Decision Making With Spreadsheet
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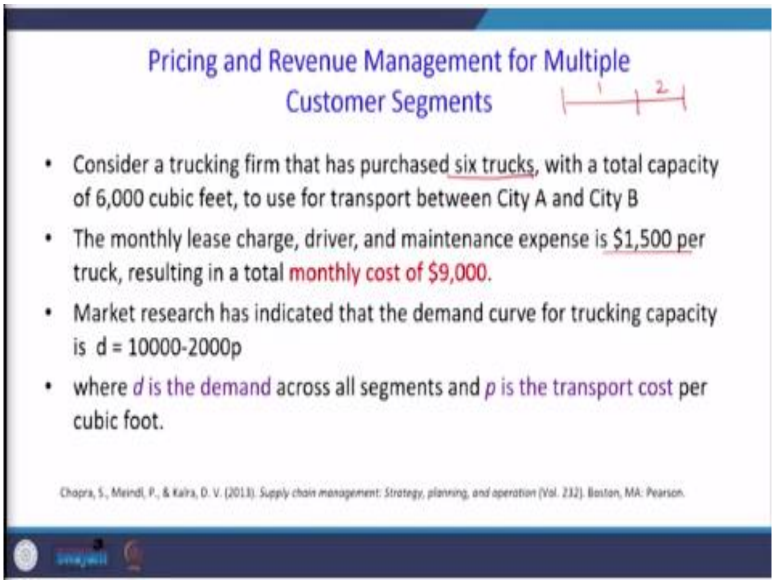
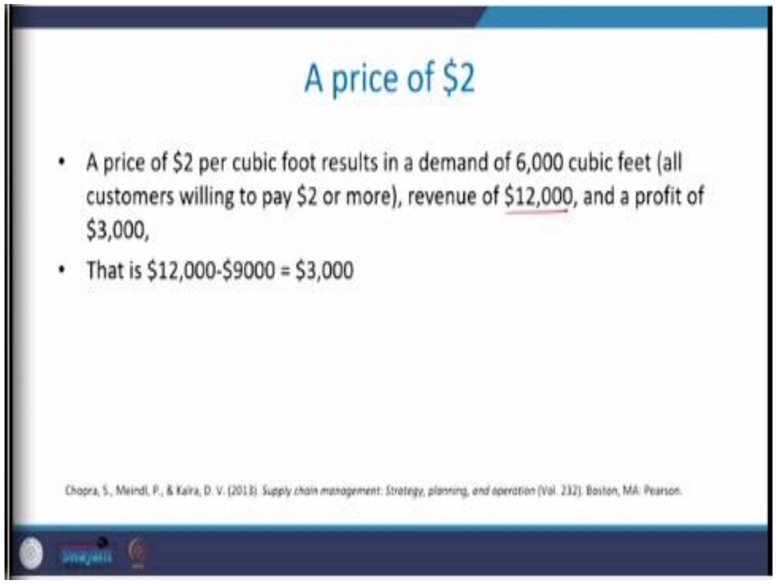
Lecture-28
Non-Linear Optimization Models-V

Dear students, in this lecture, I am going to take another problem, another application of non-linear programming, which is pricing. So, the agenda for this lecture is the application of non-linear programming for pricing.

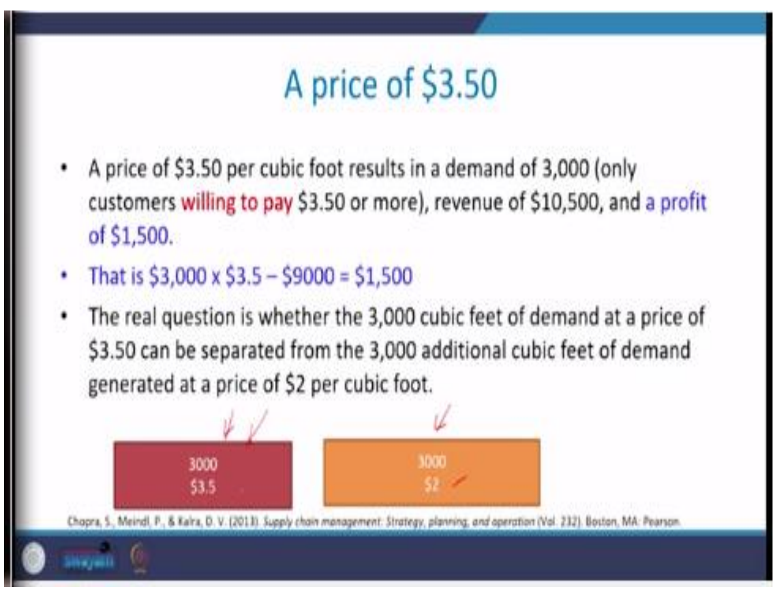


Pricing and revenue management for multiple customer segments. What is the meaning of multiple customer segments? Customers: there are different types of customers, say maybe customer one and customer 2. So, what are we going to see? We can price differently for different segments, in case if you are going for pricing differently for different segments, what should be the price for each segment? That we are going to find out.

This example is taken from Chopra et al. 's book Supply Chain Management. Consider a trucking firm that has purchased six trucks with a total capacity of 6000 cubic feet to use for transport between cities A and B. The monthly lease charge, driver, and maintenance expense is 1500 dollars per truck, resulting in a total monthly cost of 9000 because they have six trucks, so six multiplied by 1500. Market research has indicated that the demand curve for tracking capacity is d = 10000 - 2000p. Here, d is the demand across all segments, and p is the transport cost. Here, we can consider any cost because we are the contact station supply chain management; you are considering the transport cost.



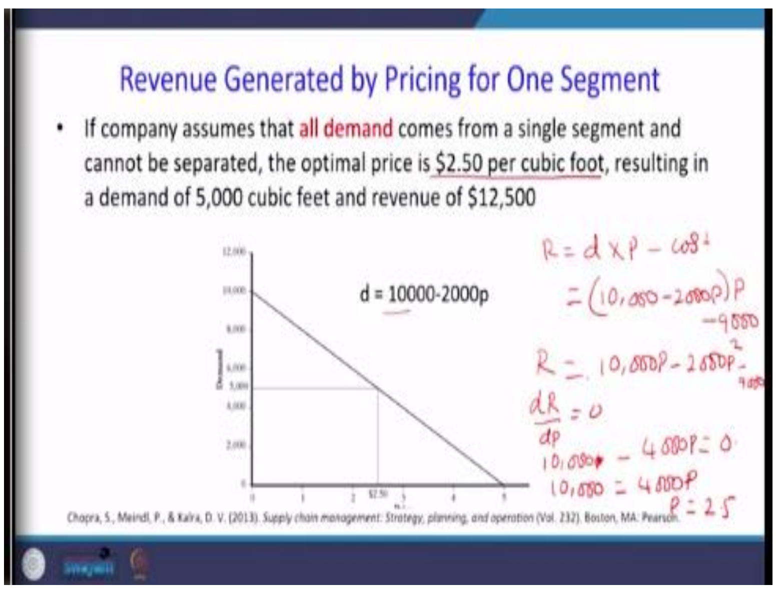
The price of 2 dollars cubic feet results in a demand of 6000 cubic feet, and all customers are willing to pay 2 dollars or more. The revenue is 12000 dollars and the profit is 3000 dollars, the demand is 6000. If you fix the price to 2 dollars, the revenue will be 12000 dollars; out of 12000, 9000 dollars is your cost, and the profit will be 3000, which is 12000 - 9000 = 3000 if you fix the selling price by 2 dollars.



What will happen if you fix the selling price by 3.5 dollars? So, a price of 3.5 per cubic feet results in the demand of 3000 customers willing to pay 3.5 dollars or more, the revenue will be 10,500 dollars and the profit of 1500 dollars. How did we get this 1500? So, the demand is 3000, the price is 3.5, so the revenue is this much minus the cost is 9, so 1500 dollars. Now you see when we keep the price at 2 dollars, we get some other figure.

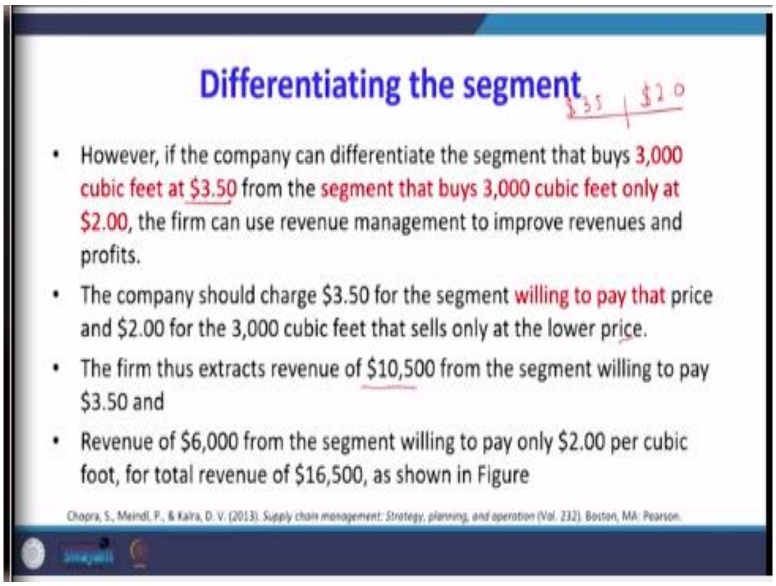
When we keep the price at 3.5 dollars, we are getting some other figures. The real question is whether the 3000 cubic feet of demand at a price of 3.5 dollars can be separated from 3000 additional cubic feet of demand generated at a price of 2000 dollars per cubic foot. The question is whether we can split the demand. What way? Say out of 6000, 3000, we can price for 3.5 dollars, the remaining 3000 for 2 dollars.

So, revenue management comes with how to split this demand. One way is if somebody is willing to book in advance, for them we can fix the lesser price, which is 2000 dollars. Some people are coming and asking to book at the last minute. For people who are willing to pay for that people, you can charge 3.5 dollars.



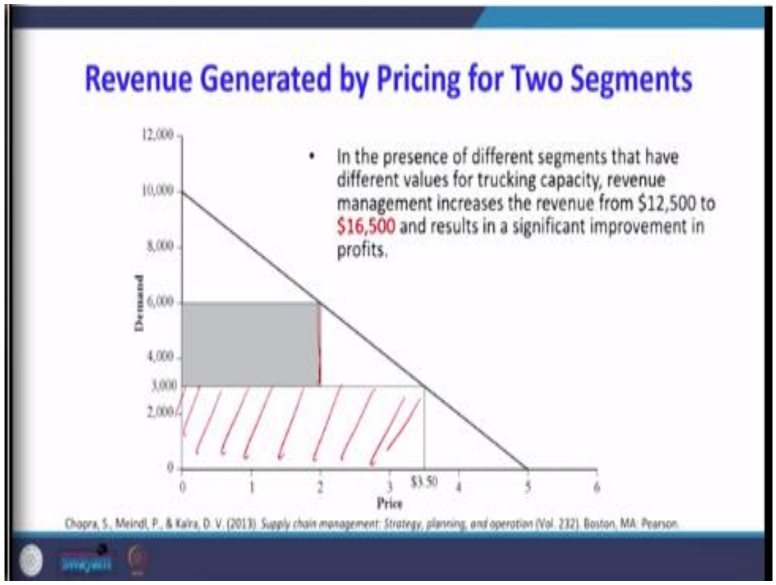
Revenue generated by pricing for one segment. If the company assumes that all demand comes from a single segment and the demand cannot be separated, the optimal price is 2.5 dollars per cubic foot. How did we get this 2.5 dollars? So, what is the revenue? Revenue is demand multiplied by price – cost. So, here, demand is the demand equation given, so 10000 - 2000p, so multiplied by p - 9000 because 9000 is cost.

So, when you simplify this revenue is 10000p - 2000p square - 9000. So, this revenue has to be differentiated with respect to p and equated to 0. So, what I am going to do? dR by dp = 0, so what will happen? This will become 10000p, so when you differentiate this with respect to p, 10000 - 4000p = 0, so 10000 = 4000p, the value of p is 2.5 dollars. So, the optimal selling price is 2.5 dollars, resulting in a demand of. When you substitute this 2.5 here, the demand will become 5000, and the revenue will become 12500, so, when you substitute in this revenue function.

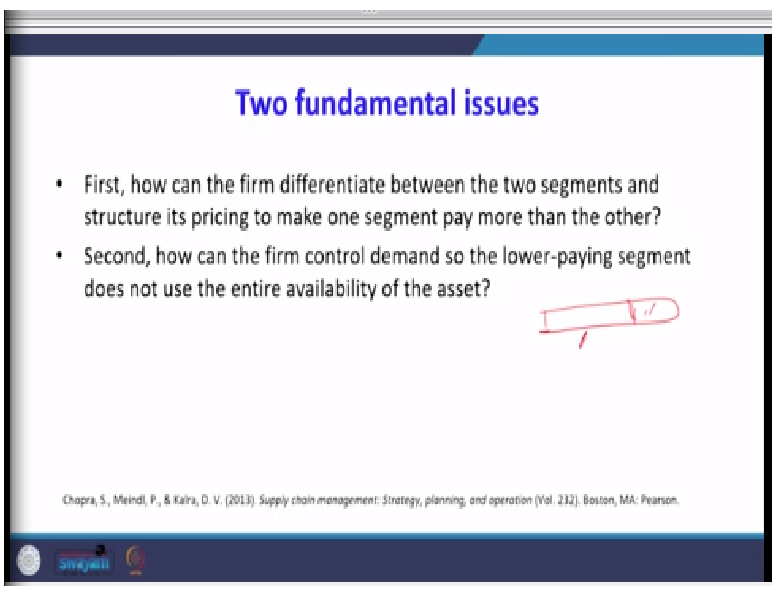


However, if the company can differentiate the segment that buys 3000 cubic feet for 3.5 dollars from the segment that buys 3000 cubic feet only at 2 dollars. So, the firm can use revenue management to improve revenue and profit, so what will we do? We are going to split into 2 segments: one is selling at 3.5 dollars, and the other one is selling at 2 dollars. The company should charge 3.5 dollars for the segment willing to pay that price and 2 dollars for the 3000 cubic feet that sell only at the lower price.

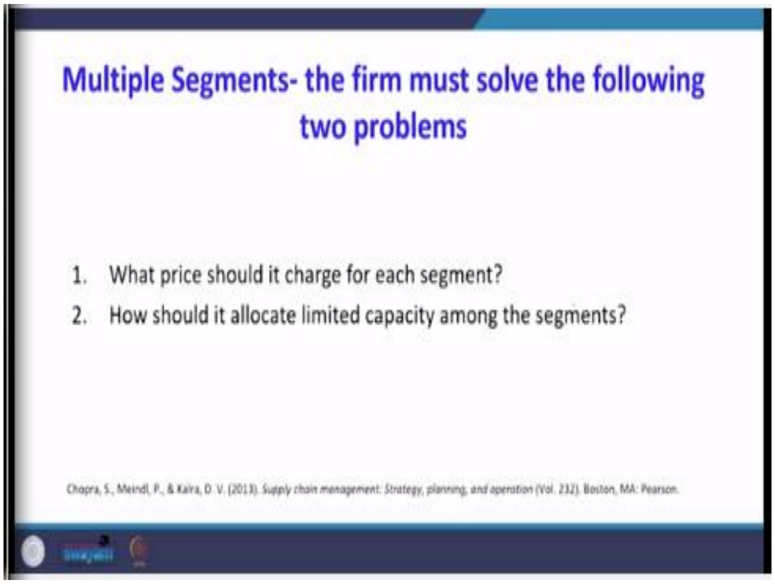
Thus, the firm extracts revenue of 10,500 dollars; how did we get these 10,500 dollars? 3000 multiplied by 3.5, from the segment willing to pay 3.5, that is the revenue. The revenue of 6000 dollars from the segment is willing to pay 2 dollars per cubic feet. So, how we got 6000 dollars? 3000 multiplied by 2, so the total revenue that is 6000 dollars + 10000 dollars will be 16,500 dollars as shown in the figure.



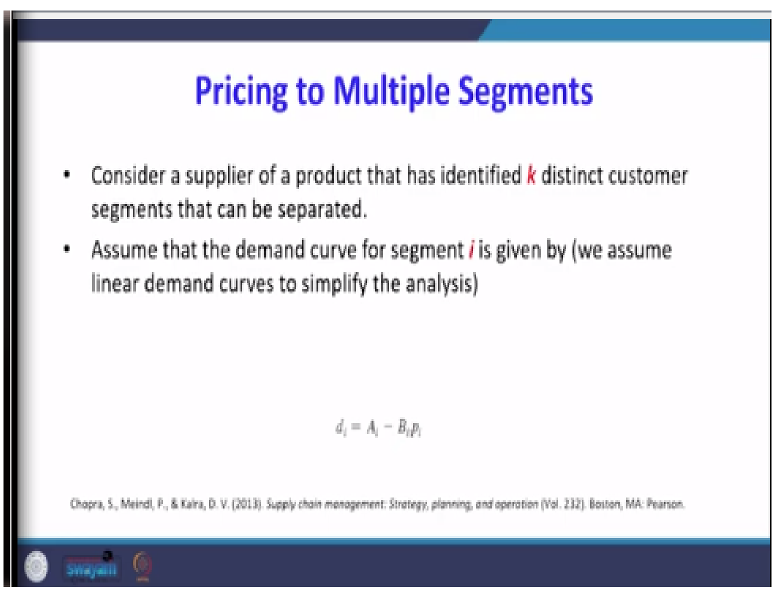
So, what we understand is that when you split the demand and charge differently for different segments, your revenue increases. You see that in the first 3000 cubic feet, the demand is sold for 3.5 dollars, and the remaining 3000 sold for only 2 dollars, so the overall revenue is 16,500 dollars. So, there is a significant improvement in profit, which is the benefit of revenue management.



There are 2 fundamental issues in revenue management. First, how can the firm differentiate between 2 segments and structure it is pricing to make one segment pay more than the other? Second, how can the firm control demand so that the lower paying segment does not use the entire availability of the asset? This is similar to our railway tatkal booking; in tatkal booking, there are a certain number of seats alerted for booking at the last minute, and they were charged at a higher price. But if you are booking in advance, you will be charged less. So, the question is how to differentiate the segment?



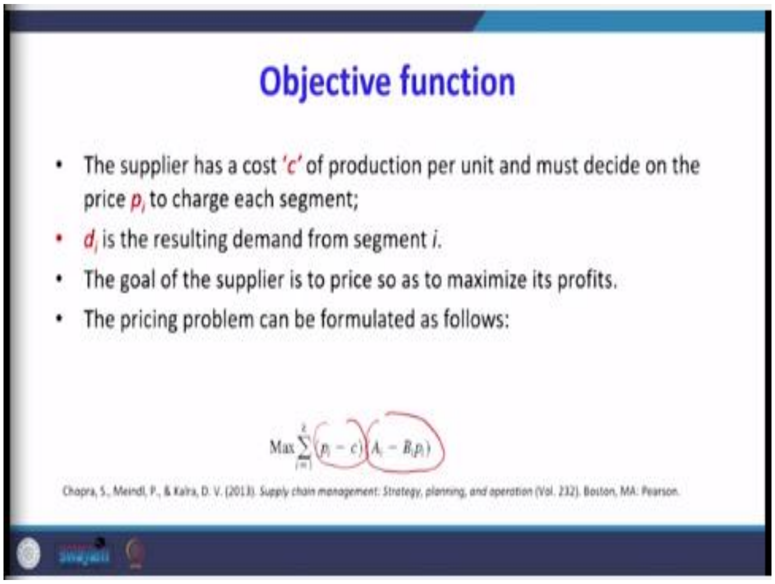
When we are solving multiple segments, there are two important questions that should be answered. One is what price should it charge for each segment? Similar to what the tatkal price should be, what should be the ordinal price in a railway ticket context? And how should it allocate limited capacity among the segments? So, how many seats should be allocated for Tatkal, and how many seats should be allotted regularly? So, these are the 2 important points.



Now let us see how to price for multiple segments. Consider a supplier of your product that has identified a distinct customer segment that can be separated. Assume that the demand curve for a segment i is given by we assume linear demand,

$$d_i = A_i - B_i p_i$$

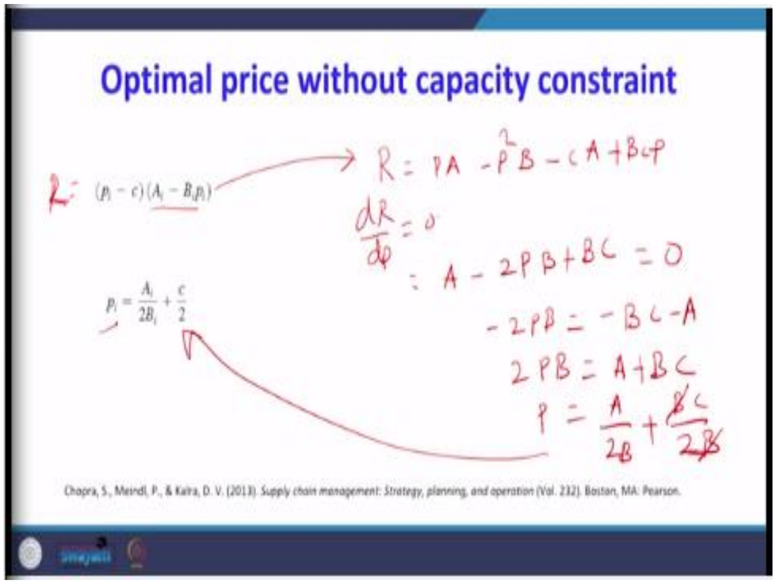
Where d is demand, A is a constant, B also coefficient of this p, price.



So, in the pricing what will be the objective function? The supplier has a cost 'c' of production per unit and must decide on the price p to charge each segment. So, d is the resulting demand from segment i. The goal of the supplier is to price so as to maximize its profit,

$$\text{Max} \sum_{i=1}^{k} (p_i - c)(A_i - B_i p_i)$$

so now the pricing problem is (p – c), this is my profit, what is the p? Selling price - cost. What is this term (A – Bp)? That is a demand. So, my profit needs to be maximized; you see that (p – c) and (A – Bp), when you multiply this objective function, become non-linear, so where we need the concept of the non-linear problem here.

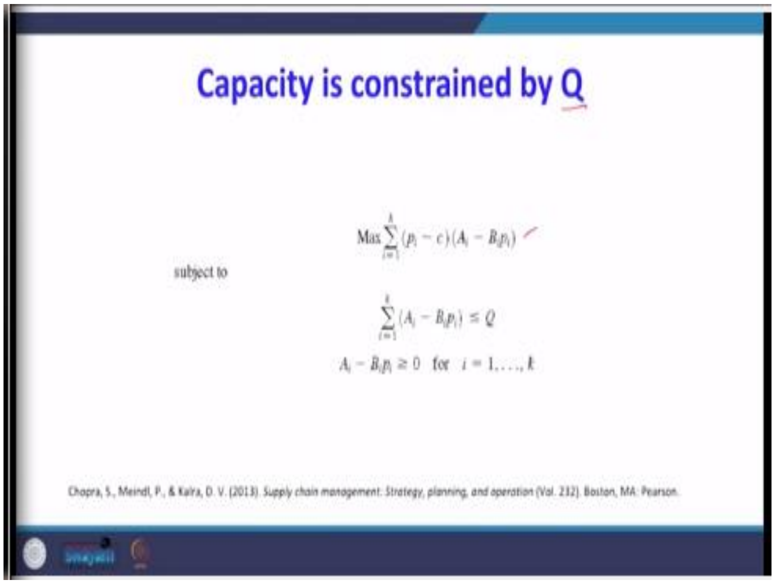


Here, we did not consider the capacity, so that is the optimal price without capacity constraint. We are not considering the capacity; we are just going to fix only the price. So, what have we done? So, this is my revenue function, so I assume that this is revenue R, so I want to know what the selling price should be. So, the revenue function R = (p – c) per unit profit is multiplied by this demand.

Now we know the optimal price p, so I am going to take this function as an R revenue function; I am cross multiplying PA - P square B - CA + BiP, and I am going to differentiate this function with respect to P and equated to 0. So, when I differentiate this will be A - 2PB, P will be 1, BC. So, I need P value, so -2PB = -BC – A, so I am removing minus on both sides 2PB = A + BC, so I am dividing by 2B on both sides,

so p = A/2B + C/2,

so this is the same equation that we are getting: p = A/2B + C/2. So, this will say for each segment what the selling price should be.



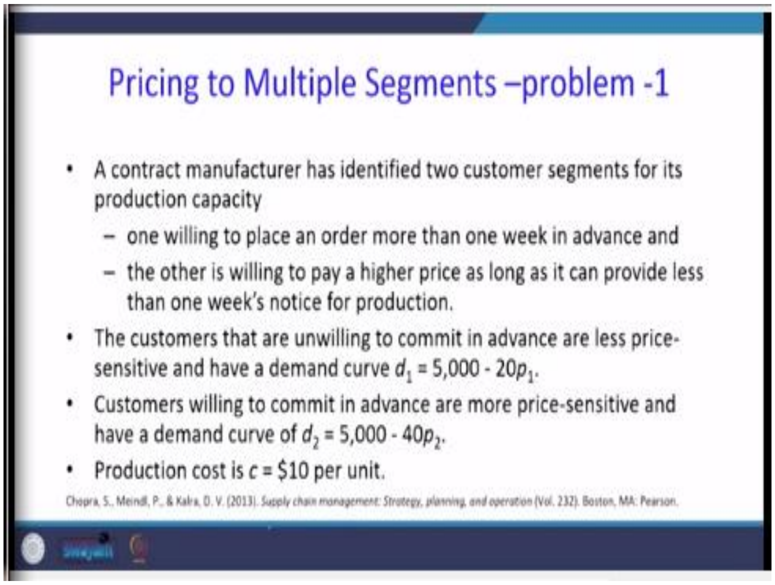
Now if you consider the constraint, capacity constraint. For example, Q is the maximum capacity, here demand. So, our objective function is maximizing our profit, so the demand is, say, less than or equal to Q, (A – Bp) is less than the demand. Suppose we are stocking the unit, and you say shop will have a constraint on storing, so that is nothing but your Q. So, now this problem is a non-linear problem because the p square is coming.

$$\text{Max} \sum_{i=1}^{k} (p_i - c)(A_i - B_i p_i)$$

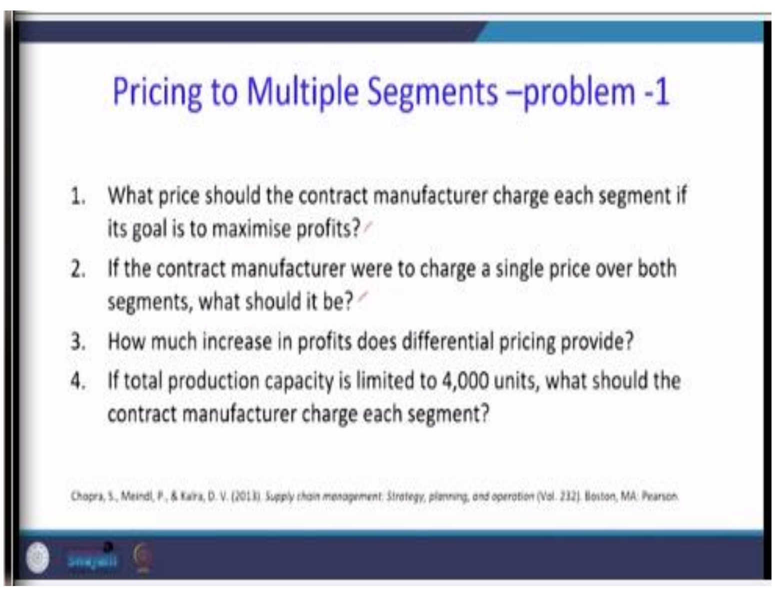
subject to

$$\sum_{i=1}^{k} (A_i - B_i p_i) \leq Q$$

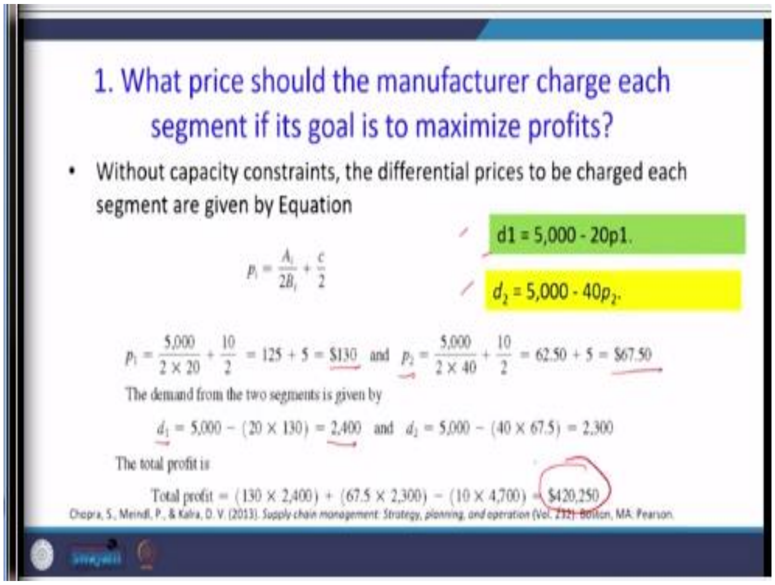
$$A_i - B_i p_i \geq 0 \quad \text{for} \quad i = 1, \ldots, k$$



Now we will take a sample problem, pricing for multiple segments. A contract manufacturer has identified two customer segments for its production capacity. One is willing to place an order more than 1 week in advance so he will be charged less; the other is willing to pay a higher price as long as he can provide less than 1 week's notice for production. Customers unwilling to commit in advance are less price sensitive and have a demand curve d1 = 5,000 - 20p1. Customers willing to commit in advance are more price sensitive and have a demand curve of d2 = 5000 - 40p2, the production cost is 10 dollars per unit.



So, what do we need to know? What price should the contract manufacturer charge for each segment if it is goal is to maximize the profit? The second one is if the manufacturer were to charge a single price over both segments, what should be the selling price? So, how much of an increase in profit does the differential pricing provide? If the total production capacity is limited to 4000 units, what should the contract manufacturer charge for each segment? So, we are going to answer these 4 questions.



The first question is what should be the price? So, without capacity constraint, the differential prices to be charged to each segment are given by this equation.

We have derived just now

$$p_i = \frac{A_i}{2B_i} + \frac{c}{2}$$

p is the price, c is cost, A is in a demand function the constant, B is the coefficient of price. So, for each segment the demand function is given d1 = 5,000 - 20p1, d_2 = 5,000 - 40p_2.

So,

$$p_1 = \frac{5{,}000}{2 \times 20} + \frac{10}{2} = 125 + 5 = \$130 \quad \text{and} \quad p_2 = \frac{5{,}000}{2 \times 40} + \frac{10}{2} = 62.50 + 5 = \$67.50$$

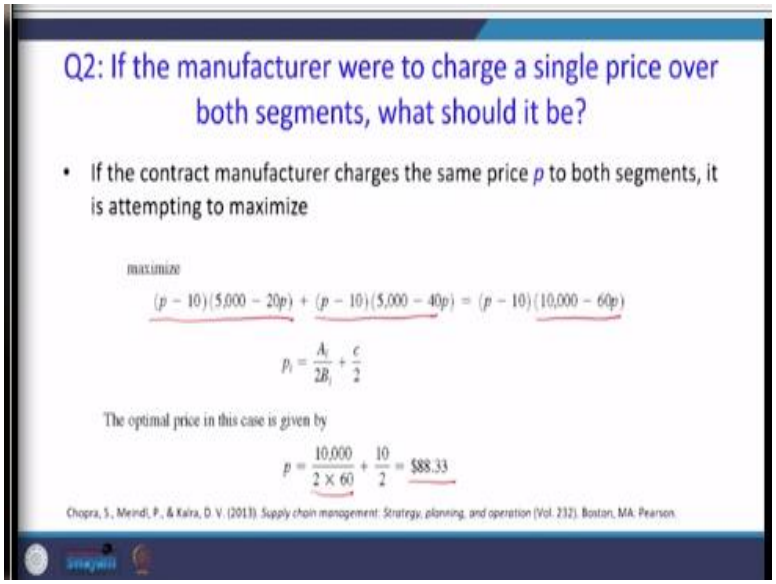
The demand from the two segments is given by

$$d_1 = 5{,}000 - (20 \times 130) = 2{,}400 \quad \text{and} \quad d_2 = 5{,}000 - (40 \times 67.5) = 2{,}300$$

The total profit is

$$\text{Total profit} = (130 \times 2{,}400) + (67.5 \times 2{,}300) - (10 \times 4{,}700) = \$420{,}250$$

That is the total profit, not cost; it is the total profit that is 420,250; otherwise, 4 lakh 20,250 dollars.



The second question is if the manufacturer were to charge a single price over both segments, what should that price be? So, what do we have to do? We have to maximize our profit,

maximize

$$(p - 10)(5{,}000 - 20p) + (p - 10)(5{,}000 - 40p) = (p - 10)(10{,}000 - 60p)$$

so, segment 1 (p – 10) is the demand; this is for segment 2; instead of p1, p2, I am going to substitute only p because the price is going to be the same. So, what can I do? This (p – 10) is constant I can bring on the left-hand side, so when you simplify, this will be (10000 - 60p). Now this is looking like our demand function.

We know that

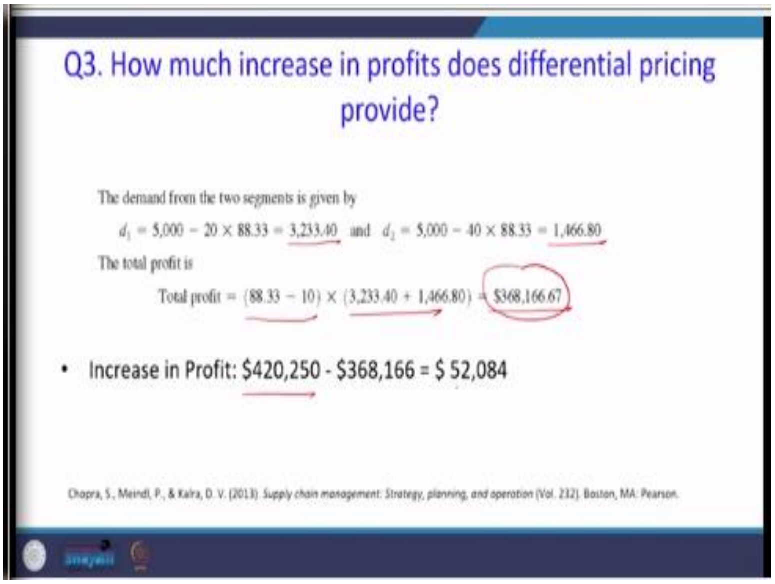
$$p_i = \frac{A_i}{2B_i} + \frac{c}{2}$$

, so here,

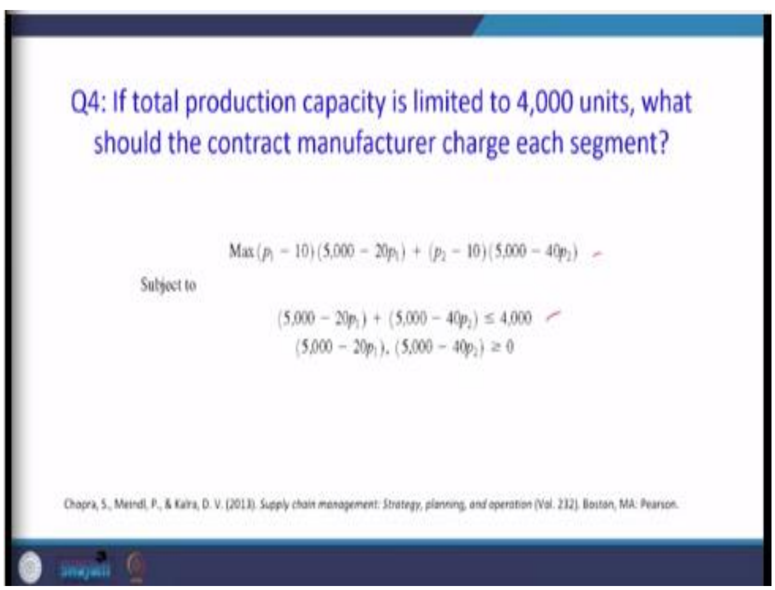
The optimal price in this case is given by

$$p = \frac{10{,}000}{2 \times 60} + \frac{10}{2} = \$88.33$$

So, if you are selling at the same price for both segments, the selling price should be 88.33 dollars.



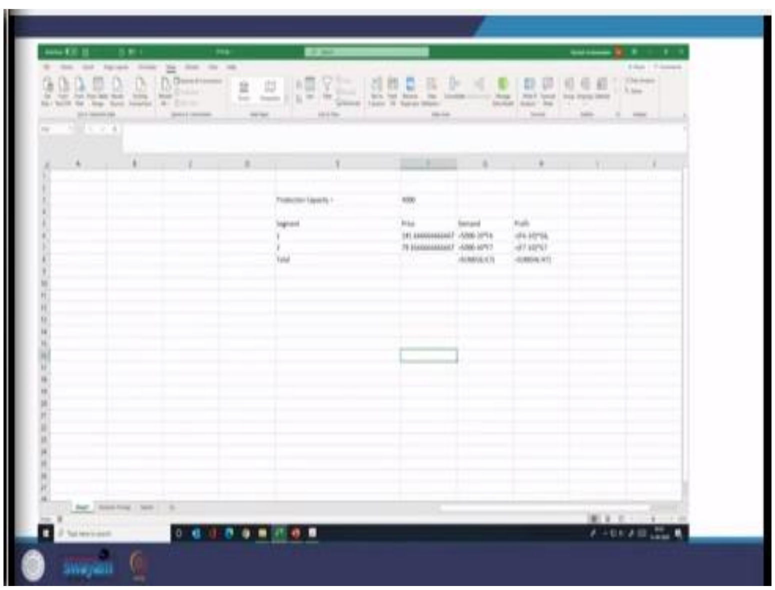
So, how much of an increase in profit does the differential pricing provide? The demand from 2 segments, so d 1 we got when we are going for the same price 3233.40, and d2 is 1466. So, the total profit is total demand multiplied by per unit profit, which is 3 lakh 68166. Initially, if you go for differential pricing, we got 4 lakh 20,250 and fifty where we got this value this one. Now, when you go for the same price, the profit is this much, so the difference in pricing is 52084 dollars. So, what we understand is that when you differentiate the price for different segments, your overall profit is increasing.



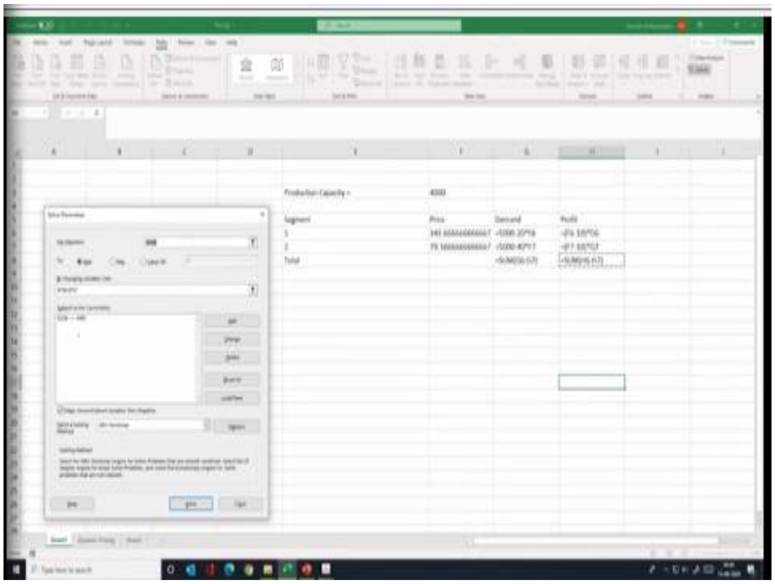
Now, if the total production capacity is limited to 4000-dollar units, what should the contract manufacturer charge? Now, in this problem, there is an objective function, there is a constraint, and this needs to be solved with the help of a solver. So, we are going to solve this problem with the help of solver. Now I will explain the formulation of this model. You see that the initial production capacity is 4000 units, so here, the F6 is your p1, and F7 is p2.

In G6, I am going to write the demand function; see, the demand function which is given to us is (5000 - 20p1), and G7 is (5000 - 40p2), so what is the profit? So, the profit is F6 - 10, and the selling price - 10 multiplied by G6 is a profit. So, when you go for H8, H8 is the sum of the profit; this needs to be maximized. The demand for the G8 is your total demand, G8 total demand, now go to the solver.

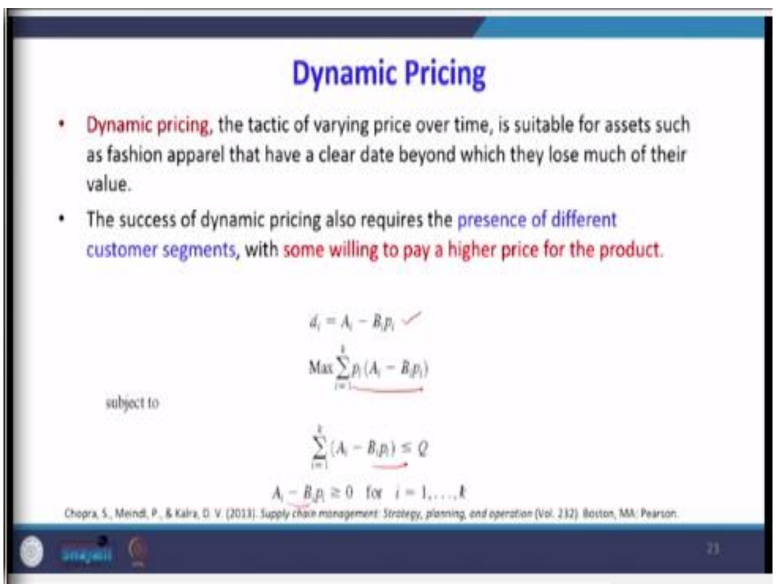
You see this: maximizing our profit has to be maximized, so the changing cell is F6 to F7 and the G8; what is the G8? The total demand should not exceed our constraint, which is 4000. So, when I solved using a non-linear model, press OK, and you see that my G8 total demand is 4000. The selling price for Segment 1 is 141.6, and for Segment 2, it is 79.16; I will return to my presentation.



So, in this slide I have explained what are the formulas which I have used it.



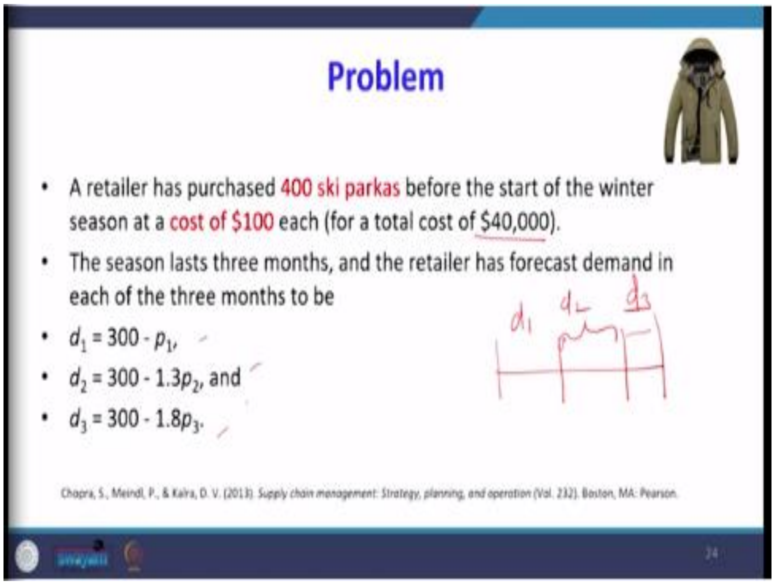
So, you see that there is a constraint. So, when I solve it, I am getting the selling price for p1 is 141, and for p2 is 79.1.



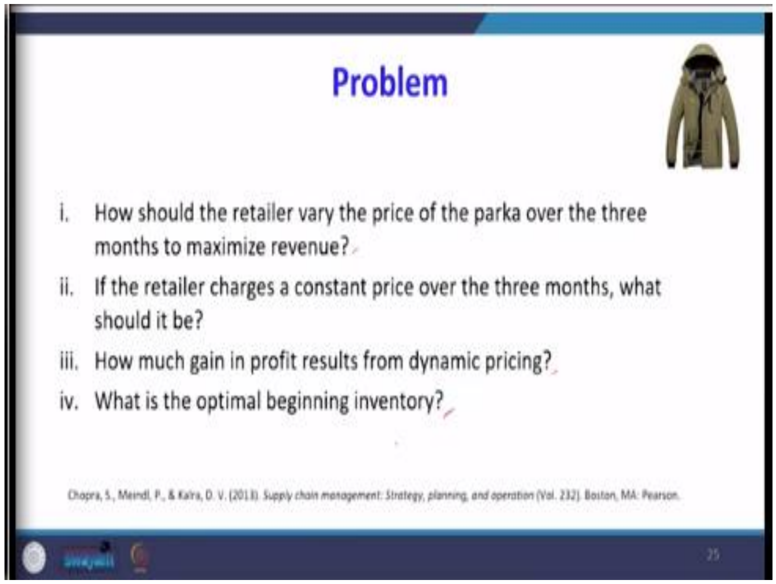
Now we are going to discuss the concept called dynamic pricing; it is nothing but charging for different segments at different prices, that is dynamic pricing. So, dynamic pricing, the tactic of varying prices over time, is suitable for assets such as fashion apparel that have precise dates beyond which they lose much of their value. For example, Roorkee, now this is the winter, the sweaters, once the season is over, will lose their value, so what will happen?

At the beginning of the winter, the shopkeepers will sell at a higher price, and when the season is over, they will go for sale, which means that they will be sold at very low prices. So, they want to know what the selling price for different seasons of this winter should be. So, this is an example of a non-linear problem. So, the success of dynamic pricing also requires the presence of different customer segments.

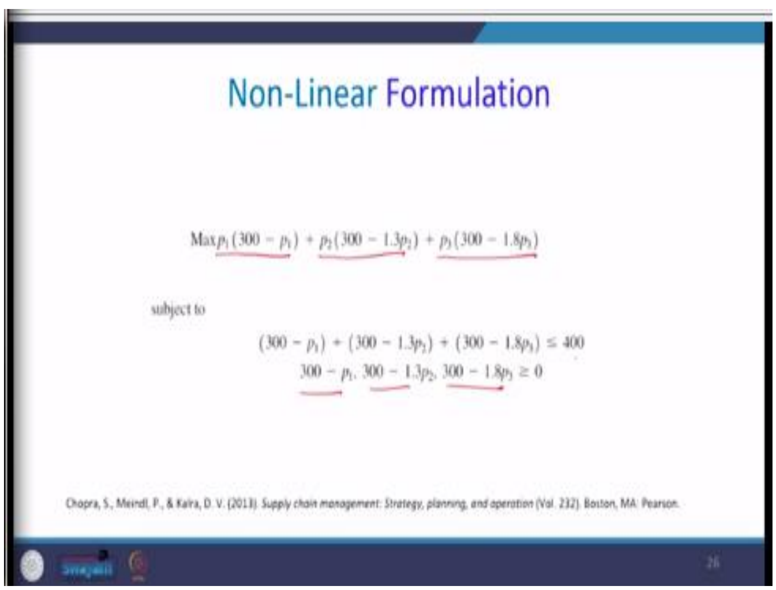
There should be different customer segments who are willing to pay higher prices for the product. So, this is a usual demand function; this is profit needs to be maximized, this is our constraint, and the demand should be positive.



Previously, we have considered the two segments; now we consider 3 segments. So, if you consider winter, maybe there may be 3 segments, the beginning of the winter, the peak of the winter, then the end of the winter. So, there will be a d1, there will be a d2, there will be a d3. Now, we are going to decide what the selling price should be for each segment. So, a retailer has purchased 400 ski parks before the start of the winter season at a cost of 100 dollars. So, the total cost is 40,000, the season lasts 3 months, and the retailer has forecast demand in each of the 3 months, say d1, d2, and d3.



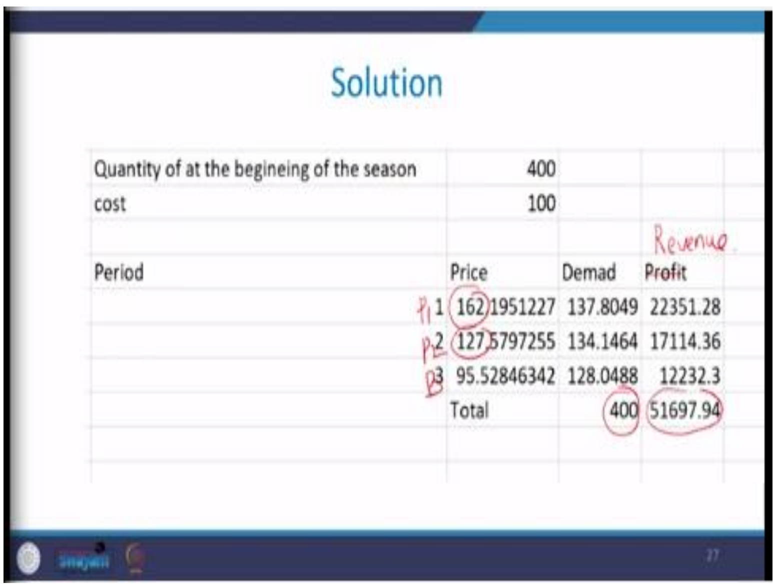
Now, how should the retailer vary the price of the parka over the 3 months to maximize the revenue? If the retailer charges a constant price over 3 months, what should it be? How much gain in profit results from dynamic pricing? What is the optimal beginning inventory? These are the questions that we are going to answer.



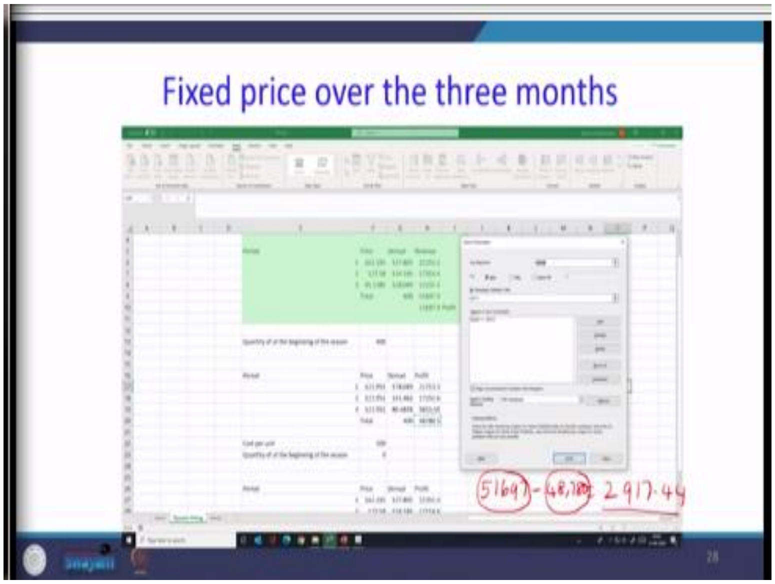
So, this is a non-linear formulation; you see segment 1, which is profit; segment 2, which is the profit; p 1, which is the selling price; this is the demand. We are maximizing revenue; actually, we need to subtract the cost; here, we are maximizing the revenue. The sum of the demand is less than 400, but all the demand should be greater than or equal to 0; why are we saying this less than 400?

The retailer may have the capacity of their shop, but he cannot store more than 400, say, parkas. Now, I will explain this Excel model. So, here, the initial inventory is 400 units, and the cost is 100. So, F6, F7, and F8 are p1, p2, p3, then G6, G7, and G8 are d1, d2, d3, and H6, H7, H8 are revenue for period 1, period 2, and period 3. At the bottom, I wrote total revenue. So, here, the objective function is I must maximize the revenue; I need to find out the value of p1, p2, and p3.

So, when I go to the solver, it is a maximization problem; the changing cell is p1, p2, p3. The total demand cannot exceed 400, which is why it is less than or equal to 400. When I solve it, the value is the p1 selling price for segment 1 is 162.19 dollars, the selling price for segment 2 is 127.57, and the selling price for segment 3 is 95.52. You see that it intuitively looks like a very interesting answer; at the beginning of the winter, the dress price is higher; at the end of this winter, the dress price that is the sweater price is much lower.

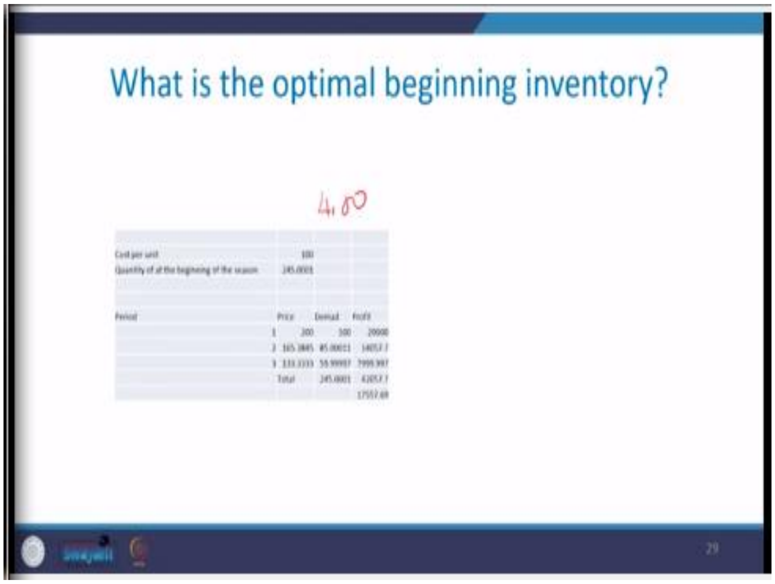


So, I have brought the screenshot, so we got p1, p2, this is p1, p2, p3, so this is our total demand, this is our total revenue, so this is revenue.



The second one is if you go for a fixed price over the 3 months then what should be that fixed price? Now we will go to the excel again. Now everything is the same; I did not make any changes. So, for the F11, please click on F11, and see that F11 is F10, F12 also F10, only that modification I have done it. So, when I put 0, 0, the first one is also 0. So, when I go to the solver when I solve it, I see that for all 3 segments, we are getting the same price, but the revenue is 4 lakh 48780.

Now, we will go to the next question in the presentation. Now, we will see the difference in selling at different prices for each segment and selling at the same price. Initially, when we are selling at different prices, you see that our revenue is 51697. Now, if you are selling at the same price, our revenue is 48,780, so the increase in revenue is 2917.



Now what is the optimal beginning inventory? We know that initially our beginning inventory is 400 units. Now the question is asked, even though it is 400 units, what should be our optimal beginning inventory? We should Is it necessary to stock 400 units, or can we stock less than 400? So that our revenue is maximized. Now, I will go back to Excel. Now I am going to explain this model to find out what is the optimal beginning inventory.

So, the first one is we know p1, p2, p3, here d1, d2, d3, now I am going to find the optimal profit. In the previous problem, we have maximized the revenue, but here, we are going to maximize the profit. For example, see the J14 is H3, what is the H3? J14 is J13, total revenue minus your H6 and multiplied by H5. Here we are not going to buy all 400 units, so we will find out the optimal beginning inventory.

So, now, when I solve it, solver, you see that one important thing is you should remember here the beginning inventory is also one of the decision variables. So, when I drag this dialog box a little down, we can see that the H6 is also one of the decision variables; this is the difference between the previous model and this model. In the previous model, it was 400, which is fixed, now this is 400 also, not 400. That cell is also going to be our decision variable.

Another difference is that we will maximize the profit, not the revenue. Because every time, supposes I am buying, for example, here, I am going to 245, so my buying cost is also going to decrease. That is why we should maximize profit, not revenue, here. So, when I solved it, now I got the optimal beginning inventory is 243.357, so the selling price is for p1= 199, p2 =166, and p3 = 133. Now, the optimal profit is 17,554.97 dollars.

In this lecture, I explained the application of non-linear programming to dynamic pricing. I have considered 2 types of problems, one is dynamic pricing without capacity constraint, and another one is pricing considering the capacity constraint. I have taken two problems where one has 2 segments, and another one has three segments. For each segment, I have found the optimal selling price; for the second problem, I have seen the optimal beginning inventory. In the next class, we will discuss the application of linear programming in project management; thank you.