Security Analysis & Portfolios Management
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Lecture 52
Capital Asset Pricing Model - II

Welcome back. So, before the break I was discussing about the capital asset pricing model, the derivation of the capital asset pricing model. And as I explained the Markowitz model requires that we maximize the Sharpe ratio that every investor maximizes the Sharpe ratio in order to arrive at his efficient frontier.

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$$\Theta_T = \frac{\bar{R}_T - R_f}{\sigma_T} = \frac{\bar{R}_P + x\bar{R}_S - (R_f + xR_f)}{\sigma_P\left(1 + x\rho\frac{\sigma_S}{\sigma_P}\right)} \quad \text{— 2(A)}$$

$$\Theta_T > \Theta_P \Rightarrow \bar{R}_S - R_f > \rho\frac{\sigma_S}{\sigma_P}\left(\bar{R}_P - R_f\right) = \beta_S\left(\bar{R}_P - R_f\right) \quad \text{— 2(B)}$$

$$\text{At equilibrium } \bar{R}_S - R_f = \beta_S\left(\bar{R}_P - R_f\right) \qquad (3)$$

And considering that particular aspect or developing on that particular aspect rather what we ended up was that if the equation number 2 B you can see here, if the left hand side of equation number 2 B is a greater than the right hand side, then what people would do is they would invest more and more in security as they would borrow at its figured and which keep on investing in security as to add it to their portfolio P to form the portfolio T.

And of course, if the left hand side is less than the right hand side of equation 2 B, then what would happen? The inverse process that is shorting security S and investing at the risk that would take place. So, the bottom line is that in this situation at equilibrium, it follows that it is absolutely necessary that the equality should hold. It is only when there would be equality that the market would be a stable, market would be in equilibrium.

So, and this expression that we have here forms the premise of the CAPM model on the basis of the assumptions that we have already made as I will explain now. Now, under the CAPM assumptions, all investors want to maximize the Sharpe ratio. This is the input from the Markowitz model because the CAPM model builds on the Markowitz model. It assumes that investors behave as per the Markowitz framework, the investors take Markowitz model for granted, for constructing of their optimal portfolios.

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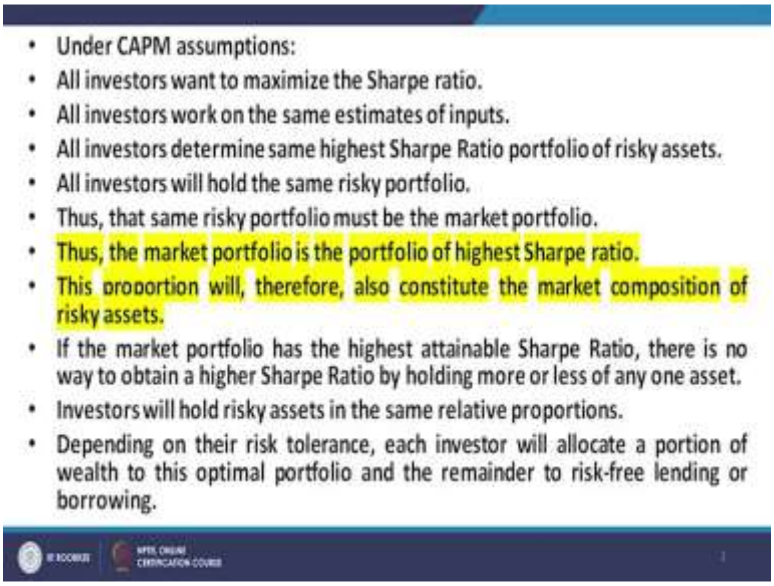

So, this input is from the Markowitz model that all investors want to maximize the Sharpe ratio. All investors work on the same estimates of input. This we have explained explicitly assumed, when we talk about the assumptions in the CAPM model that all investors in so far as the estimates that go into the development of the Markowitz model, development of computation of the efficient frontier are assumed to be same by all the investors.

That means all the investors because they have the same inputs and they have the same objective function that is the Sharpe ratio, they have the same objective function, they have the same inputs. This is the assumption of the CAPM model. The objective function is the assumption of the makeovers model and on the premise that the 2 hold together, it means that every investor, now this is important, every investor will end up with the same higher Sharpe ratio.

Because the inputs that are going into the calculation of the Sharpe ratio is the same. So, the output must be the same and the maximum Sharpe ratio as calculated by each and every investor would be the same. That means what? All investors will hold the same risky portfolio. Why? Because they are all of them are maximizing the Sharpe ratio. So, all of them have the same inputs, all of them have the same objective functions, they end up with the same output, that is they maximize the same combination or they invest in the same combination of risky assets.

So, all investors have the same risky portfolio. That means what? That means, because each and every investor is, no this is important you need to understand this. Each and every investor is holding the same set of securities in the same proportion. What does it mean? That means the market outstanding of that each and every security must also be in the same proportion.

For example, let me take an example. Let us say we have got 3 investors x, y and z and let us say we have got 3 securities a, b and c. Let us say, each of these investors x, y and z are holding the securities in the ratio 3 is to 2 is to 1, they hold security a in b and c, x hold security a, b and c in the ratio 3 is to 2 is to 1, y holds the security a, b and c in the ratio 3 is to 2 is to 1, z holds the securities a, b and c in the ratio 3 is to 2 is to 1. What will be the market outstanding of securities a, b and c? They also to have to be in the ratio 3 is to 1 that is what we are trying to convey here.

That because each and every investor is holding the same proportion of risky assets, that proportion of risky assets must be replicated or must be the market portfolio must represent the outstanding that are there in the market of each of those securities. Thus the same risky portfolio must be the market portfolio. Thus the market portfolio is a portfolio with the highest Sharpe ratio, because each investor is holding that portfolio which gives them the highest Sharpe ratio and the highest Sharpe ratio is the same for all the investors.

Therefore, all the investors are holding the same proportion of risky securities. And therefore, the same proportion of risky securities must be reflected in the market in outstanding, market outstanding must consists of the same proportion of securities of risky assets. And therefore, the market portfolio must also be the portfolio which has the highest Sharpe ratio. The proportion will therefore, also constitute the market composition of risky assets that I have already explained.

Now, if the market portfolio as its highest attainable Sharpe ratio, there is no way to obtain a higher Sharpe ratio by holding more or less of any one asset. We have already said that the market portfolio is a portfolio with the highest Sharpe ratio. So, you cannot improve upon that ratio by either buying or selling of any security. And therefore, you cannot obtain a higher Sharpe ratio by holding more or less of any asset. Now, as I menti1d, investors will hold the risky assets in the same relative proportion because of they have the same inputs that go into the Markowitz objective function depend.

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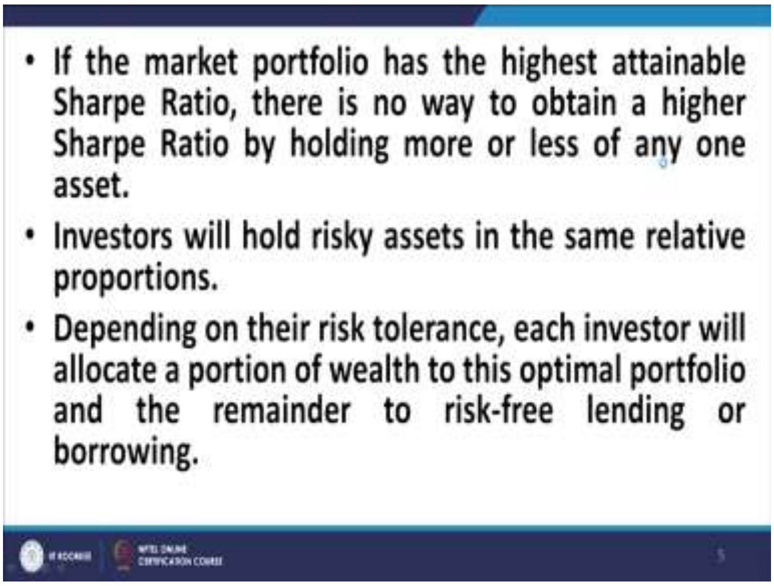

Now, how does it actually operate? You will contend that how can it be that every investor has the same portfolio. If they do not have the same portfolio, they have the same proportion of risky assets in their risk comp1nt of their portfolio, the actual portfolio will comprise of two parts, the risk free asset and the risky portfolio. The proportion of risky assets in the risky portfolio will be the same for each investor that is the important part. The proportion of risky assets, risky securities in the risky portfolio will be the same across all the investors.

But they do not hold the risky portfolio alone, what they hold is a combination of the risky portfolio and the risk free asset either long or short whether riskless lending or riskless borrowing, what does it depend on? It depends on the risk return profile, it depends on the risk return indifference curve, how risk taking or risk averse?

The investor is will determine what is the composition of the risk free asset and what is the composition of the risky portfolio? The basic thing is as far as the risky portfolio is concerned the content of all the securities in the risky portfolio remains uniform across all the investors. That is the outcome of the Sharpe model. It is not that everybody is investing in just the risky portfolio al1, no, no, no. It is the risky portfolio plus the risk free asset.

A combination would be determined by the risk profile or the risk attitude of the investor, risk averse investor will hold more content of a risk free asset and less content of risky asset.

A risk taking individual will invest more in the risky portfolio, less in the risk free asset, he may even borrow at the risk free rate and invest in the risky asset.

So, that is how the optimal portfolio for a particular investor will be determined, it will be determined by the interaction of his indifference map or his utility function with the efficient frontier. So, applying the CAPM improvement rule, what we end up with is because the portfolio P that we have talked about is now the market portfolio because it is the market portfolio yield by all the investors.

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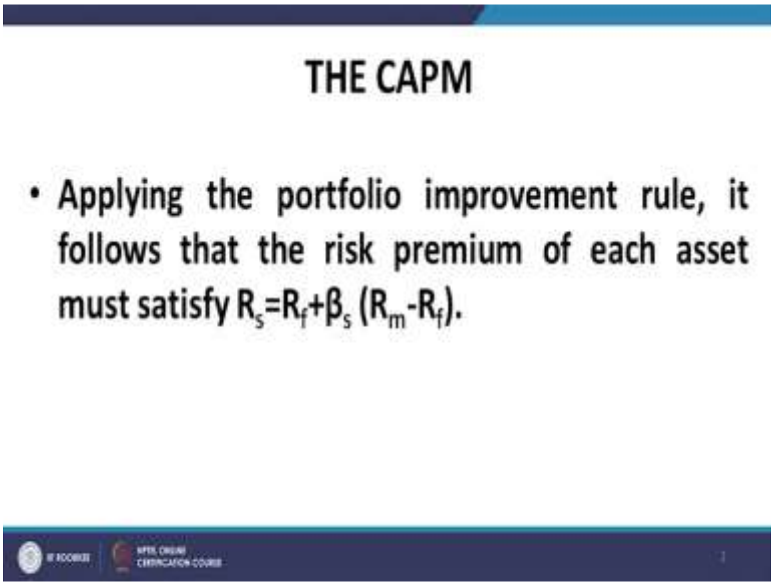

So, what we end up with that applying the portfolio improvement rule, it follows that the risk premium on each asset must satisfy this linear equation Rs is equal to Rf plus beta s into Rm minus Rf. Where Rs is the expected return please note this on security s and Rm is the expected return on the market portfolio.

Now, I take up another approach a slightly more direct approach to the derivation of the CAPM model. This the earlier approach that I have taken up was retained to bring to you a certain rational behind the behind the CAPM approach which this particular derivation probably camouflages to some extent, it covers up to some extent.

But the earlier approach was more may not be so direct but it was more educative, more informative, more logical, this approach is more mathematical and more precise you may say, but it misses out on the nuances of the CAPM model. But nevertheless, let us take it up.

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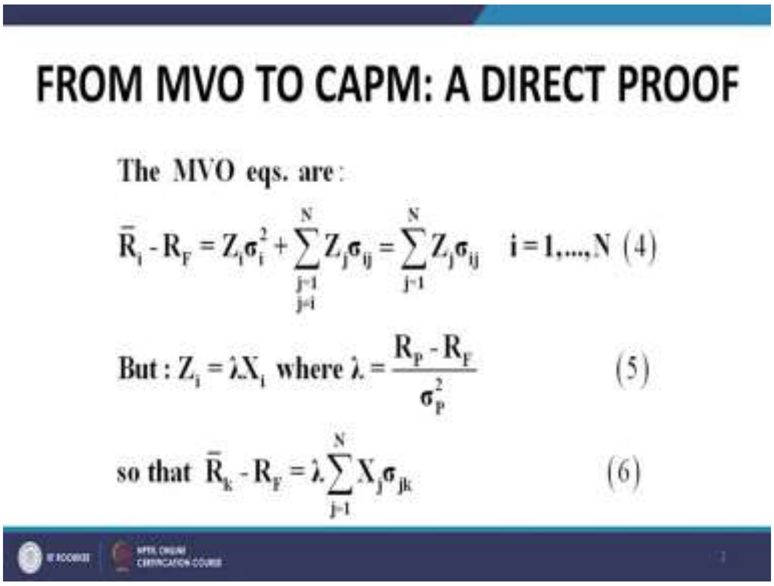

The mean variance optimization equations are the equations that we are now very much accustomed to, this is equation number 4 on this slide. We have Zi is equal to lambda Xi, where lambda is equal to ERP that is expected return on portfolio P minus RF divided by sigma P square. We have discussed this in lot of detail when we talked about the three security problem. So, RK. R bar K minus RF is given by lambda summation X, this is simply substituting Zi equal to lambda Xi in equation number 4, what we get is equation number 6.

Please note, I have written equation number 4, the right hand side of equation number 4 in a more concise form where I have removed the constraint j unequal to i because when j is equal to i, what do we get? We get Zi, sigma i square which is nothing but the first term on the right hand side this term, and that means what?

That means that if I remove this constraint j unequal to i, I can incorporate the first term within the summation itself. That is precisely what I have d1. So, in the second term or the second equation that is equation number 5 we have used Zi is equal to lambda Xi and using this expression, we have rewritten equation number 4 as equation number 6.

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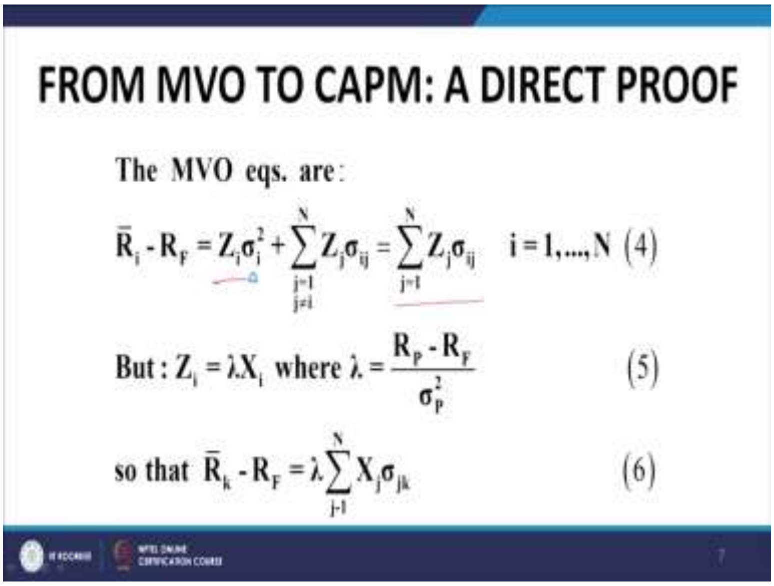

Now, by definition the covariance between securities j and k is given by the covariance between the returns on securities j and k is given by the expression that is the top equation on this slide, sigma jk is equal to expected value of Rj minus Rj bar into Rk minus Rk bar. Multiplying by j or rather Xj, I am sorry multiplying by Xj and summing over j equal to 1 to n, what we get is equation number 7. Where we have assumed the, where we have used the property of expectation that it distributes over the excess

So, that being the case what we have now here is j equal to 1 to n summation Xj sigma jk is given is equal to the expression that is equation number 7 here.

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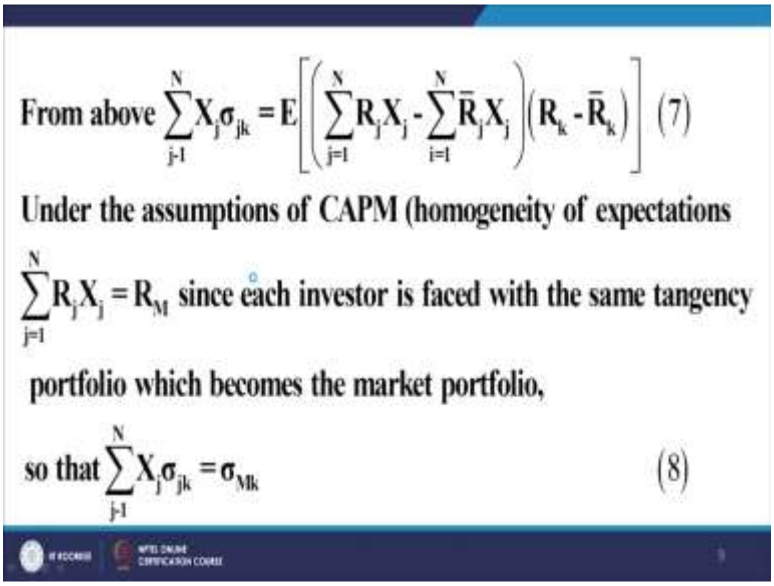

Now this is what we have from the earlier slide equation number 7. Now under these CAPM assumptions which I discussed in a lot of detail few minutes back, summation of Rj, Xj over all the securities because everywhere let me repeat once more because every investor in the market is holding the risky securities in the same proportion. Therefore, that proportion must also be the proportion of market outstandings. We use this property here and we write, what do we write?

We write R summation j equal to 1 to n Rj, Xj as RM, a w8ed average. Because now, you see the Xj so far is the proportion of risky securities in the investor's portfolio. But because that same proportion is the market outstanding, I can take this Xj as the market outstanding multiplied by the respective return, expected return and what do I get? I get the expected return on the market that is the that is the rational underlying this equation summation j equal to 1 to n, Rj Xj is equal to RM.

So, that what we end up with is a using equation number 7 and using this particular property what we get is summation j equal to 1 to n Xj sigma jk, Xj sigma jk is equal to sigma Mk. This is equation number 8, where we are simply substituted for summation RJ Xj and R bar j Xj in terms of the market portfolio.

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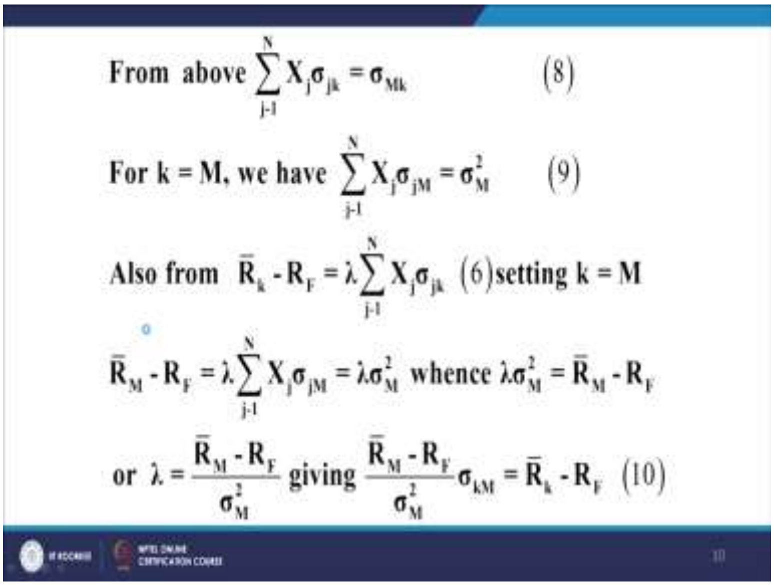

Now, we return to equation number 8 which was there on the previous slide. Suppose, we take k is equal to M, then what we end up with is j equal to 1 to n summation Xj sigma jM is equal to sigma M square. Also from equation number 6 what we have, Rk minus RF is equal to lambda summation j equal to 1 to n Xj sigma jk. Setting k equal to M in equation number 6, what do I get? R bar M minus RF is equal to lambda summation Xj summation jM.

Now, if substituting from equation number 9, here, what I get is RM minus RF is equal to lambda sigma M squared, where I have substituted from equation number 9 in equation number, let us call this equation number 9 A. So, I have substituted from equation number 9, in equation number 9 A on the right hand side and what I get is R bar M minus RF is equal to lambda sigma M square. Therefore, what is lambda? Lambda is equal to R bar M minus RF divided by sigma M square, lambda is equal to RM minus RF divided by sigma M square.

So, now substituting in equation number 6, what I get is Rk minus RF is equal to lambda into what is this expression? This expression is sigma Mk or sigma kM. This is summation j equal to 1 to n Xj sigma jk is equal to sigma Mk and lambda is equal to what? Lambda is equal to R bar M minus RF divided by sigma M square. So, we have got both these terms on the right hand side. We have got lambda right hand side of equation number 6, we have got lambda is equal to R bar M minus RF upon sigma M square and we got sigma Xj sigma jk is equal to sigma Mk. So, substituting these values, what we get is equation number 10.

Now, sigma kM divided by sigma M squared is nothing but beta kM or the regression coefficient of the returns on security k regressed upon the returns on security or mark it M. So, that is nothing but the CAPM model.

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Now, $\sigma_{jk} = E\left[\left(R_j - \bar{R}_j\right)\left(R_k - \bar{R}_k\right)\right]$ so that

$$\sum_{j-1}^{N} X_j \sigma_{jk} = \sum_{j-1}^{N} X_j E\left[\left(R_j - \bar{R}_j\right)\left(R_k - \bar{R}_k\right)\right]$$

$$= E\left[\left(\sum_{j=1}^{N} R_j X_j - \sum_{i=1}^{N} \bar{R}_j X_j\right)\left(R_k - \bar{R}_k\right)\right] \qquad (7)$$

Now, a quick relook at the relationship between CAPM and SIM model, single index model, the single index model is represented by the first equation on this slide. And the CAPM model is represented by this second equation. If you look at the second equation, the second equation can be written in the form of the third equation. This let us call this equation number 1, let us call it 2, let us call it 3.

And from the third equation, what we find is that the CAPM model on the basis of the assumptions as to the market and the investor behavior of the various constituents that interplay in the investment decision. What we end up with that alpha of the single index model in the long run should approach 0. And what does it mean? It means that in the long run, the excess return on any security in the long run would be determined by its exposure to the market or the systematic risk, you may say and times the excess return on the market.

Let me repeat, the excess return on any security in the market will be determined by its relationship with the market returns multiplied by the excess return on the market, which is equation number 3. And for any particular observation, we can write the CAPM model as equation number 4. And equation number 5, gives you the various assumptions that underlie both the single index and the CAPM model.

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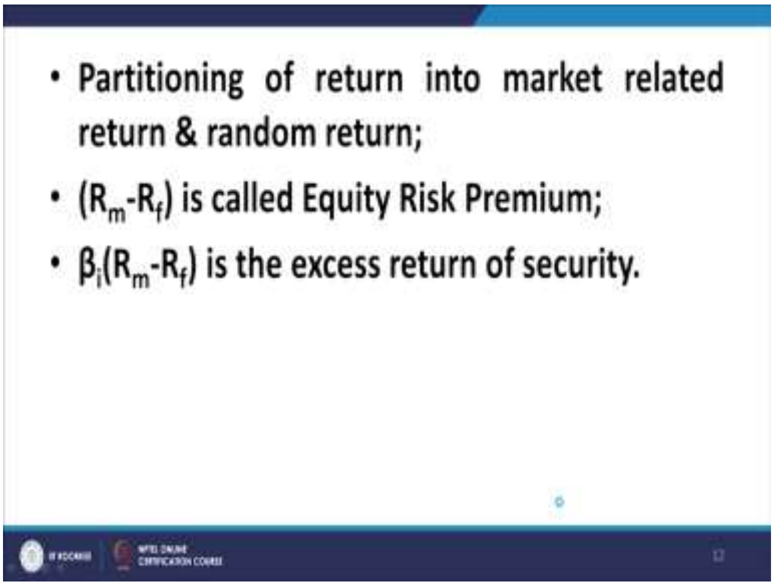

So, partitioning of return into market related return and random return that is the assumption of the single index model and Rm, the expression Rm minus Rf is called the equity risk premium and beta Rm minus Rf is called the excess return of security i. So, these are what you call the inputs that go into the CAPM model.

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What does the CAPM model tell us? The CAPM model tells us number 1, that standalone risk can be measured by variance. In fact, the standalone risk should be measured by variance. Why? Because it is the total risk of a security, the total risk of a security is likely to contain a significant component of the residual risk. And therefore, it is appropriate that when you are handling individual securities, you should focus on total risk because the component of unsystematic risk or the residual risk would also be significant when we are talking about an isolated security.

So, in so far as an individual security is concerned, we should focus on the measuring of the risk of individual securities by the standalone risk which is the variance or the standard deviation risk. But, standalone risk is not the appropriate measure of risk in the stock market. You can see here in this equation for this CAPM model, equation number 2, if you look at equation number 2 carefully, the expected return on a security S is determined only by its beta and what is beta? Beta is the regression of the securities returns and we service the market returns.

In other words, we depicts the systematic relationship between the security returns and the market returns. So, that means what? That means, the expected returns on a given security are influenced by or a related to the market returns rather than the total, market risk I am sorry, rather than the total risk. The risk on a particular security, individual security is, I am sorry, the return on a particular security, individual security, the expected return on an individual security is related to the expected return on the market portfolio.

That is that means what? That means, we are focusing on the systematic relationship between the security and the market and it is the systematic relationship which generates returns for the individual security and it is the component of risk which is random, which is unsystematic, which is not rewarded by the market, why does the market not do so, why does the market not reward unsystematic risk, the market does not reward unsystematic risk because the market feels that all the market players have good enough portfolios, sufficiently diversified portfolios have undertaken sufficient diversification to eliminate the unsystematic risk to the minimum level to the insignificant level.

And therefore, the market says that you I will reward you only if you take the systematic risk. If you or the return that is going to be derived by a security, the expected return that is going to be derived by security would be determined by how much exposure, it has to the market in terms of the market fluctuations, how large is the fluctuations in the amplitude of the given security in relation to the market rather. So, that is the important thing. So, a standal1 risk is not the appropriate measure of risk in the stock market.

I repeat as far as the stock market is concerned, it is the market risk that is important as per the CAPM assumptions. The larger is the systematic risk that you are willing to take, a larger would be your expected return. But the larger the total risk you are going to be willing to take may not necessarily be rewarded by larger expected returns. Let me repeat this fundamental statement.

The larger is the systematic risk that you are willing to take. The risk associated with the market fluctuations, the larger would be the expected return on your portfolio or depending on the value of beta there (())(23:31) the regression coefficient. And the, not necessarily so, that larger is the total risk that you are taking may be rewarded by larger expected returns. So, beta measures, beta offers a method of measuring the risk of an asset that cannot be diversified away. That is the market risk.

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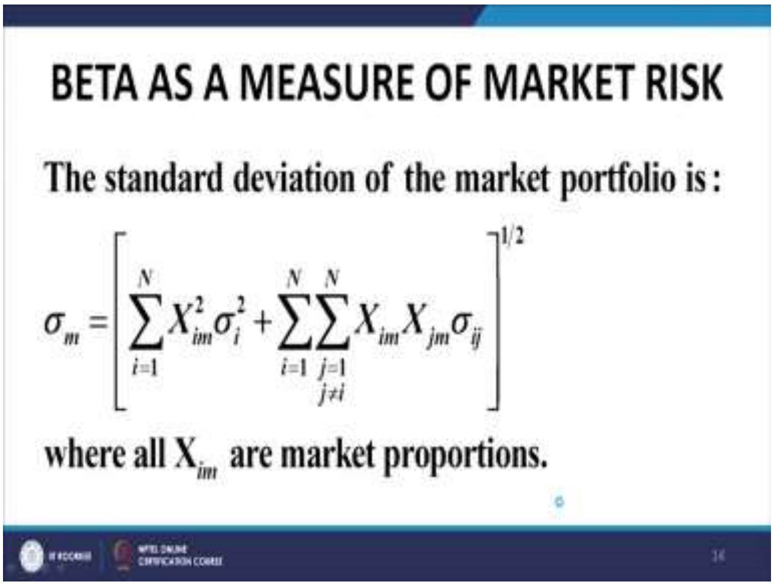

Now beta is a measure of market risk. I just said that beta offers a method of measuring the market risk let me try to justify this statement. The standard deviation of a market portfolio is given by this expression that you have on this slide. We have all Xim’s are market proportions. I repeat, the standard deviation of the market portfolio, you are talking about the market portfolio. And because you are talking about the market full portfolio, the standard deviation of the market portfolio would be determined by the market proportions and therefore, all the Xim’s that are included here are the market proportions.

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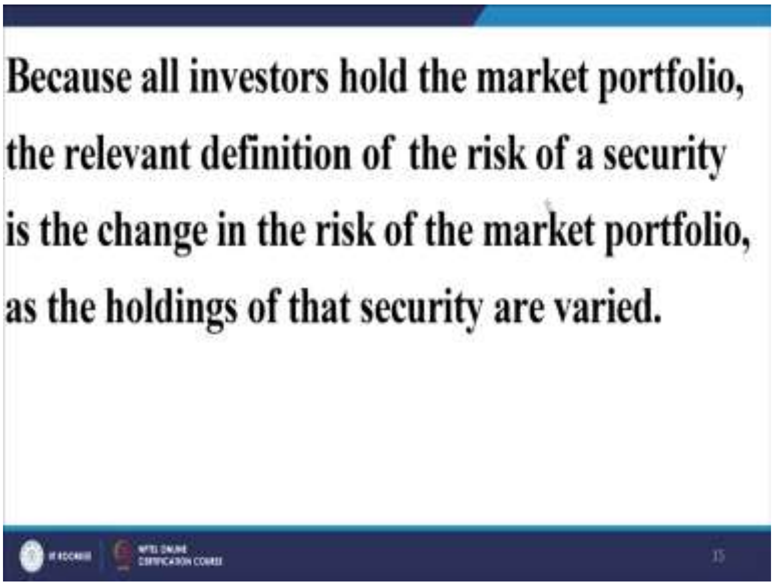

Now because all investors hold the market portfolio, this is fundamental because all investors or the market portfolio, the relevant definition of the risk of a security is the change in the risk of the market portfolio as the holdings of that security are varied. Let me repeat this statement because all investors hold the market portfolio, the relevant definition of risk of a security is the change in the risk of the market portfolio as the holdings of the security are varied.

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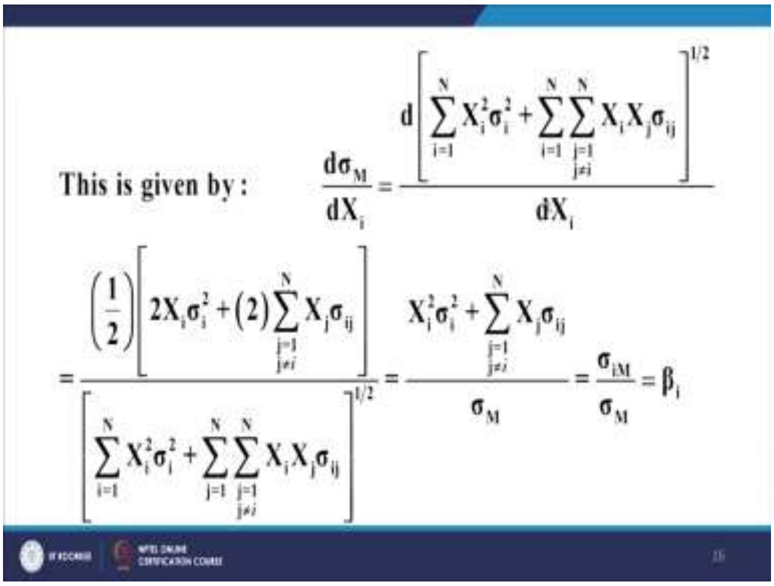

So, what do we do? We differentiate the market standard deviation by the composition of various securities. In other words, we work out d sigma M upon d Xim and what we end up with after some algebra is the value of beta i. So, beta i it is that represents the market risk in the CAPM framework. Therefore, it is the CAPM framework usually is also called the beta risk framework. It is the framework where the beta encapsulates the relationship between the expected returns and the expected returns on the market and the expected returns on the security.

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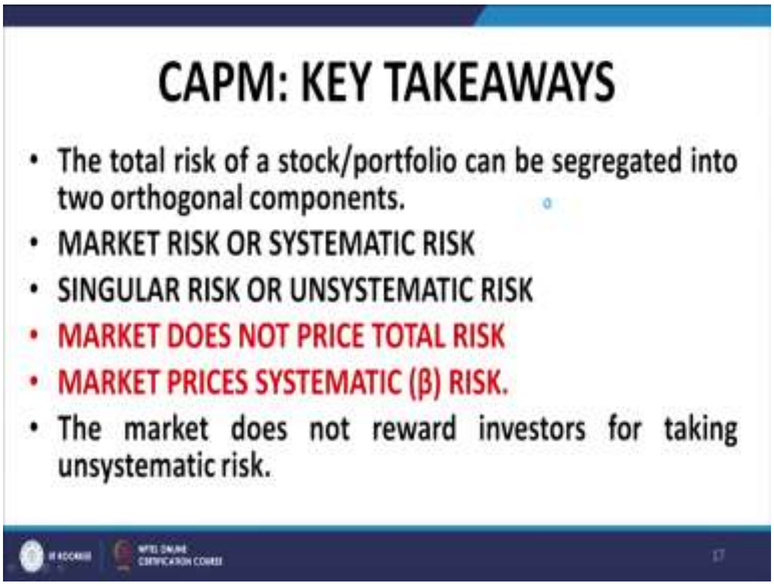

So, let us know recap the key takeaways from this gap model, the total risk of a stock oblique portfolio can be segregated into 2 orthogonal components, market risk or systematic risk and singular risk or unsystematic risk. Market does not prize total risk, I repeat the statement market does not prize total risk, if you take more total risk, you may not necessarily be rewarded by our expected return. But if you take higher systematic risk, you would be rewarded by higher expected returns. This is the philosophy of the CAPM model. Market does not reward investors for taking unsystematic risk.

Let us understand the implications of the above takeaways. Consider a portfolio P having sigma P equal to 6 percent and beta equal to 1. Let market sigma, sigma M be 4 percent, let Rf be 3 percent and Rm be 12 percent. Let us assume that we have a portfolio and a market having given parameters. Then we have using the CAPM model, what do we get? We get RP is equal to 12 percent and sigma systematic of portfolio P is 4 percent and the unsystematic risk is 4.47 percent.

Thus on the average portfolio P will give expected return of 12 percent. Of course, every observation would not return would not yield twelve percent, there would be a strong random comp1nt that contributes to the standard deviation of 4 point 4.47 percent. Now, compare this portfolio P with another portfolio Q that has sigma Q equal to 6 percent, sigma Q is 6 percent, what a sigma P? Sigma P was also 6 percent. So, P and Q have the same total risk, but what is special about O? Beta is 1.50, what was beta of P? Beta of P was 1.00.

So, the portfolio Q has higher systematic risk than portfolio P but portfolio Q has the same total risk as portfolio P. I repeat portfolio Q has higher systematic risks in portfolio P but portfolio Q has the same total risk as portfolio P, what is the implication? The expected return on the portfolio Q it will be 16.5 percent, the expected return on portfolio Q will be 16.5 percent. What was the expected return on portfolio? It was 12 percent. So, notwithstanding that fact that portfolio P and Q have the same level of total risk. Portfolio Q is awarding you a higher expected return of 16.5 percent compared to portfolio P which will giving you 12 percent.

This example clearly brings forth the fact that market may not reward total risk with higher expected return. The total risk on P and Q is the same. So, if market was to reward total risk than expected returns on P and Q would not have to be the same or that is not the case. What is happening here is that the expected returns in portfolio Q are higher than the expected return on portfolio P and why is that? Because the systematic risk of portfolio Q as captured by beta is more compared to the systematic risk of portfolio P because if beta is lower.

And because Q has a higher systematic risk and because market rewards systematic risk, so because Q has higher systematic risk if Q ends up with a higher expected returns. So, that is the relationship between P and Q. So, an investor who is investing in Q can expect a higher return because he is taking a higher systematic risk which is concerned by the market to be relevant risk. Let us look at another portfolio S, what is the (())(30:26), what are the Cardinals of S? The Cardinals of S are RS is equal to 12 percent, sigma systematic, I am sorry, the Cardinals of S are sigma S is equal to 4 percent and beta is equal to 1.

What does it mean? It means that it means RS is equal to 12 percent but what about the systematic risk of S? A systematic of risk of S is only 4 percent and it has no unsystematic risk. So, what does it mean? It means that the total risk of S is only 4 percent. What was the total the total risk of P? The total the total risk of P is 6 percent. What does it mean? Let us try to understand this. It means that the total risk of P and S is the same, I am sorry, the return on S, P and S is the same, both are giving you 12 percent returns, expected returns. But if you look at the total risk, the total risk of P is higher at a 6 percent, the total risk of S is only 4 percent.

So, in other words, we are able to achieve the same expected return with a lower total or with a lower what to call total risk. It means that there is greater certainty of portfolio S achieving that return of 12 percent compared to the return that is compared to the certainty of the generation of returns by the portfolio P. So, again, we find that portfolio S is superior to portfolio P. Portfolio S is having same expected return as portfolio P but it is having a lower total risk.

As you can see here a sigma systematic is only 4 percent and unsystematic is 0. So, the total risk is only 4 percent. Whereas, in the case of the portfolio P, the total risk was 6 percent. So, again we end up with this with the very fact that S will give you the same average return as P but with lesser fluctuations, lesser chance or lesser uncertainty, lesser chance of not realizing of those returns.

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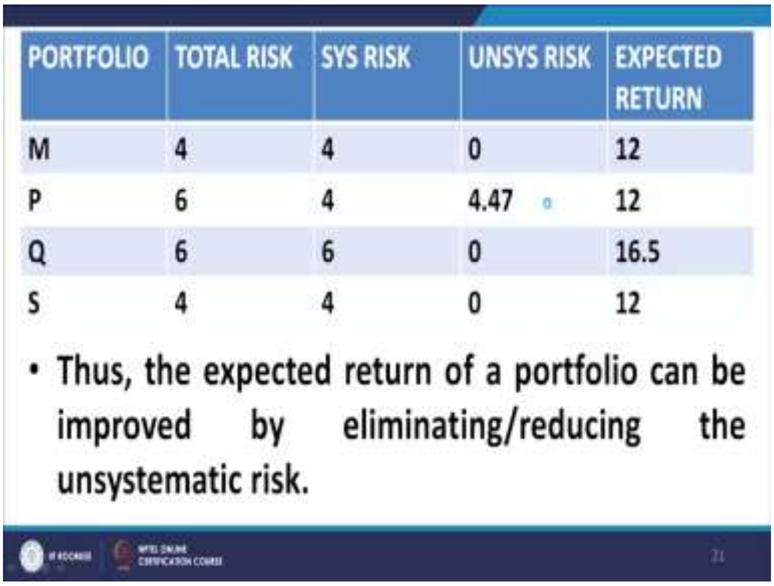

PORTFOLIO	TOTAL RISK	SYS RISK	UNSYS RISK	EXPECTED RETURN
M	4	4	0	12
P	6	4	4.47	12
Q	6	6	0	16.5
S	4	4	0	12

- Thus, the expected return of a portfolio can be improved by eliminating/reducing the unsystematic risk.

So, this is a table which illustrates whatever I have explained in the last few minutes. You have the portfolio M which is the market portfolio, which is obviously an efficient portfolio which has no unsystematic risk, the portfolio P that had a comp1nt of systematic error and systematic risk. Portfolio Q, which had the same total risk at portfolio S portfolio P but at no unsystematic risk. And therefore, because market rewarded systematic risk Q had a higher systematic risk, it ended up with a higher return.

And then we put 4 we had portfolios, which had the same expected return as portfolio P but it has a lower total risk. So, again, what we end up with is that market rewards only systematic risk, it does not reward unsystematic risk.

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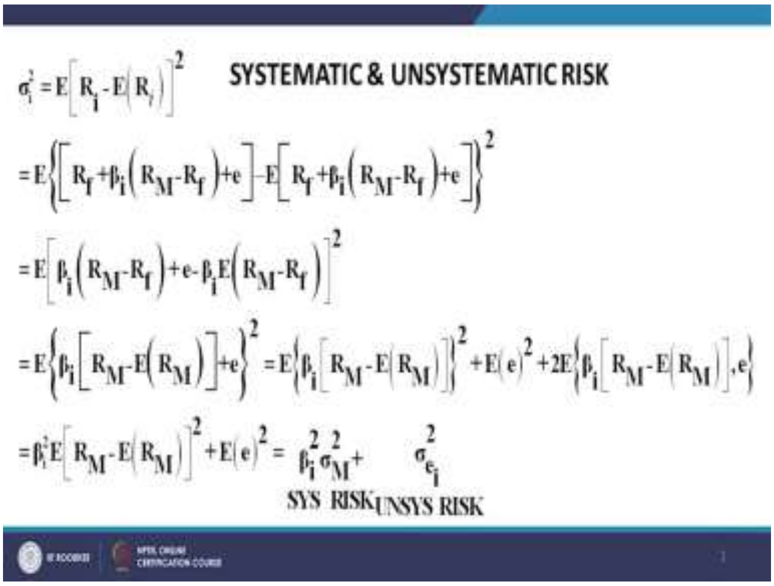

So, this derivation of systematic and unsystematic risk is absolutely parallel to the corresponding derivation that, in fact, we also touched upon right at the beginning of this lecture, beginning of the prior lecture, in fact, absolutely parallel, so I will not spend time on it. The net result is that beta squared sigma M squared gives you the systematic risk. And sigma ei squared gives you the unsystematic risk. Absolutely similar expression to what we have for the single index model.

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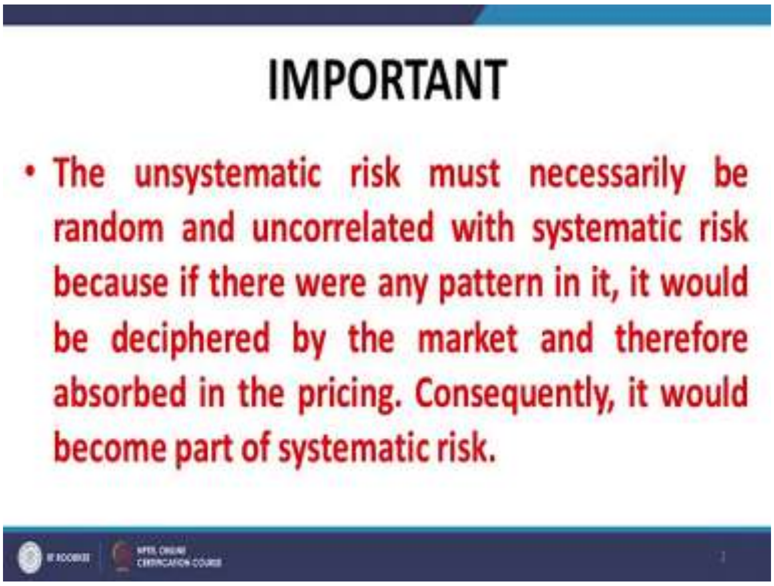

There is one important observation that I would like to give the unsystematic risk must necessarily be random, the unsystematic risk must necessarily be random and uncorrelated with the systematic risk, why? Because, if there was any pattern in it, if there was any pattern in the unsystematic risk, it would immediately be deciphered by the market players and it would be captured by the market price of the relevant asset. It would be absorbed in the pricing process. The consequently it would become a part of this systematic risk.

So, let me repeat the unsystematic risk must be random and unsystematic risk should not be associated with a systematic risk. Because if it says what happens, if the unsystematic risk has some kind of a pattern, it could be captured by the market in its pricing process and thereby, it would become a part of the systematic risk.

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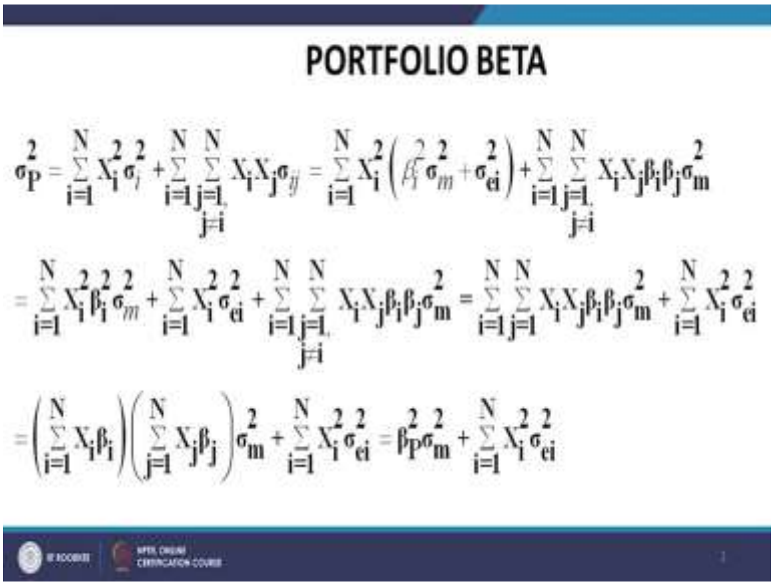

This is as far as the portfolio beta is concerned. This is a proof that the portfolio beta, beta of a portfolio is equal to the weighted average beta of its constituents. I repeat, the beta of a portfolio is equal to the weighted average beta of its constituents. It is algebraically proved. Again, the derivation is quite straightforward, so let us not spend time on it.

In the next lecture, I will start with the capital market line and the security market line and then we will move to the arbitrage pricing theory. Thank you.