convexity of the price-yield curve.

**Non linearity.**  
**First derivative is negative.**  
**Slope is negative,  $\theta$  is in second quadrant.**  
**Second derivative is positive.**  
**Hence, the curve is convex to the origin.**



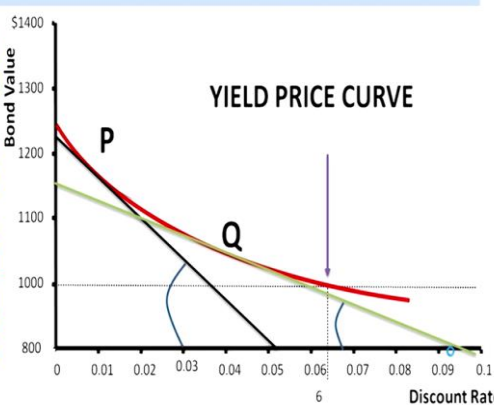


This was explicitly established by calculating the differential coefficients the first derivative and the second derivative using the cash flow discounted price of the bond and we found explicitly that the first derivative turns out to be negative and the second derivative turns out to be positive that is, the slope increases, the yield increases and therefore, the curve is a convex curve towards the origin.

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### YTM AND BOND PRICE: CONVEXITY

**Negative first derivative:**  
**Price and YTM are inversely related.**  
**Positive 2<sup>nd</sup> derivative:**  
**Slope increases with YTM.**  
**These two together imply that the magnitude of the slope is decreasing with YTM thereby establishing convexity.**



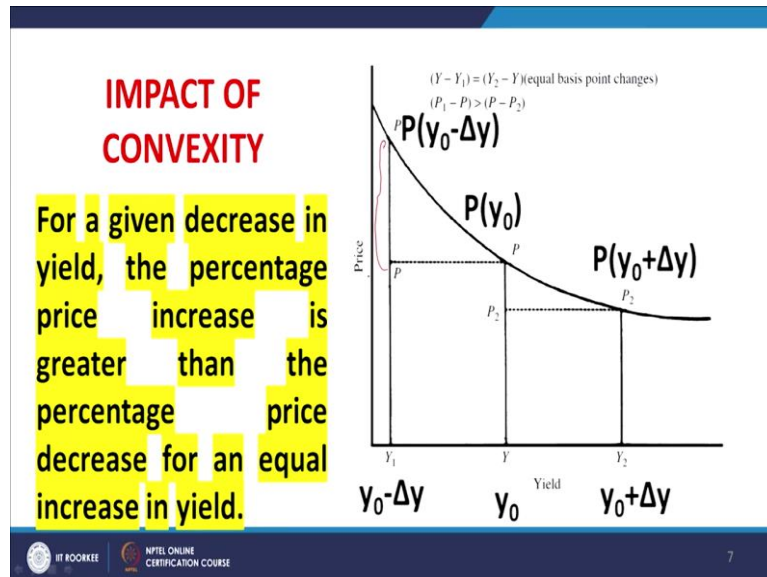
YIELD PRICE CURVE

6  
3/8



Then, we display and this figure in this diagram explicitly explained these two points that I have elucidated just now number one that the yield curve is negatively sloping. And number two, it is convex.

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The impact of convexity is captured here is shown on this slide and that is the rise in price corresponding to a certain change in YTM, certain decrease in YTM exceeds the fall in price corresponding to an equivalent increase in YTM. Let me repeat the rise in price corresponding to a certain decrease in YTM is more than the fall in price corresponding to an equivalent increase in YTM. This is clearly shown in this diagram, we find that  $P(y_0 - \Delta y) - P(y_0)$  that is this particular length of the ordinate is more than the corresponding length between  $P(y_0)$  and  $P(y_0 + \Delta y)$ .

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**MEASURES OF INTEREST RATE RISK**

- DOLLAR VALUE PER BASIS POINT (DV01)
- **MACAULAY'S DURATION & CONVEXITY**
- MODIFIED DURATION
- INTEREST RATE ELASTICITY

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Then when I introduce the concepts are the various measures of interest rate risk, the dollar value per basis point in Macaulay's duration and convexity the modified duration and we are here to talk about the interest rate elasticity which we shall do today.

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**DOLLAR VALUE PER BP**

- **DOLLAR VALUE PER BASIS POINT (DV01) IS THE CHANGE IN BOND PRICE CORRESPONDING TO A CHANGE OF ONE BASIS POINT IN THE YIELD**
- It is given by the negative slope of the price/yield curve:

$$DV01 = -\frac{dP(y)}{dy}$$

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The dollar value per basis point is defined. It is a straightforward definition it is the change in price or the negative of the change in price corresponding to a change in 1 basis point in the YTM. It is the negative of the change in price corresponding to a 1 basis point change in the YTM, that is DV01 is equal to minus dP y upon dy, the minus sign is introduced into the definition for convenience for convention in order to return a positive figure for the dollar value for basis point.

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**DURATION**

$$P_0 = \sum_{t=1}^T \frac{C_t}{(1+y_0)^t}; P_0 = P(y_0); P_0 + dP = P(y_0 + dy)$$

Expanding  $P(y_0 + dy)$  as a Taylor series around  $y_0$ , we have



$$P(y_0 + dy) = P(y_0) + P'(y_0)dy + \frac{1}{2}P''(y_0)dy^2 + \dots$$

$$\left. \frac{dP}{P} \right|_{y_0} = \frac{P(y_0 + dy) - P(y_0)}{P(y_0)} = \frac{P'(y_0)}{P(y_0)}dy + \frac{1}{2} \frac{P''(y_0)}{P(y_0)}(dy)^2 + \dots$$



10

$$\text{Duration } D = - \frac{(1+y_0)P'(y_0)}{P(y_0)} = \frac{\sum_{t=1}^T \frac{tC_t}{(1+y_0)^t}}{P(y_0)},$$

$$\text{Convexity } C = \frac{(1+y_0)^2 P''(y_0)}{2P(y_0)} = \frac{\sum_{t=1}^T \frac{t(t+1)C_t}{(1+y_0)^t}}{2P(y_0)}$$



11

We then talked in detail about duration in the expression for the duration, the expression for the duration is given here on this slide it is given by  $\frac{1}{1+y_0} \frac{P'(y_0)}{P(y_0)}$ . And when you explicitly differentiate the DCF formula, you arrive at the expression that is given in the right-hand side extreme right-hand side of the first equation, similar expression for the convexity is given in the bottom right hand corner of your slide.

Now, the important thing here, the units of duration are time, the dimensions of duration are time as you can calculate using this formula here. And however, the units of convexity are time squared, as can be seen from this formula because it is a second derivative in the case of duration, we have a first derivative. So, we have the change in price per unit time and

therefore, when it gets inverse we get duration in terms of time units and we get convexity in terms of time square units.

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$$\frac{dP}{P}\bigg|_{y_0} = \frac{P(y_0 + dy) - P(y_0)}{P(y_0)} = \frac{P'(y_0)}{P(y_0)} dy + \frac{1}{2} \frac{P''(y_0)}{P(y_0)} (dy)^2 + \dots$$

$$= -D \frac{dy}{(1+y_0)} + C \left( \frac{dy}{1+y_0} \right)^2$$

Then, this is the formula for the percentage change in price in terms of duration and convexity when both, we incorporate or we retain the second order derivative terms in the Taylor expansion. Then we end up with this formula, which is here on your slide. The first term is the duration is the straight-line formula the straight-line approximation for the percentage change in price, and the second term introduces the correction, which incorporates the effect or the impact of convexity of the yield price curve.

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### DURATION AS DCF WEIGHTED AVERAGE TIME

$$\text{Duration } D = \frac{\sum_{t=1}^T \frac{tC_t}{(1+y_0)^t}}{P(y_0)} = \sum_{t=1}^T t \times \left[ \frac{\frac{C_t}{(1+y_0)^t}}{P(y_0)} \right] = \sum_{t=1}^T t \times \left[ \frac{\frac{C_t}{(1+y_0)^t}}{\sum_{t=1}^T \frac{C_t}{(1+y_0)^t}} \right]$$

- A bond's (annual) Macaulay duration is calculated as the weighted average of the number of years until each of the bond's promised cash flows, where the weights are the present values of each cash flow as a percentage of the bond's full value.

Durations representation as a weighted average time as shown on this slide, I shall not go into the details again, but the formula can be explained in words as the bond's annual Macaulay's duration is calculated as a weighted average of the number of years until each of the bonds promised cash flows that is  $t$  where the weights are the present values of each cash flow as a percentage of the bond's full value that is the DCF value of the bond.

In other words, the weights or the fraction of the cash flow, to which  $t$  relates expressed and normalised with respect to the full price of the bond or the total discounted cash flow value of the bond.

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## DURATION BETWEEN COUPON DATES

- Between coupon dates, the Macaulay duration of a coupon bond decreases with the passage of time and then goes back up significantly at each coupon payment date.



Then the duration between coupon dates the Macaulay's duration of a coupon bond decreases with the passage of time and then goes back up significantly on the next coupon payment date, it is some sort of a sawtooth pattern.

(Refer Slide Time: 08:24)

**IF WE IGNORE CONVEXITY, THEN DURATION AS LINEAR APPROXIMATION**

$$D = -\frac{(1+y_0)}{P(y_0)} P'(y_0); P'(y_0) = -D \frac{P(y_0)}{(1+y_0)}$$

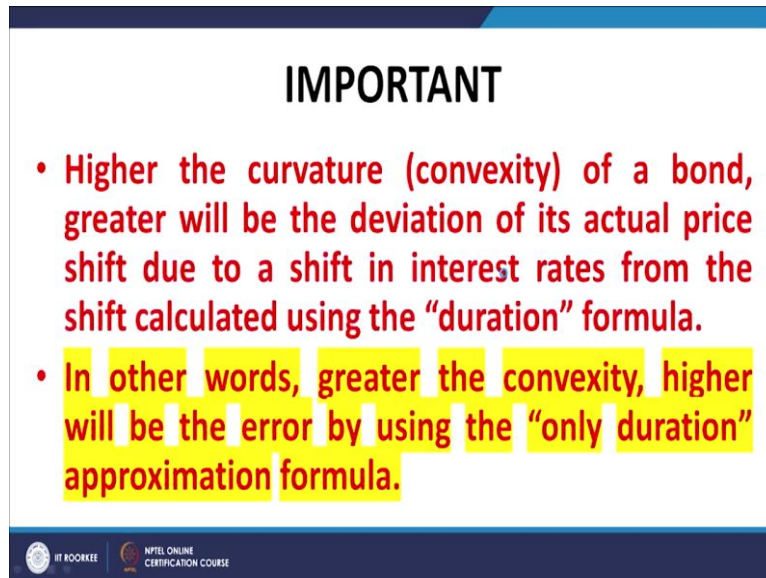
$$D = \frac{\sum_{t=1}^T \frac{tC_t}{(1+y_0)^t}}{P(y_0)} \text{ is fixed for given } y_0;$$

or  $\left. \frac{dP}{dy} \right|_{y_0} = P'(y_0) = \text{constant at given } y_0 \text{ showing that "duration" is a linear approximation of the yield-price curve around the point of reference.}$

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Now, if we incorporate... if we ignore convexity as I mentioned duration happens to be the linear approximation of the yield price curve at the point at which the duration has been calculated at  $y$  equal to  $y_0$ . The point at which we are calculating duration or convexity are the price change relating to minimal or an infinitesimal shift in the YTM duration gives you the linearized impact assuming that the yield price curve is a straight line around that mark in the infinitesimal neighbourhood of that point at which we are calculating the impact of our infinitesimal shift in the YTM. So, duration is the linear impact I repeat and convexity adds on to it the impact due to the curvature of the yield price curve.

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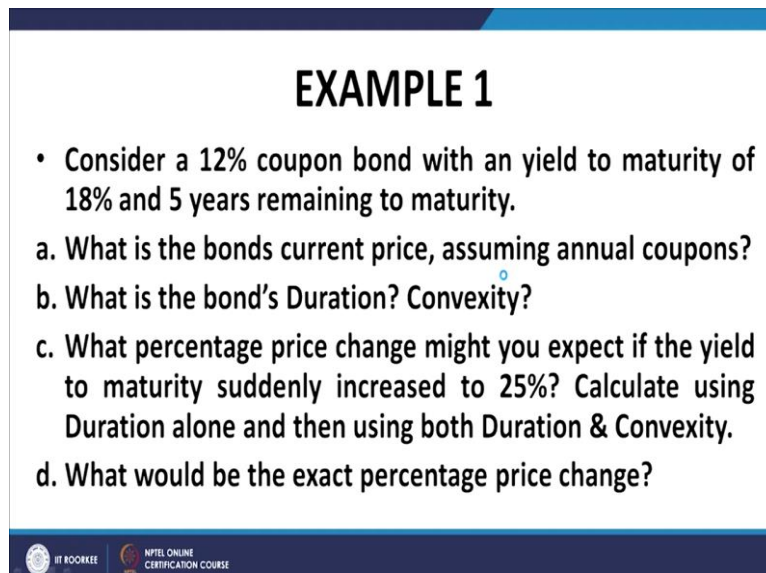
**IMPORTANT**

- Higher the curvature (convexity) of a bond, greater will be the deviation of its actual price shift due to a shift in interest rates from the shift calculated using the “duration” formula.
- In other words, greater the convexity, higher will be the error by using the “only duration” approximation formula.

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So, higher the convexity of the yield price curve greater will be the inaccuracy of the percentage price change that you arrive at using the duration alone. In other words, the error that would be introduced if we use, if you work out the percentage price change using duration alone or using the first ordered expansion in the Taylor expansion would be significant if the convexity of the yield price curve is significant. So, the convexity becomes all the more relevant when the curve or the yield price curve is more convex.

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**EXAMPLE 1**

- Consider a 12% coupon bond with an yield to maturity of 18% and 5 years remaining to maturity.
  - a. What is the bonds current price, assuming annual coupons?
  - b. What is the bond's Duration? Convexity?
  - c. What percentage price change might you expect if the yield to maturity suddenly increased to 25%? Calculate using Duration alone and then using both Duration & Convexity.
  - d. What would be the exact percentage price change?

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TIMELINE	0	1	2	3	4	5
YTM		0.18	0.18	0.18	0.18	0.18
DISC FACTOR		0.84745763	0.71818	0.60863	0.5157889	0.4371
CASH FLOW		12	12	12	12	112
DCF	81.23697	10.1694915	8.61821	7.30357	6.1894665	48.956
tC(t)		12	24	36	48	560
DISC tC(t)	318.8557	10.1694915	17.2364	21.9107	24.757866	244.78
DURATION	3.925007					
t(t+1)C(t)		24	72	144	240	3360
DISC t(t+1)C(t)	1752.167	20.3389831	51.7093	87.6428	123.78933	1468.7
CONVEXITY	10.7843					

Then we took up this example in detail, again, I should not go through it again. But this shows various things, this shows the competition of duration, the competition of the current market price. That is the first thing that is done here, the competition of duration that is the second step. And finally, the competition of convexity, all these terms, all these three calculations are explicitly shown on this Excel worksheet, for the example or for the parameters that are contained in the example.

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ACTUAL REVISED PRICE	0	1	2	3	4	5
TIMELINE						
YTM		0.25	0.25	0.25	0.25	0.25
DISC FACTOR		0.8	0.64	0.512	0.4096	0.3277
CASH FLOW		12	12	12	12	112
DCF	65.03936	9.6	7.68	6.144	4.9152	36.7
ORIGINAL PRICE	81.23697					
ACTUAL % CHANGE	-0.199387					
<b>GIVEN CHANGE IN YTM</b>					<b>0.07</b>	
<b>% CHANGE IN PRICE USING DURATION</b>					<b>-0.232839</b>	
<b>CONVEXITY CORRECTION</b>					<b>0.037951</b>	
<b>NET % CHANGE IN PRICE</b>					<b>-0.194888</b>	

Then we work out the price shift on various counts, we work out the actual percentage price shift, that is the percentage price shift when the YTM changes from 18 percent to 25 percent. On the basis of the DCF formula itself, we also work out the percentage price change using the linearized duration version. And we also work out the convexity correction, and we find

that in the absence of the convexity correction, the result or the outcome of the duration valued percentage price shift is quite far removed from the actual percentage price shift.

And however, when we incorporate the effect of convexity, the values tend to converge. This is for a significant price shift of 7 percent in the YTM. Of course, in practice, this would never happen, but it is incorporated in the example to magnify the outcomes for the practical range of shifts in YTM. Therefore, if we use duration and convexity both, we are very much in line with the actual price shift using the DCF formula.

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**MACAULEY'S AND MODIFIED DURATION**

$$\text{Modified Duration } (D_{\text{MOD}}) = \frac{D_{\text{MAC}}}{(1 + y_0)} = - \frac{P'(y_0)}{P(y_0)}$$

$$= \frac{1}{(1 + y_0)} \sum_{t=1}^T \frac{tC_t}{(1 + y_0)^t}$$

$$\frac{dP}{P} = -D_{\text{MOD}} dy \text{ so that } D_{\text{MOD}} = - \frac{dP/P}{dy}$$

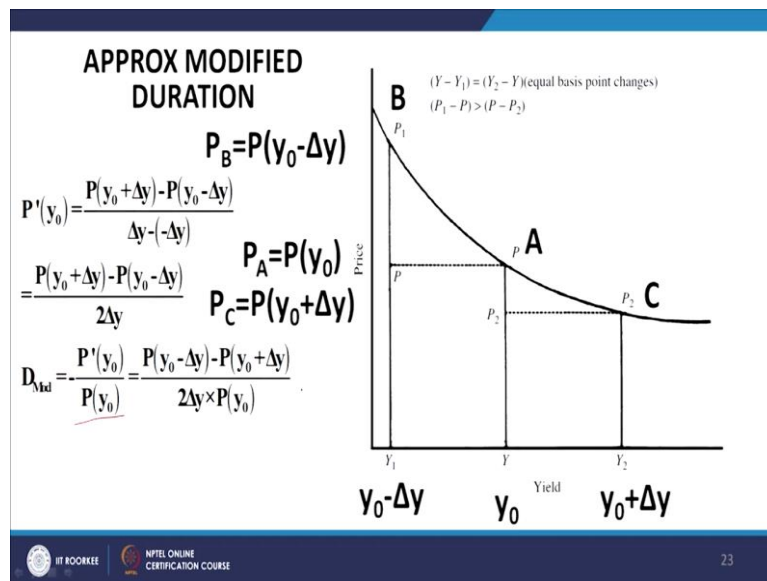
**Percentage Change in price for a 1% change in a bond's YTM**

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Then, I introduced the concept of modified duration, which is in some sense, rescaling of the Macaulay's duration by the factor or normalisation by the factor of 1 plus  $y_0$ . In other words,  $D_{\text{mod}}$  is equal to  $D_{\text{mac}}$  divided by 1 plus  $y_0$ , where  $y_0$  is the YTM at which the current market price of the bond is assumed, when the process or the calculations are being performed.

And there is another equivalent expression for the modified duration, which is the percentage change in price the negative of the percentage change in price per unit change in YTM for a 1 percent or for a single unit change in YTM the percentage change in price that is given a measure of, that will give you a measure of the modified duration as well. The two are equivalent as can be seen from the derivation that is contained in this slide.

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Here we start today's lecture, we were in the process of working out the approximation or the approximate formula for the modified duration. Again, we returned to this figure the yield price curve. And let us assume that we want to calculate the modified duration at the point A at which the current market price is P of  $y_0$  and the current YTM is  $y_0$ , the current YTM is  $y_0$  and the current market price is P of  $y_0$ .

So, what we do is, we take infinitesimal or very small shifts in the YTM to the left that is a decrease in YTM as well as an increase in YTM. Let us assume that the shift is of magnitude  $\Delta y$ . So, we have another point to the left of  $y_0$  which is  $y_0$  minus  $\Delta y$  at which the price is  $P_B$  or P of  $y_0$  minus  $\Delta y$  and we have another point to the right of  $y_0$  which is  $y_0$  plus  $\Delta y$  at which the corresponding price of the bond  $P_C$  or P of  $y_0$  plus  $\Delta y$ .

Now, if we try to work out the approximate slope of the curve BC by approximating it with a straight line, what we get is the slope of this particular straight line is  $P_C$  minus  $P_B$  divided by  $\Delta y$ ,  $2\Delta y$  because the distance between the between the abscissa at the point B and the point C is  $2\Delta y$ . So, the slope that is  $P'$  of  $y_0$  or  $P'$  of  $y$  at the point  $y_0$  can be approximated as  $P$  of  $P_C$  minus  $P_B$  divided by  $2\Delta y$ .

So, now using the formula of duration, modified duration, so what we get is  $P$  of  $y_0$  minus  $\Delta y$ , minus  $P$  of  $y_0$  plus  $\Delta y$  upon  $2\Delta y$  into  $P$  of  $y_0$ . So, this is how the approximate formula can be used and this is the background of this approximate formula, I can repeat it briefly again. Suppose, we want to work out the value of the duration modified duration at the point A, when the bond is being priced at a YTM of  $y_0$  with a price of P of  $y_0$  a  $P_A$ . What we

do is we take infinitesimal small shifts in  $y_0$  to  $y_0$  minus  $\Delta y$  on the left hand side and  $y_0$  plus  $\Delta y$  on the right hand side.

The corresponding points are B and C, the corresponding prices are  $P_B$  and  $P_C$  which  $P_B$  is equal to  $P$  of  $y_0$  minus  $\Delta y$  and  $P_C$  is equal to  $P$  of  $y_0$  plus  $\Delta y$ . Now, we approximate the slope of the yield price curve at the point A by the slope of the straight line BC. The slope of the straight line BC is given by  $P_C$  minus  $P_B$  divided by the different shifts in the YTM the total shift in the YTM which is  $\Delta y$  minus minus  $\Delta y$  that is  $2 \Delta y$ .

So, what we get as the slope is  $P_C$  minus  $P_B$  divided by  $2 \Delta y$ . Now, the modified duration as per the formula that we saw a few minutes back is given by  $P'_{y_0}$  upon  $P_{y_0}$  that is with a negative sign of course. So, it is equal to  $P_B$  minus  $P_C$  divided by  $2 \Delta y$  into  $P$  of  $y_0$  or  $P_A$ . So, this is the approximate formula for the working out of the duration.

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## MACAULEY'S AND MODIFIED DURATION

$$\text{Modified Duration } (D_{\text{MOD}}) = \frac{D_{\text{MAC}}}{(1 + y_0)} = - \frac{P'(y_0)}{P(y_0)}$$

$$= \frac{1}{(1 + y_0)} \sum_{t=1}^T \frac{tC_t}{(1 + y_0)^t}$$

$$\frac{dP}{P} = -D_{\text{MOD}} dy \text{ so that } D_{\text{MOD}} = - \frac{dP/P}{dy}$$

Percentage Change in price for a 1% change in a bond's YTM

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### APPROX MODIFIED DURATION

$$P_B = P(y_0 - \Delta y)$$

$$P_A = P(y_0)$$

$$P_C = P(y_0 + \Delta y)$$

$$P'(y_0) = \frac{P(y_0 + \Delta y) - P(y_0 - \Delta y)}{\Delta y - (-\Delta y)}$$

$$= \frac{P(y_0 + \Delta y) - P(y_0 - \Delta y)}{2\Delta y}$$


$$D_{\text{Mod}} = - \frac{P'(y_0)}{P(y_0)} = \frac{P(y_0 - \Delta y) - P(y_0 + \Delta y)}{2\Delta y \times P(y_0)}$$

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Here is the formula for the modified duration  $D_{\text{MOD}}$  and this is what we have used. I have enclosed it in a box.  $D_{\text{MOD}}$  is equal to minus  $P'$  dash  $y$  upon  $P$  dash  $y$ .  $dP$  upon  $dy$  that is  $P'$  dash  $y$  divided by the current price. So, this is the formula that is given here.  $D_{\text{MOD}}$  is equal to minus  $P'$  dash  $y$  upon  $P$  dash  $y$ . Substituting the value of  $P'$  dash  $y$ , we get the formula that is on the right-hand side of this equation.

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- $P(y_0 - \Delta y)$  is the price of the bond if YTM is *decreased* by  $\Delta y$  and
- $P(y_0 + \Delta y)$  is the price of the bond if the YTM is *increased* by  $\Delta y$ .
- $P(y_0)$  is the current price of the bond.



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So, these are the explanations for the terms that we have used  $P$  of  $y_0$  minus  $\Delta y$  is the price corresponding to the YTM decreasing by  $\Delta y$  from  $y_0$  to  $y_0$  minus  $\Delta y$ . Similarly,  $P$  of  $y_0$  plus  $\Delta y$  and  $P$   $y_0$  is the current market price or the current market price of the bond corresponding to a YTM of  $y_0$ .

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- Note that  $P(y_0 - \Delta y) > P(y_0 + \Delta y)$
- Because of the convexity of the price-yield relationship, the price increase  $P(y_0 - \Delta y)$ , for a given decrease in yield, is larger than the price decrease  $P(y_0 + \Delta y)$ .
- Then,

$$D_{\text{Mod}} = -\frac{P'(y_0)}{P(y_0)} = \frac{P(y_0 - \Delta y) - P(y_0 + \Delta y)}{2\Delta y \times P(y_0)}$$

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And now on these are some more details about the figure, which we have already discussed, because of the convexity of the yield price curve, what happens is  $P$  of  $y_0$  minus  $\Delta y$  will be greater than  $P$  of  $y_0$  plus  $\Delta y$  we have already discussed this thing and this arises because of the convexity and then this formula also I have explained.



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- The formula uses the average of the magnitudes of the price increase and the price decrease, which is why  $P(y_0 - \Delta y) - P(y_0 + \Delta y)$  (in the numerator) is divided by 2 (in the denominator).
- $V_0$  is in the denominator to convert this average price change to a percentage, and
- the  $\Delta y$  term is in the denominator to scale the duration measure to a 1% change in yield by convention.

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Now, the important thing is that in arriving at this approximation, we approximated the yield price curve or the change in the price between the point B and the point C as being represented by a straight line and we have assumed that the slope of that particular straight line is given by PC minus PB divided by 2 delta y. So, that is a approximation that we have introduced as we know the change in the change in the price is not linear it is nonlinear it is a convex curve between the YTM and the price.

(Refer Slide Time: 19:00)

### INTEREST RATE ELASTICITY

$$\frac{P(y_0 + dy) - P(y_0)}{P(y_0)} = \frac{P'(y_0)}{P(y_0)} dy = -D \left( \frac{dy}{1 + y_0} \right)$$

since  $P'(y_0) = -D \frac{P(y_0)}{(1 + y_0)}$  (IGNORING CONVEXITY)

$$\text{or } IE_{y_0} = \frac{dP/P_0}{dy/y_0} = \left( \frac{P'(y_0)}{P(y_0)} \right) y_0 = -D \left( \frac{y_0}{1 + y_0} \right) = -D_{\text{Mod}} y_0$$

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Then we will come to the interest rate elasticity, we define interest rate elasticity in as more or less similar to the expressions that are used in economics in macroeconomics. So, we

define interest rate elasticity as the percentage change in price and divided by the percentage change in the YTM expressed as a percentage of the initial YTM.

So, let me repeat, it is the percentage change in price expressed as divided by the percentage change in YTM on the basis of the original YTM  $dy$  upon  $y$ ,  $dy$  is the percentage change in YTM divided by  $y_0$  which is the initial YTM. So, this is the interest rate elasticity  $dP$  upon  $P_0$  is the percentage change in price, percentage change in price divided by percentage change in YTM expressed on the basis of the original YTM.

So, in terms of duration, you can see substituting the value of duration, substituting the expression of duration modified duration is what it is  $P$  dash  $y$  upon  $P_y$ . So, if you substitute this by the, if you substitute this expression by the modified duration, then what we end up with is that the interest rate elasticity is equal to minus of  $D_{mod}$   $y_0$ ,  $D_{mod}$   $y_0$  and which in terms of the Macaulay's duration is minus  $d$  into  $y_0$  upon one plus  $y_0$  using the relationship between  $D_{mod}$  and  $D_{mac}$ . I repeat, when we talk about  $D_{mod}$  we have a negative sign here.

So, we have to account for the negative sign also. And that is why this negative sign is appearing, when we are working out the interest rate elasticity. Interest rate elasticity is equal to minus  $D_{mod}$  into  $y_0$ ,  $y_0$  is the point at which we are working out the interest rate elasticity and the earlier expression here, which I have underlined is the expression for the interest rate elasticity in terms of the Macaulay's duration.

(Refer Slide Time: 21:16)


### A RECAP

$$D_{MOD} = -\frac{P'(y_0)}{P(y_0)} = \frac{1}{P(y_0)} \sum_{t=1}^T \frac{tC_t}{(1+y_0)^t} = \frac{D_{MAC}}{(1+y_0)}$$

$$DV01 = -\frac{dP}{dy} = D_{MOD} \times P = \frac{D_{MAC}}{1+y} \times P$$

$$\frac{\Delta P}{P} \Big|_{y_0, P_0} = -D_{MAC} \frac{dy}{(1+y_0)} + C \left( \frac{dy}{(1+y_0)} \right)^2 = -D_{Mod} dy + C \left( \frac{dy}{(1+y_0)} \right)^2$$

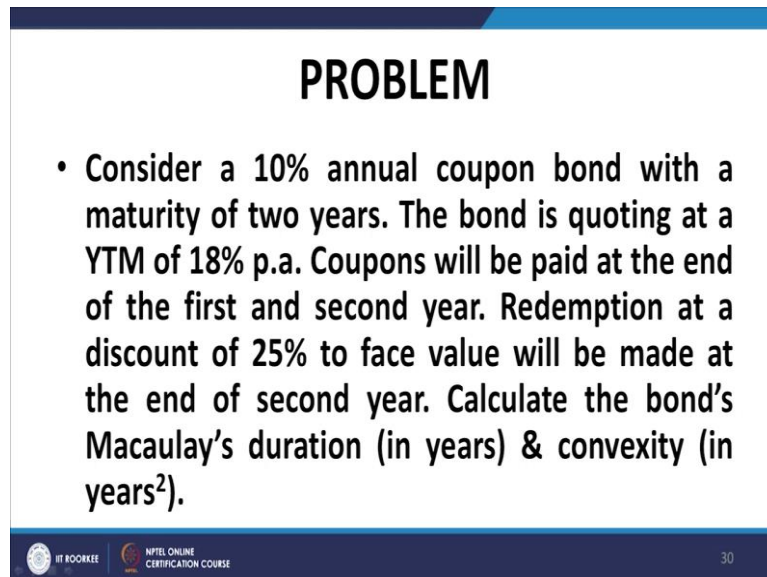
$$IE = \frac{dP/P_0}{dy/y_0} = -D_{MAC} \left( \frac{y_0}{1+y_0} \right) = -D_{Mod} y_0$$



So, this is a recap of what all we have done so far the various definitions of interest rate risk measures the DV01,  $D_{Mac}$ , the Macaulay's duration, modified duration and the interest rate

elasticity and I have also placed here the expression for the percentage change in price expressed in terms of the duration and convexity of the bond.

(Refer Slide Time: 21:44)



**PROBLEM**

- Consider a 10% annual coupon bond with a maturity of two years. The bond is quoting at a YTM of 18% p.a. Coupons will be paid at the end of the first and second year. Redemption at a discount of 25% to face value will be made at the end of second year. Calculate the bond's Macaulay's duration (in years) & convexity (in years<sup>2</sup>).

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So, let us take up a problem, consider a 10 percent annual coupon bond with a maturity of 2 years, consider a 10 percent annual coupon bond with the maturity of 2 years. The bond is quoting at a YTM of 18 percent per annum coupons will be paid at the end of the first year and the second year redemption at a discount of 25 percent of face value will be made at the end of the second year. Calculate the bonds Macaulay's duration in years and convexity in year squared.

Let us read the problem again. Consider a 10 percent annual coupon bond with a maturity of 2 years, the bond is quoting at a YTM of 18 percent. Coupons will be paid at the end of the first year and second year redemption at a discount of 25 percent to face value will be made at the end of the second year. Calculate the bond's Macaulay's duration in years and convexity in years square.

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$$D = \frac{\sum_{t=1}^T \frac{tC_t}{(1+y_0)^t}}{P(y_0)} = \frac{\sum_{t=1}^T \frac{tC_t}{(1+y_0)^t}}{\sum_{t=1}^T \frac{C_t}{(1+y_0)^t}} = \frac{\frac{1 \times 100}{1.18^1} + \frac{2 \times (100 + 750)}{1.18^2}}{\frac{100}{1.18^1} + \frac{(100 + 750)}{1.18^2}}$$

$$= \frac{84.75 + 1220.91}{84.75 + 610.46} = \frac{1305.66}{695.21} = 1.8781$$

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## PROBLEM

- Consider a 10% annual coupon bond with a maturity of two years. The bond is quoting at a YTM of 18% p.a. Coupons will be paid at the end of the first and second year. Redemption at a discount of 25% to face value will be made at the end of second year. Calculate the bond's Macaulay's duration (in years) & convexity (in years<sup>2</sup>).

This is quite straightforward quite simple, we assume the face value to be 1000 although it is not necessary, you can see from here that the face value whatever you assumed will be cancelled between the numerator and the denominator, but for convenience, we may assume that the face value is 1000. I repeat the outcome that you get the result that you get for duration and convexity are independent of face value.

So, whatever face value you take 100, 1000 whatever you may take or you may take F as the face value, you will end up with the same expression for the duration. So, 1 into... the formula for the duration is given on the left-hand side summation of TCT divided by 1 plus y0 to the power t divided by P y0. Now, y0 here is given at 18 percent you can see here in this problem YTM is 18 percent, coupon rate is 10 percent.

So, why we have 1 into 100 assuming a face value of 1000 and 2 into 100 plus the redemption value which is 75 percent of the face value. So, I take it as 750 and you discount these quantities for 1 year and 2 year respectively. And in the denominator, you use the DCF base price which is equal to 100 divided by 1 plus 1.18 plus 100 plus 750 divided by 1.18 whole square. So, when you simplify this expression, you get the duration as 1.8781 years. Please note the year and the dimensions of duration are those of time. So, because all the expressions are in annualised, so we shall get the duration in years.

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$$C = \frac{\sum_{t=1}^T \frac{t(t+1)C_t}{(1+y_0)^t}}{2P(y_0)} = \frac{\frac{1 \times 2 \times 100}{1.18^1} + \frac{2 \times 3 \times (100 + 750)}{1.18^2}}{2 \times \left( \frac{100}{1.18^1} + \frac{(100 + 750)}{1.18^2} \right)}$$

$$= \frac{169.50 + 3662.73}{84.75 + 610.46} = \frac{3932.23}{2 \times 695.21} = 2.7562$$

The convexity also we can do similarly, it is shown here the t, for t equal to 1 we see, we multiply the discounted cash flow 100 upon 1.18 by t into t plus 1 that is 1 into 2 and for t equal to 2, we multiply the discounted cash flow that is 750 divided by 1.18 whole square by 2 into 3, the denominator contains 2 into the current market price. When you simplify all this expression, you get a convexity of 2.7562 percentage square, I am sorry year squared.

(Refer Slide Time: 25:16)

LET FACE VALUE BE	1000		
YEAR		1	2
COUPON PAYMENT	0.1	100	100
REDEMPTION VALUE	0.75		750
TOTAL CASH FLOWS		100	850
YTM	0.18	0.18	0.18
DISCOUNT RATE		0.847457627	0.71818443
CURRENT PRICE	695.20253	84.74576271	610.456765
TIME WEIGHTED CASHFLOWS	1305.6593	84.74576271	1220.91353
DURATION	1.8780992		
t(t+1) WEIGHTED CASHFLOWS	3832.2321	169.4915254	3662.74059
CONVEXITY	2.7561983		

And this is the Excel sheet working out the same expressions in Excel, we get a duration of 1.87, which is precisely what we got in the earlier calculation and we get the convexity as 2.756, which is also coinciding with the value that we got in the earlier expression.

(Refer Slide Time: 25:35)

## PROBLEM

- On the basis of the value of duration & convexity arrived at in the previous question,, calculate the price change (in %) corresponding to a 5% increase in the ytm from the above figure.

Then next problem which is based on the results that we have obtained in the previous problem, on the basis of the value of duration and convexity arrived at in the previous few calculation in the previous question, calculate the price change in percentage corresponding to a 5 percent increase in the YTM from the above figure, the above figure means 18 percent which was given in the previous problem.

(Refer Slide Time: 26:00)

<b>GIVEN YTM (y)</b>	<b>0.18</b>
<b>CHANGE IN YTM (dy)</b>	<b>0.05</b>
<b>HENCE, dy/(1+y)</b>	<b>0.0423729</b>
<b>% CHANGE IN PRICE DUE TO DURATION</b>	<b>-0.0795805</b>
<b>% CORRECTION DUE TO CONVEXITY</b>	<b>0.0049486</b>
<b>TOTAL PRICE CHANGE</b>	<b>-0.0746318</b>

So, here are the calculations, the given YTM is 18 percent, the change in YTM is 5 percent. So,  $dy$  upon  $1 + y$  is equal to  $0.05$  divided by  $1.18$  and that comes to  $0.0423$ . So, the percentage change in price due to duration is given by  $-d$  into  $dy$  upon  $1 + y$  which works out to minus  $0.795$  percent.

And the correction due to convexity works is given by  $C$  into  $dy$  upon  $1 + y$  whole squared, and which works out to  $0.004948$  and the total price change if we incorporate the correction on account of convexity in that figure arrived at using duration works out to  $7.46$  percent negative.

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<b>ACTUAL PRICE CHANGE</b>			
<b>LET FACE VALUE BE</b>	<b>1000</b>		
<b>YEAR</b>		<b>1</b>	<b>2</b>
<b>COUPON PAYMENT</b>	<b>0.1</b>	<b>100</b>	<b>100</b>
<b>REDEMPTION VALUE</b>	<b>0.75</b>		<b>750</b>
<b>TOTAL CASH FLOWS</b>		<b>100</b>	<b>850</b>
<b>YTM</b>	<b>0.23</b>	<b>0.23</b>	<b>0.23</b>
<b>DISCOUNT RATE</b>		<b>0.81300813</b>	<b>0.66098222</b>
<b>CURRENT PRICE</b>	<b>643.1357</b>	<b>81.30081301</b>	<b>561.834887</b>
<b>% PRICE CHANGE</b>	<b>-0.0749</b>		

This is the actual price change and the actual price change we work out on the basis of actual calculation of the revised price at the revised YTM of 23 percent. And we get a figure of 7.49 percent negative as the percentage price change, this is the actual price change worked out on the basis of the DCF formula.

So, again we see that notwithstanding the fact that this is such a massive price change, or sorry YTM change of 5 percent we still get very close approximation or very close congruence between the figures that we get on the basis of the actual calculation and the calculation based on duration and convexity 7.46 percent versus 7.49 percent.

(Refer Slide Time: 28:03)

**IMMUNIZATION**

**Consider a case when the interest rates increase immediately after a bond issue.**

Obviously, the income from reinvested coupons will increase due to this increase in interest (reinvestment) rates.

However, the anticipated price at the end of the holding period will decrease and hence, the expected capital gains would decrease.

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Now, we come to a very, very important concept, the practical use of duration. Now, as I have mentioned right at the beginning of this class and also in the previous lecture, that if a person holds a bond for a period which is not equal to its maturity, then he gets exposed to two types of risk the interest rates change its reinvestment income or the income that he would derive from the reinvestment of his coupon payments would change on the one hand. And number two, the selling price of the bond at which it would exit the investment on the date of maturity of his holding period would also change.

Now, the interesting part is the most interesting part is that suppose there is an increase in interest rates, then the reinvestment income increases on the one hand, but the price at which you liquidate the investment on the end of the holding period decreases. So, while there is a positive effect for the investor on an increase in interest rates, due to an increase in reinvestment income, there is also a negative effect due to the fall in the price at which he would be able to liquidate his investment.



The converse is the case. If the interest rates decrease, if the interest rates decrease, his reinvestment income will decline which will operate to its detriment. However, the price at which he would liquidate the investment would increase. And again, the two factors would oppose each other. Now, the important thing is because they are opposing each other, they would annul each other to some extent, the extent of annulment here is the important part, the extent of annulment depends on the holding period.

If the holding period of the investor is very close to the maturity, the impact of a increase in interest rates on reinvestment income would be much higher than the impact on the price at which he would be able to liquidate his investment. Why? Because the reinvestment periods number much more. However, the discounting periods if you are very close to the maturity of the bond, the discounting period should be much less.

And as a result of which overall, if there is an increase in interest rates and the holder holds the bond for a long period close to its maturity is reinvestment income would increase, the fall, the decline in his capital gains would be marginal. On the other hand, the converse is the case if he is holding the bond for a short period, the change in price would be significant the change in price at which he will exit the investment, at which he will liquidate the investment would be significant.

Why? Because there are a number of discounting periods yet in the life of the bond. However, the reinvestment income effect or the increase in reinvestment income would be marginal because he would have had little time to reinvest his coupon payments for up to the date of his holding period. So, the annulment first thing, number one takeaway, that the effect of any change in interest rates whether it is an increase or a decrease, operates in two opposing manner.

Number 1, the reinvestment income, number 2 the capital gains, an increase would increase reinvestment income, decrease capital gains and vice versa. So, they operate opposite to each other and annul each other to some extent. The second takeaway is that the extent of annulment, the extent of neutralisation of one by the other depends on how long the investor will remain invested in that particular bond.

Shorter the holding period, shorter would be the impact of reinvestment income, shorter would be the benefit of an increase in interest rates. But if the longer the holding period, then what happens the reinvestment income increases significantly if the interest rates increase, however the loss on account of decline in the price of the bond is marginal. So, these are two

factors which you take away, which we will carry forward when we talk about the quantitative analysis of what I have just explained.

I repeat number 1, the impact of an increased change in interest rates is two-fold. One, on the reinvestment income two on the capital gains, the impact is opposite to each other, one may increase the investment income decrease in price and vice versa. And number two, the amount of annulment or the extent of annulment due to these two adverse opposite impacts depends on the holding period of the investor. I shall continue from here after the break.  
Thank you.