

Security Analysis and Portfolio Management
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Lecture 60
Financial Derivatives-3

Welcome back. So, let us recap where we stand. The theoretical forward prices or the no arbitrary forward prices are given by $S_0 \exp(rT)$ we proved that. In the case when there are no dividends or other carrying costs et cetera.

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A BRIEF RECAP: KEY TAKEAWAYS SO FAR

- Theoretical Forward Prices $F(0,T)=S_0 \exp(rT)$
- Forwards are private contracts, futures are exchange traded.
- Futures carry no default risk, performance guaranteed by clearing house.
- Clearing house protects itself against loss due to default by **marking to market & margining**.
- Due to MTM, maximum loss due to default restricted to one day's price change.
- Incidence of MTM default detected on very next day.
- This default risk also covered by adequate margins.

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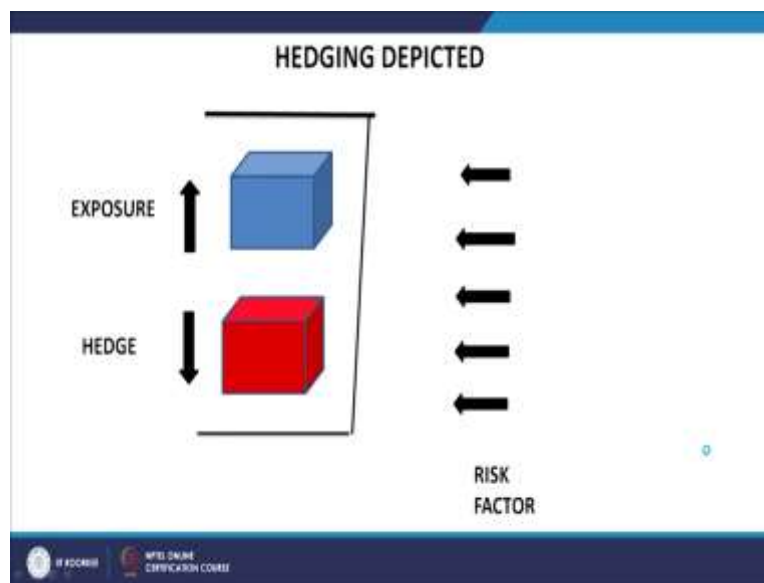
Forwards the private contracts futures exchange traded futures carry no default risk because the performance is guaranteed by the clearing house. The clearing house protects itself against loss due to default by marking to market and margining. Due to marking to market maximum loss due to default is restricted to one day's price change incidence of marking to market default detected as on is detected on the very next day.

Incidence of MTM default is detected on the very next day. The default risk is covered by the adequate margining amount or adequate margin. So, whatever default risk is there and notwithstanding the fact that marking to market has taken place on a daily basis that is captured or that is taken care of by the margin amount. Now, we talk about future hedging. The most important application of futures contracts is the hedging process.

In fact, there are 3 fundamental applications first is speculations where you take a naked position in the futures contract on the anticipation that the price of futures moves in a direction which is as per your perception in which is different from your and your perception is different from the market and as a result of which if you outdo the market you earn profits on that in that situation or that position that is called speculation based speculation.

The hedging is the process where you have an exposure already where certain risk factors impact the value of your portfolio or the value of your assets in an adverse manner. And you want to protect yourself against the possibility of that adverse movement by taking a position in the futures or other derivatives markets. And then there is a possibility of making arbitrary profits. So, this is how hedging is depicted you have an exposure which is represented by the blue box.

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And this exposure is susceptible to price changes which may affect you adversely or due to the impact of the risk factor which is represented by the arrows. So, what you do is to protect yourself against these adverse changes or possible adverse changes in the value of your portfolio, in the value of your exposure you take a counter position in the futures market and this counter position in the futures market operates in such a way as to be able to insulate you. Or substantially reduce your exposure to be substantially reduced the value changes due to the impact of this risk factor. This is called hedging.

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HEDGING

- A hedge is a fence or boundary formed by closely growing bushes or shrubs.
- A hedge is an investment position intended to offset potential losses or gains that may be incurred by a companion investment.
- Hedging is the practice of taking a position in one market to offset and balance against the risk adopted by assuming a position in a contrary or opposing market or investment.




Let us look at the formal definition of hedging. Hedging is an investment position intended to offset potential losses or gains that may be incurred by a companion response. Hedging is the practice of taking a position in one market to offset imbalance against the risk adopted by assuming a position in a contrary or opposite market or investment. So, that is what I explained as the meaning of hedging.

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BASIS & BASIS RISK

- Basis at any instant of time t is given by $b_t = S_t - F_t$. Since both S_t and F_t are stochastic processes and evolve in time with a random element, the basis is also a stochastic process.
- The basis is, therefore not precisely predictable at a future instant of time or that it will remain constant.



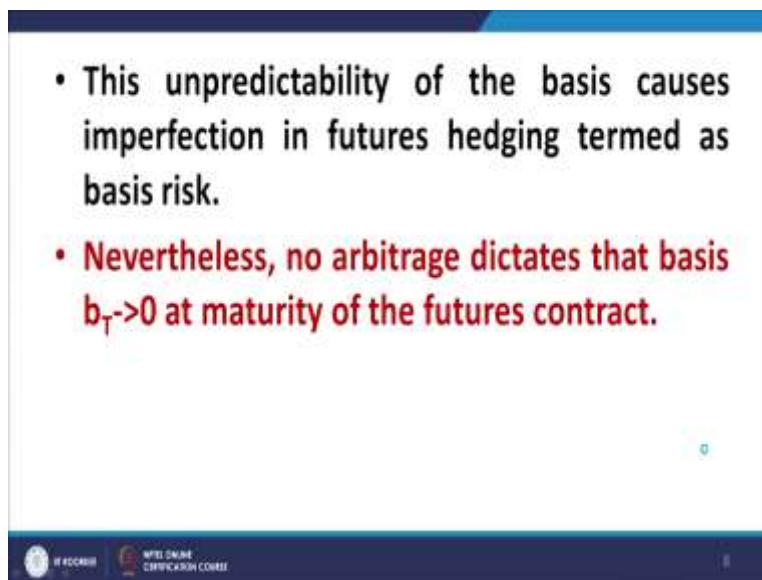
Now, a very important term associated with the use of futures for hedging is called a basis. What is the basis? The basis at any instant of time small arbitrary instant of time small t is the

difference between the spot price prevailing at that point in time and the futures price prevailing at important time in risk factor for any given underlying asset. Now, since both S_t and F_t are random variables they are they evolve in time as random variables they are in essence stochastic processes.

So, both S_t F_t being stochastic processes they evolve in time randomly when without perfect prediction. And as a result of it the basis also operates as a stochastic process. It evolves randomly in time. However, the basis although not being precisely predictable at a future instant of time is subject to a particular constraint. What is the constraint, that constraint is introduced by the impact of arbitrary requirements or the by the requirement of no arbitrary hedge. What is that implication of no arbitrary is?

The unpredictability of the basis causes imperfection in futures as in terms it basis is nevertheless, this is the important part.

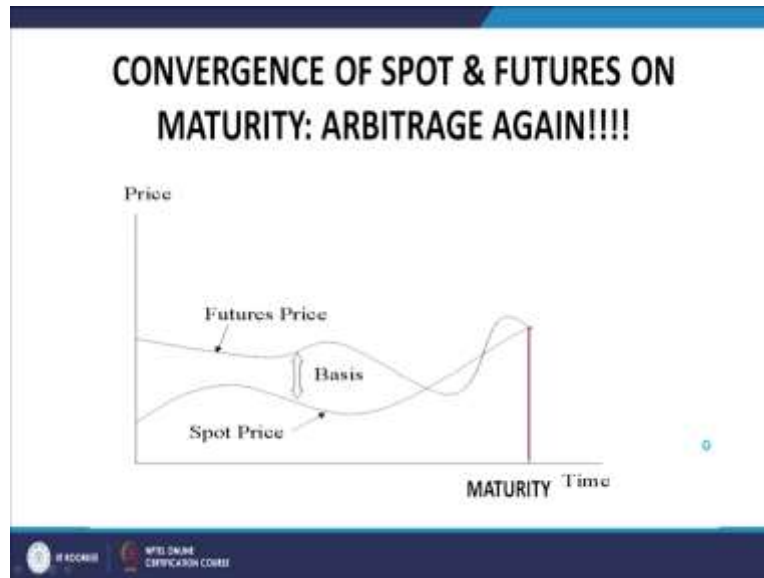
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Nevertheless, no arbitrary dicates that the basis b_T tends to 0 at the maturity of the futures contracts. This is required by arbitrage considerations. So, there are two important points the unpredictability of the basis as I mentioned. The basis is a random process is a stochastic process that evolves randomly in time because it is the difference of two stochastic variables the spot price and the futures price.

And therefore, the basis itself is a stochastic process nevertheless, no arbitrage dictates that the basis at the point in maturity of the futures contract must necessarily approach 0.

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This is the diagram that shows how the basis approaches 0. It is quite obvious in fact because if the futures market moves the spot market price is lower in the futures market prices are you buy the asset from the spot market and can deliver it against the short position in the futures market and make an arbitrage profit. Then, if the other way around you do the other the converse transaction the net result is that on the date on with the futures meta immatures it is necessarily to be true that the futures price must equal the spot price.

I repeat on the date that the futures immature the futures price must necessarily converge to the spot price. Furthermore, and basis risk as I mentioned is the risk arising of the arising out of the imperfection in the futures hedging due to the randomness embedded in the basis. How does it relate? How does this peer basis risk arise in the context of futures as in?

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HOW DOES A FUTURES HEDGE OPERATE?

- There, usually, exists a positive correlation between spot and futures prices.
- Hence, if one has a portfolio consisting of opposite positions in the spot and futures markets, the price changes in one market will substantially offset the price changes in the other market.
- The degree of offsetting will depend on the level of correlation between the price processes in the two markets.

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Let us look at that. But before we do that let us try to understand how a futures hedge operates. There is usually a positive correlation between the spot prices and the futures prices. I repeat, usually, it is not always true it is usually true. That there is a positive correlation between spot prices and futures prices. Therefore, if one has portfolio consisting of opposite positions in the spot and futures markets, what will happen?

The price changes in one market will substantially offset the price changes in the other market. Because if you have a long position say in long set of stocks comprising a long portfolio. And if you take a short position on some on similar stocks or same or similar stock. Then what would happen if the price of this stocks goes down? Then the value of your portfolio will decrease but the value of your futures position will increase.

Because the short position and the spot prices if they decrease it is very likely that the futures prices will also decrease and future prices will decrease means what the value of your short position in the future contracts will increase. And as a result of which it would offset the losses in your long portfolio.

Now, the degree of offsetting will depend on the level of correlation between the price versus in the two markets that is the spot market and the futures market. Let me read this slide again. It is for fundamental importance there usually exists a positive correlation between spot and futures prices.

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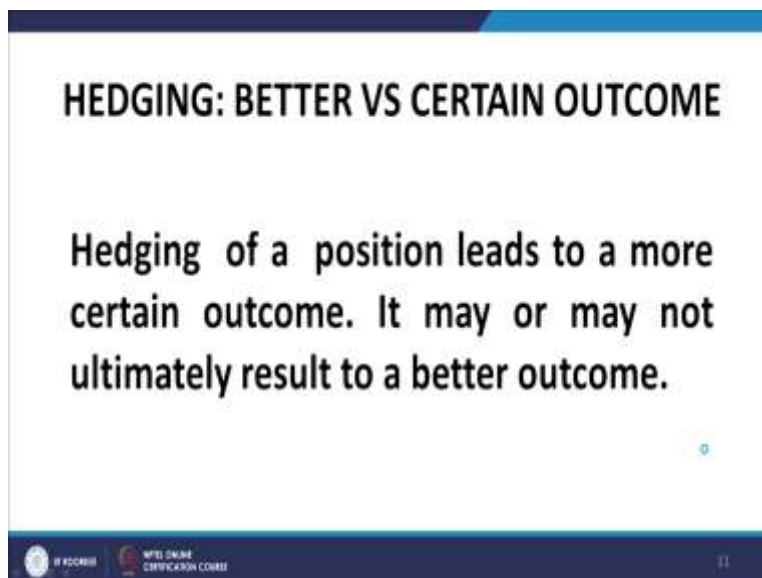
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Hence, if one has a portfolio consisting of opposite position in the spots in futures markets. The price changes in one market will substantially offset the price changes in the other market. The degree of offsetting will depend on the level of correlation between the price processes in the two markets.

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HEDGING: BETTER VS CERTAIN OUTCOME

Hedging of a position leads to a more certain outcome. It may or may not ultimately result to a better outcome.

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Now, this is a very interesting point. Say hedging would normally as I mentioned what happens in hedging let us recap. If you have a long position in a particular let us say you have long position in a particular asset. What you do is? You take a short position in a futures or with the

underlying as that same asset, you have a long position in asset a you take a short position in the futures on asset a.

Now, it is empirically observed that in most cases the spot prices of a in the spot markets and the prices of a in the futures market operate in tandem that is the move in the same direction. So, what happens? Suppose the price of a goes down you are worried that this price of a would go down you are long in that asset. What would happen in the futures market? If the spot price goes down, it is very likely because of the positive correlation between the spot market and the futures market. The price in the futures market will also go down.

But you short in the futures position. So, a decline in price in the futures contract will result in profits on the futures position. And as a result of it what would happen the net impact or the net impact of the test market stimulus on the combination of your hedged asset the asset plus the hedge would be minimal. And that is how the hedge operates.

Now, it is clear from what I have argued that the hedge operates in a manner in which it tries to annul the price changes of the primary position. Your long position in a s price changes will be countered by the price changes in your in the hedge because you are taking a short position in the hedge. And that being the case the net result of taking up a hedge or creating a hedge on a position is that the overall outcome or the overall pricing is minimal.

And you have more certain outcomes. For example, if you hedge your purchase of raw material against price escalation your impact on your profitability would be would be less volatility. Because the purchase of raw materials price would be known beforehand. So, naturally the volatility translating into your profit and loss account would be less. So, it is more the bottom line is hedging makes your outcomes more certain.

But that does not mean that the hedging would always be profitable whether a hedge is profitable or not depends on the risk return trade off of the hedger. Whether is he happy with the possibility of losing out on higher profits by hedging and thereby having a position where these profits are stable? Let me illustrate this with an example you look at this example. Now, let us leave aside whatever is given upfront.

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HEDGING: BETTER VS CERTAIN OUTCOME			
SPOT PRICE t=0	60	60	60
QUANTITY	100	100	100
SPOT PRICE t=T	50	60	70
FUTURES PRICE t=0	62	62	62
FUTURES PRICE t=T	50	60	70
FUTURES QUANTITY	100	100	100
CHANGE IN SPOT VALUE	-1000	0	1000
CHANGE IN FUTURES VALUE	-1200	-200	800
NET CHANGE	200	200	200

The two important things are the highlighted rows. Now, if you do the hedging what is happening here? There are 3 possible outcomes and each of the possible outcomes your net value of your asset is 200 in each case so, it is stable it is the fluctuations are absent. Now, but you look at this look at the other situation which is the grey and the brown highlighted part. If the, if you do not have the exposure of what do you end up with?

You end up with 3 possible scenarios or loss of 1000 0 breakeven that is in 0 profits 0 loss or a profit of 1000. So, in effect what is happening is by the phenomenon of hedging what you are doing is you are losing out on the possibility of getting a profit of 1000. Because if your unhedge position yielded a profit of 1000 or would have yielded a profit of 1000 by hedging your cut out that profit to only 200.

So, that is the impact of hedging that it makes you lose out on potentially higher income opportunities. But it also protects you in the adverse case when there is a dip or there is a fall in prices. As you can see in the first column, that there was a possibility of a loss of 1000 but that loss of 1000 has been curtailed to a profit of 200. So, that possibility of loss has also been eliminated but the possibility of profit has also been eliminated.

So, it operates both ways. Now, whether you do hedging or not is dependent on your risk your risk return profile how adverse you are to taking risk. If you are not adverse to taking risk you would pretty much to no hedging in this scenario. But if you are adverse to taking risk you would

obviously like to hedge your exposure notwithstanding the fact that the opportunity to earn supernormal profits have been lost.

So, although the hedge leads to a more certain outcome the hedger loses out on potential profit that is scenario 3 perfect versus imperfect hedge.

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PERFECT VS IMPERFECT HEDGE

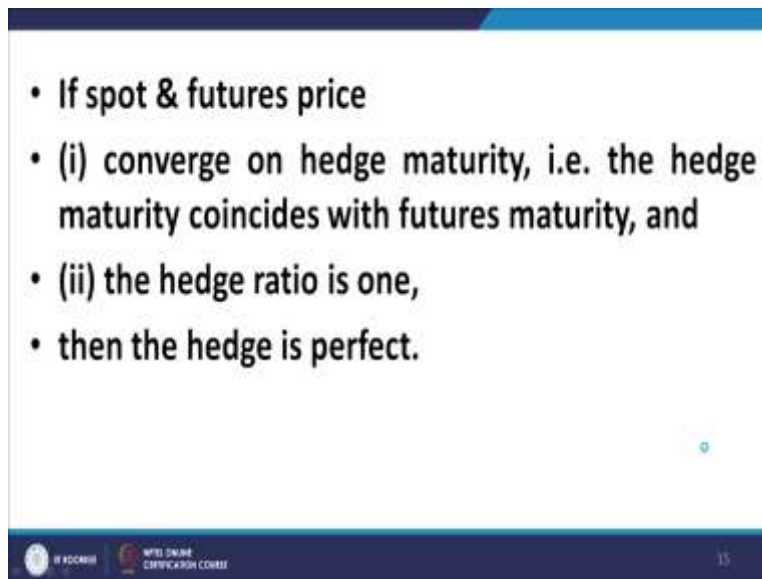
- A perfect hedge is a hedge where the cash flows at hedge maturity of the hedged portfolio are certain.
- Forward is a perfect hedge.
- Futures is imperfect hedge.

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A perfect hedge is the hedge where the cash flows at hedge maturity of the hedge portfolio are certain. Perfect hedge is a hedge where the cash flows at hedge maturity of the hedge portfolio are certain forward contract is a perfect hedge because as you know the forward contract and envisages the delivery of the asset at a predetermined price at a predetermined date. And therefore, and assuming that the forwards are default free. The cash flows are certain on the data maturity of the hedge futures are not a perfect hedge.

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And future hedge would could be a perfect hedge under two situations where the hedge period coincides with the maturity of the futures contract. I repeat where the hedge period coincides with the maturity of the futures contract. And as a result of it the spot and futures price converge as on the date of maturity of the futures contract which is also the date of the end of the hedge period.

So, if the end of the hedge period coincides with the maturity of the futures contract, then that enables that the futures prices coincide with this spot prices as on the date of the hedge end of the hedge period and number 2 if the hedge ratio is 1. So, if both these conditions are satisfied, then the future hedge can turn out to be a perfect hedge.

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LONG HEDGE

- Hedges that involve taking a **long position in a futures contract** are known as long hedges.
- A long hedge is appropriate when a company knows it will have to **purchase a certain asset in the future** and wants to lock in a price now.

Now, what is the long hedge? A long hedge is a hedge in aware the taking of the hedge involves a long position in the futures contract when would somebody do a long edge and his original position original position is short that is when he wants to or when he intends to purchase a certain asset in the future and wants to lock in the price right now.

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SHORT HEDGE

- A short hedge is a hedge that involves a **short position in futures contracts**.
- A short hedge is appropriate when the **hedger already owns an asset and expects to sell it at some time in the future**.
- A short hedge can also be used when an asset is not owned right now but will be owned at some time in the future.

The short edge is the inverse of that, that is a shortage involves taking a short position in the futures market and the short hedge is appropriate when the hedger already owns an asset like the

examples I just gave you of holding the share or the asset and expects to sell it at some time in the future.

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SHORT HEDGING & BASIS

- Price at which underlying is sold in the spot market at maturity of hedge period = S_T
- Profit from the short futures position, that was created at F_0 at $t=0$ and closed out at F_T at $t=T$ is $F_0 - F_T$
- Net proceeds from the hedged transaction = $S_T + F_0 - F_T = F_0 + (S_T - F_T) = F_0 + b_T$
- Thus, if the basis strengthens unexpectedly, short hedger gains & vice versa.

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Now, the relationship between hedging and basis this is interesting this is important the price at which the underlying is sold we are talking about short hedging. So, we are considering a situation where the person already owns the asset and he wants to sell the asset is worried about a decline in price. So, it takes a short position in the futures market the price at which the underlying is sold in the spot market at the maturity of the hedge period let us call it S capital T .

The profit from the short futures position that was created at the price F_0 at t equal to 0. And liquidated at the price of F_T at equal to capital T which is the end of the hedge period. This is given by F_0 minus S_T . Why F_0 minus F_T because it is a short position. Normally what we have is the final price minus initial price. If it is a long position and if it is a short position the thing gets reversed it is F_0 minus F_T . So, the profit on the hedge is equal to F_0 minus F_T .

So, the net price that is realized by the short hedged is given by S_T plus F_0 minus F_T that is equal to F_0 plus S_T minus F_T that is equal to F_0 plus b_T . So, the net price realized by the short hedged is given by F_0 plus b_T that b_T is no please note this b_T is what? b_T is the basis at the point at which the hedge is lifted that is t equal to capital T . I repeat b_T is the basis at the point at which the hedges lifted.

So, now, please note b_T can be can have an element of randomness in the event that the hedge period or the closing of the hedge period the lifting of the hedge period does not coincide with the maturity of the futures in that case. The basis can have an element of randomness and therefore, this hedge would not be a perfect hedge. But however, if the maturity of the hedge period coincides with the maturity of the futures contract then the basis will be 0.

And the net price that is realized by the by the short hedger is equal to F_0 which is known at t equal to 0 and there is no randomness. And the second implication is that if the basis strengthen is unexpectedly that is if the basis increases the shortage hedger gains and vice versa.

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- While F_0 is known upfront, b_T being the basis at $t=T$, is not. It is b_T that captures the randomness element.
- If hedge maturity coincides with futures maturity, $b_T > 0$ and the net price received is F_0 which is known upfront and thus there is no randomness due to maturity mismatch.

I repeat this while F_0 is known upfront b_T being the basis at t equal to t is not it is b_T that captures the randomness or the randomness element and that leads to the imperfection in the futures hedging if hedge maturity coincides with the futures maturity then b_T approaches 0. Because futures maturity spot and futures coincides of basis is equal to 0. And the net price is received as F_0 which is known upfront. And thus there is no randomness due to maturity mismatch.

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LONG HEDGE & BASIS

- Similarly, the net price paid by a long hedger= -
 $S_T + F_T - F_0 = -F_0 - (S_T - F_T) = -(F_0 + b_T)$
- Thus, if the basis strengthens unexpectedly, long hedger loses & vice versa.

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Long hedge and basis this is pretty much the same as on similar lines as we discussed in the earlier case of short hedging.

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WHY FUTURES HEDGE IS NOT A PERFECT HEDGE?

- Marking to market.
- Basis risk due to:
- Different underlying asset.
- Non-identical maturity.
- Lot size issue.

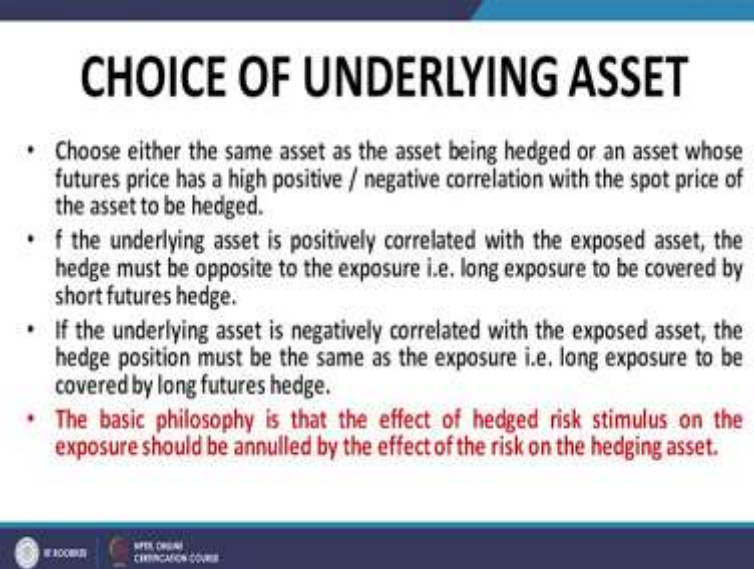
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Why futures hedge is not a perfect hedge there are so, many reasons. There are 3 basic reasons in fact number 1 you could have a situation where the is your spot exposure and the position that you have taken in the futures market have different underlyings. In other words, the under the asset in which you have this stock I am sorry, in which you have the spot exposure does not constitute the underlying asset of the futures contract.

Let me repeat the situation in which where the asset in which you have this spot exposure does not coincide with the asset which is the underlying asset of the futures contract. The second is what you discuss just now the possibility of the hedged maturity not coinciding with the futures maturity which gives rise to randomness in the basis which is called basis. And then there could be the issue of lot size.

For example, if you have an exposure of for 50,000 pound sterling in relation to US dollars. But the lot size in the US dollars and a pound sterling market is 62,500. Therefore, the lot size may not match and if the lot size does not match obviously you are exposed to basis again once again.

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CHOICE OF UNDERLYING ASSET

- Choose either the same asset as the asset being hedged or an asset whose futures price has a high positive / negative correlation with the spot price of the asset to be hedged.
- If the underlying asset is positively correlated with the exposed asset, the hedge must be opposite to the exposure i.e. long exposure to be covered by short futures hedge.
- If the underlying asset is negatively correlated with the exposed asset, the hedge position must be the same as the exposure i.e. long exposure to be covered by long futures hedge.
- The basic philosophy is that the effect of hedged risk stimulus on the exposure should be annulled by the effect of the risk on the hedging asset.

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So, choice of the underlying asset now, if you do not have the underlying asset being the same as the asset in which you have the exposure you should have you should look for an asset which has a strong correlation the price of which asset as a strong correlation with the price process of the asset in which you have the exposure. I repeat the futures contracts are not available on the asset on which you have the exposure.

Then what you should do is you should look for an asset which has a price process which is strongly correlated with the price process of the of the asset in which you have the exposure. strongly correlated means it could also of course, be strongly anti correlated.. But the point is if it is strongly correlated we will take an opposite position in the futures market. If you are long in that asset will take the short position in the futures.

And if it is strongly anti correlated then a long position will be counterbalanced by a long position in the futures market on the underlying.

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NUMBER OF CONTRACTS

NOTATION

Given a random variable X , we given its expectation $E(X)$ as:

$$E(X) = \sum_{i=1}^n p_i x_i$$

Now, the number of contracts this is the expression for the expected value the learners would be knowing this by now. Now, the second point is that the variance of the linear combination of two random variables. I repeat the variance of a linear combination of two random variables alpha x plus beta y where alpha and beta are real numbers. And x and y are random variables

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Let $Z = \alpha X + \beta Y$, then

$$\begin{aligned} \sigma_{Z=\alpha X+\beta Y}^2 &= E[Z - \bar{Z}]^2 = E[\alpha X + \beta Y - E(\alpha X + \beta Y)]^2 \\ &= E[\alpha X + \beta Y - \alpha E(X) - \beta E(Y)]^2 \\ &= E\{\alpha[X - E(X)] + \beta[Y - E(Y)]\}^2 \\ &= E\{\alpha[X - E(X)]\}^2 + E\{\beta[Y - E(Y)]\}^2 \\ &\quad + 2E\{\alpha[X - E(X)]\beta[Y - E(Y)]\} \\ &= \alpha^2 E[X - E(X)]^2 + \beta^2 E[Y - E(Y)]^2 \\ &\quad + 2\alpha\beta E\{[X - E(X)][Y - E(Y)]\} \\ &= \alpha^2 \sigma_X^2 + \beta^2 \sigma_Y^2 + 2\alpha\beta \sigma_{XY} \\ &= \alpha^2 \sigma_X^2 + \beta^2 \sigma_Y^2 + 2\alpha\beta \rho_{XY} \sigma_X \sigma_Y \end{aligned}$$

It can be shown to be equal to alpha squared sigma x squared plus beta squared sigma y squared plus two alpha beta rho x y sigma x sigma y. So, these this is a result which we shall use in arriving at the number of contracts.

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$$I_{H,0} = Q_S S_0 - Q_F F_0; \quad \tilde{I}_{H,T} = Q_S \tilde{S}_T - Q_F \tilde{F}_T$$

$$\Delta I_H = \tilde{I}_{H,T} - I_{H,0} = Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F} \text{ where } \Delta \tilde{S} = \tilde{S}_T - S_0$$

$$\sigma_{\Delta I_H}^2 = Q_S^2 \sigma_{\Delta S}^2 + Q_F^2 \sigma_{\Delta F}^2 - 2\rho Q_S Q_F \sigma_{\Delta S} \sigma_{\Delta F} \quad \text{--- (1)}$$

$$\text{For minima } \frac{d\sigma_{\Delta I_H}^2}{dQ_F} = 0 = 2Q_F \sigma_{\Delta F}^2 - 2\rho Q_S \sigma_{\Delta S} \sigma_{\Delta F}$$

$$Q_F = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} Q_S = \beta_{\Delta S, \Delta F} Q_S = h Q_S \text{ since } \frac{Q_F}{Q_S} = h = \text{hedge ratio}$$

So, here is a derivation of the number of contracts I shall not go through it in detail it is quite straightforward. We look at the at a hedge position IH at T equal to 0 which involves a quantity in the spot market QS S0 as this price in the spot market QF is the quantity in the futures market or the quantity that is used as the hedging. Hedging the apparatus and F0 is the futures price at T equal to 0 of that particular H.

So, the net value of the hedged position at t equal to 0 is given by QS S0 minus QF F0 in the negative sign is there because we are assuming that the positions in the spot market and the futures market are opposite. And the value of the hedged position at t equal to capital T is given by QS S tilde t minus QF F tilde t where the tilde represents the fact that both ST and FT are random variables.

They are parts of this stochastic process represents the evolution of prices of the spot markets and the futures markets. So, the change in value of the hedge change in value of the hedge position is given by IH T minus I S0 that is QS delta S tilde minus QF delta F tilde where delta S tilde is equal to S tilde minus S0 please note S0 is not a random variable S0 is the for price known at t equal to 0 which is known which is not random.

Similarly, F_0 is not random. So, $\Delta \tilde{S}$ is equal to $\tilde{S} - S_0$, $\Delta \tilde{F}$ is equal to $\tilde{F} - F_0$. The variance of this change in changing the hedge position is given by the expression that is here using the formula that I mentioned just now let us call it equation number 1. What we do is? We differentiate this variance with respect to QF ; please note QF is the only 3 degree of freedom in this expression QS is already known.

It is like is the quantity of the exposure which is known to us and all the variance. The covariance is are given to us as externally inputs. So, the only variable that we can adjust that we can manipulate to do the hedging or to do a good hedge is the amount that we take up in the futures market. So, we differentiate this variance with respect to QF . And we equate it to 0. And the result that we get is here in this last equation QF is equal to $\beta \Delta S / \Delta F$.

And that is equal to H / QS where the definition of the hedge ratio is equal to the exposure in the futures market divided by the exposure in this spot market. So, it has the delta H , I am sorry the hedge ratio h small h is the ratio of the positions taken in the futures markets and the spot markets. So, we get the hedge ratio is equal to $\beta \Delta S / \Delta F$. This is the case where we are looking at a minimum variance hedge.

Now, please note let me clarify one thing here. H is the hedge ratio it is defined as the ratio of the position taken by the hedger in the futures market and the position that the hedger has in this spot market. This is a universal definition of hedge ratio in this special case in the very special case that we are talking about the optimization of a minimum or of optimization of the hedge in terms of its minimum variance that is we are considering variance as a measure of risk.


And we are considering that hedge switch minimizes the variance of my total hedge position that in that special case in that special scenario the hedge ratio turns out to be equal to the regression coefficient between ΔS and ΔF .

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In the special case of minimum variance hedge

$$\frac{Q_F}{Q_S} = h = \text{hedge ratio} = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = \beta_{\Delta S, \Delta F}$$

Also $\frac{d^2 \sigma_{\Delta I}^2}{dQ_F^2} = 2\sigma_{\Delta F}^2 > 0$ confirming minimality of variance



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So, in this special case of minimum variance hedge the hedge ratio is equal to beta delta S comma delta F. And otherwise as a general scenario as a general definition the hedge ratio is equal to QF upon QS.

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IMPORTANT

- The regression is with changes in prices and not prices themselves.
- The futures price changes is the independent variable and the spot price changes is the dependent variable.
- We have not accounted for the integrality of the no of contracts.
- We are only considering minimising variance over the hedge period.



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So, certain observations here number 1: the regression is with respect to changes in prices and not prices themselves. Number 1: the regression is with respect to changes in prices and not prices themselves. Number 2: the futures price changes the independent variable and the spot

price changes is the dependent variable. You have got beta delta S delta f. So, beta is the regression coefficient with delta F being the independent variable.

And delta S being the dependent variable. We have not accounted for the integral at t or the integral nature of the number of contracts. We are already considering minimize variance over the hedge period.

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Now, we talk about index futures. Index futures are the most popular futures in fact so I devote some time to talking about index future let us recap this scupper model. Number one the market rewards prices only is a systematic risk the market prices only a systematic risk it does not price unsystematic risk.

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THE CAPM MARKET

- The market rewards (prices) only systematic risk.
- It assumes that unsystematic risk can be managed (eliminated) by diversification to any level chosen by the investor.
- Systematic risk is measured by beta.
- Unsystematic risk is measured by the variance of the residual (random) term.
- Beta of a portfolio scales in the same manner as expected return. Hence, there is conformity between risk and return in this (CAPM) framework.

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In other words it assumes that unsystematic risk can be managed or eliminated by diversification to any level chosen by the investor. So, it need not be rewarded by the market. Systematic risk is measured by the regression coefficient beta unsystematic risk is measured by the variance of the residual or the random term beta of a portfolio scales now, this is important beta of a portfolio scales in the same manner as expected return. Hence, there is conformity between risk and return in this CAPM framework.

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THE MARKET PORTFOLIO

- Market portfolio is the universal risky portfolio.
- Market portfolio is efficient.
- Market portfolio is fully diversified and hence devoid of any unsystematic risk.
- The total risk of market portfolio is its systematic risk.
- Beta of market portfolio is unity. Why

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What are the features of the market portfolio which is relevant in the context of index futures. Number 1: market portfolio is the universal risky portfolio. Number 2: market portfolio is efficient. Number 3: market portfolios fully diversified and hence devoid of any unsystematic risk. Number 4: the total risk of a market portfolio is that systematic risk because it has 0 unsystematic risk. Beta of the market portfolios is unity let us look at the basic theory of index futures.

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BASIC THEORY OF INDEX FUTURES

- Consider Rs V worth of investment in share S . Let the beta of S be β_s .
- Let the beta of another stock M^* be 1.00. Add a short position in Rs $\beta_s V$ worth shares of M^* to the portfolio..
- The beta of the portfolio is $(\beta_s * V - \beta_s V * 1) / (V - \beta_s V) = 0$.
- Thus, we have constructed a portfolio with no systematic risk.

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Consider rupees V worth of investment in share S . Do you have a share S and you invested an amount in rupees V in the share S . Let us assume that the beta of this particular share S is beta S . Let the beta of another stock M^* as some other stock M^* . And we assume or that stock M^* is such that its beta is equal to 1. I repeat, we look forward and we find a stock M^* such that the beta of the stock M^* is equal to 1.

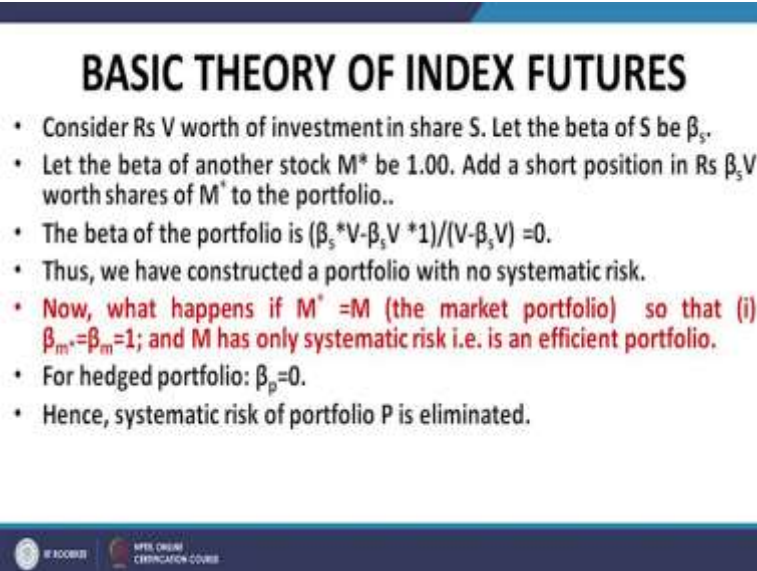
What we do is? We add a short position of rupees $\beta_s V$ into V in the stock M^* to our original portfolio. We have our original portfolio P which comprises of the shares of Company S at a value total value of V . The beta of this share S is beta S . We look for and find share M^* which has a beta of 1. And what we do is? We add an this share M^* short the share M^* of the value of $\beta_s V$ into V and add this to our portfolio P .

So, our portfolio P now consists of two shares share S of the value of V with a beta of beta S share M^* with a value of $\beta_s V$ and a beta of 1. What is the beta of this portfolio? You

can work out the beta of this portfolio it is equal to 0. Because your position in M star is equal to is a short position your position in M star is a short position. So, the beta of your portfolio is equal to 0.

Therefore, we have been able by taking the short position and the share M star to eliminate completely the systematic risk of our portfolio p. What happens if M star is nothing but the market portfolio?

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BASIC THEORY OF INDEX FUTURES

- Consider Rs V worth of investment in share S. Let the beta of S be β_s .
- Let the beta of another stock M* be 1.00. Add a short position in Rs $\beta_s V$ worth shares of M* to the portfolio..
- The beta of the portfolio is $(\beta_s * V - \beta_s V * 1) / (V - \beta_s V) = 0$.
- Thus, we have constructed a portfolio with no systematic risk.
- Now, what happens if M* = M (the market portfolio) so that (i) $\beta_{m^*} = \beta_m = 1$; and M has only systematic risk i.e. is an efficient portfolio.
- For hedged portfolio: $\beta_p = 0$.
- Hence, systematic risk of portfolio P is eliminated.

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We know that the market portfolio is the portfolio with a beta of unity. We know that the market portfolio has a beta of 1. And we know that M star is such a share that we have found out to be a beta of 1. So, if we substitute M star by the market portfolio M which has the same property that it has a beta of 1. But there is another special property of M and that is that M is an efficient portfolio and what does it mean?

It means that the unsystematic risk of M is 0 the total risk of M is confined towards the systematic risk the unsystematic risk of M is 0. Now, what do we have? We have the situation the systematic risk of our portfolio P which is a combination now of the stock S and the market portfolio in the ratio V is to V is to V into beta S short of it of course negative because it is short. If you use this ratio to use this portfolio then the systematic risk of this portfolio is 0.

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- **What about unsystematic risk???**
- Unsystematic risk $\sigma_{\text{unsys}}^2 = \sum_{i=1}^N X_i^2 \sigma_{\epsilon_i}^2$
- Since M does not have any unsystematic risk, it follows that the unsystematic risk of S remains unchanged by the addition of M.
- Taking an appropriate position in the market index (portfolio) enables the investor to manage the systematic risk (beta) of his portfolio.
- Since the market portfolio is efficient i.e. it has no unsystematic risk, taking a position in the market portfolio does not affect the unsystematic risk of the portfolio.



What about the unsystematic risk? The unsystematic risk of this portfolio remains unchanged. Why it remains unchanged? Because the market portfolio that we have added to our portfolio P does not have any unsystematic risk. Therefore, by the addition of this market portfolio M to our portfolio P we are not changing the unsystematic risk of Portfolio P. So, let us do a quick recap now.

Taking an appropriate position in the market index market portfolio enables the investor to manage the systematic risk bracket beta of the portfolio. Since the market portfolio is efficient that is it has no unsystematic risk taking a position in the market portfolio does not affect the unsystematic risk of the portfolio.

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INFERENCE

- Hence, by taking an appropriate position in a suitable no of units of the market portfolio, the investor has been able to completely hedge the systematic risk of the portfolio while keeping the unsystematic risk unchanged.

Inference: hence, by taking an appropriate position in a suitable number of units of the market portfolio. We into beta S the investor has been able to completely hedge the systematic risk of the portfolio while keeping the unsystematic risk unchanged. Now, is the important part.

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- Since futures prices and spot prices are expected to move in tandem due to no arbitrage, a position in futures on the market portfolio in lieu of spot tantamounts to a similar impact in the portfolio cardinals.
- Hence, use of futures on M will yield similar effects as positions in M itself i.e. **reduce systematic risk and leave unsystematic risk unchanged.**
- Furthermore, futures have the additional advantage of requiring **NO INITIAL INVESTMENT** for taking positions.

Since futures prices and spot prices are expected to move in tandem due to no arbitrage considerations a position in the futures market portfolio in lieu of the spot market portfolio tantamount to a similar impact on the portfolio P cardinals. Let me repeat this fundamental statement. Since the futures prices and the spot prices are at expected to move in tandem due to

no arbitrage a position in the futures on the market portfolio in lieu of the spot market portfolio tantamount to a similar impact on the portfolio's cardinals.

That is the P portfolio is cardinal. Hence use of futures on M will yield similar effects as positions in M itself. I repeat because there is a strong correlation between the price processes of M the market portfolio. And the price processes of the futures on the market portfolios the addition of futures to on the market portfolio to the portfolio P in lieu of the addition of the spot market portfolio or the spot market shares or stock or indices you may call it what you like amounts to the same situation.

Therefore, it reduces at such an addition reduces systematic risk and without affecting the unsystematic risk. But the use of futures on the market portfolio in lieu of the market portfolio itself has an advantage and a very fundamental advantage. And that advantage is that the futures have the additional advantage of requiring no initial investment for taking positions except of course the margin requirements.

So, for taking positions and futures on the market portfolio you do not require full fledged investments. You only require my investments in the margin account and therefore they are much more convenient than taking positions in the market portfolio themselves. So, this is what gives rise to this stock index futures.

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STOCK INDEX FUTURES

- Stock index futures are futures contracts on any stock index e.g. S & P BSE SENSEX that constitutes the underlying asset.
- A long/ short position in a SIF is equivalent to taking a long/short position in a futures contract on a portfolio of stocks that is equivalent to the portfolio of stocks constituting the stock index.
- The underlying asset of such futures is the index portfolio or simply the index.
- Settlement on index: $z^*(F_t - F_0)$

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The stock index futures are futures contracts on any stock index like the S and P BSE index. This stock index constitutes the underlying asset of the futures contract. A long or short position in stock index futures is equivalent to taking a long or a short position in the futures contract on a portfolio of stocks that is equivalent to the portfolio of stocks constituting the stock index. The underlying asset of such futures is the index portfolio or simply the index.

The settlement on this index future is done in terms of z into F_h minus F_0 . Where z is called the contract multiple or the lot size.

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- A futures contract on a stock market index represents the right and obligation to buy or to sell a portfolio of stocks characterized by the index.
- Stock index futures are cash settled.
- The contracts are marked to market daily.
- **Final Settlement: On the last trading day, the futures price is set equal to the spot index level and there is a final mark to market cash flow.**

A futures contract on a stock market index represents the right and obligation to buy or to sell a portfolio of stocks characterized by the index. Stock index futures are as is obvious would necessarily cash settled you cannot deliver an index. So, the settlement of the stock index futures has to be done through cash if futures are marked to market daily like other futures the final settlement on the stock index futures takes place on the last trading day by equating the futures price.

Or the future index value to the spot index value prevailing on that day at the point of settlement and on the basis of which a final mark to market cash flow takes place.

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The applications of stock index futures most important application is hedging to manage the beta the systematic risk of a portfolio which I have explained in just now. Other applications include speculation and index futures. So, with this I come to the end of this course. And I do hope the learners will enjoy pursuing this course. And I request the learners that it students to interact with me on the discussion forum with their queries. I should always be willing to answer their questions on any topics which I covered in this course. Thank you very much. All the best.