Decision Making With Spreadsheet
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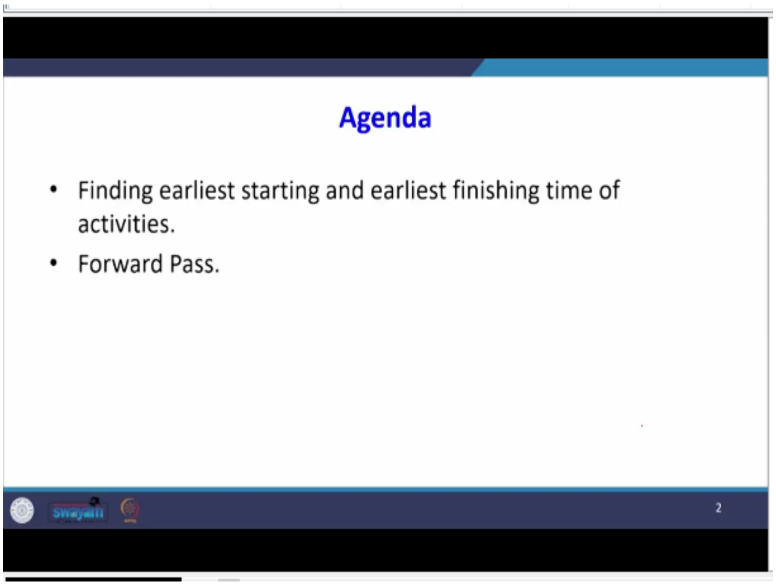
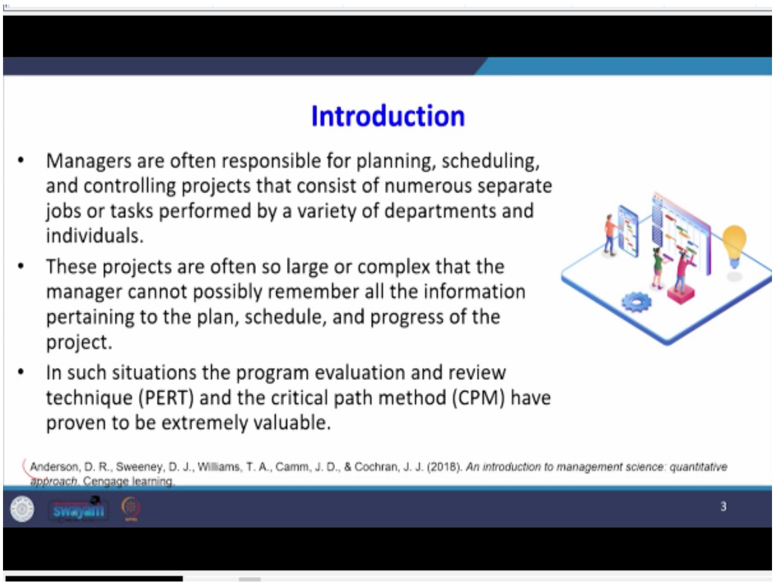
Lecture-29
Project Scheduling: PERT/CPM-I

Dear students, in this lecture, I am going to discuss the project scheduling that is a PERT/CPM.

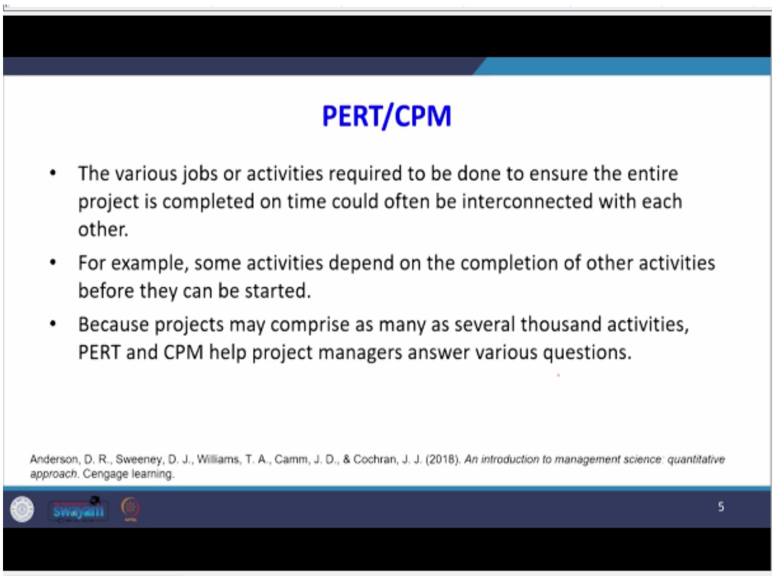


So, the agenda for this lecture is to find the earliest starting and earliest finishing time of activities. That technique is called a forward pass.

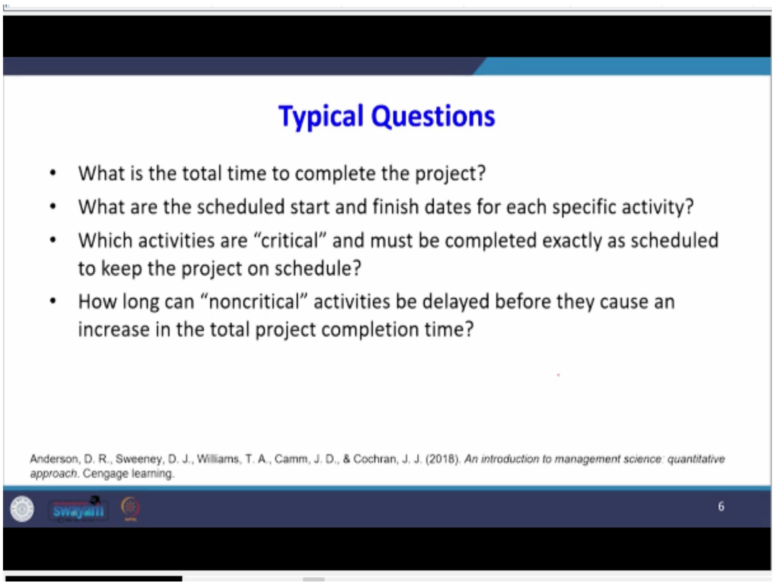


The reference for this lecture is from the book Anderson et al. Managers are often responsible for planning, scheduling, and controlling projects that consist of numerous separate jobs or tasks performed by a variety of departments and individuals. These projects are often so large or complex that the manager cannot possibly remember all the information pertaining to the plan, schedule, and progress of the project. In such situations, the program evaluation and review technique called PERT, and the critical path method CPM have proven extremely valuable.

What are the applications of this PERT and CPM? PERT and CPM can be used to plan, schedule, and control a wide variety of projects. Common applications include research and development of a new product and processes and the construction of plants, buildings, and highways. Another application is the maintenance of large and complex equipment, and another application is the design and installation of the new system. Like this, the concept of PERT and CPM has numerous applications.



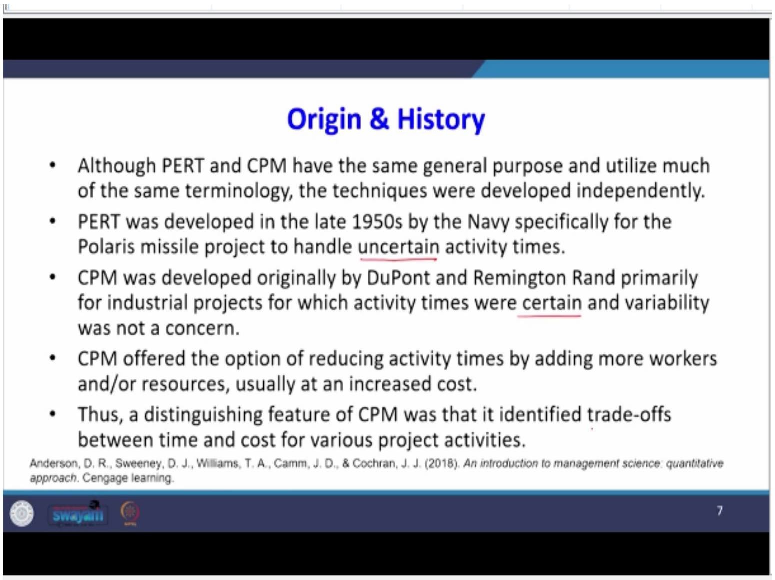
The various jobs or activities required to be done to ensure the entire project is completed on time could often be interconnected with each other. For example, some activities depend on completing other activities before they can be started. Because projects may comprise as many as several thousand activities, PERT and CPM help project managers answer various questions.



What are the typical questions that the PERT and CPM can answer, what is the total time to complete the project? It is a very important question. How much time will it take to complete the project? What are the scheduled start and finish dates for each specific activity? If a project has a number of activities, you should know when a project should be started, when an activity will end, when an activity should be started, and when an activity will be finished.

So, which activities are critical and must be completed exactly as scheduled to keep the project on schedule? So, those activities are critical activities; later, we will be studying if there is a small delay in critical activities that will affect or that will delay the whole project. So, those activities are called critical activities. How long can non-critical activities be delayed before they cause an increase in the total project completion time?

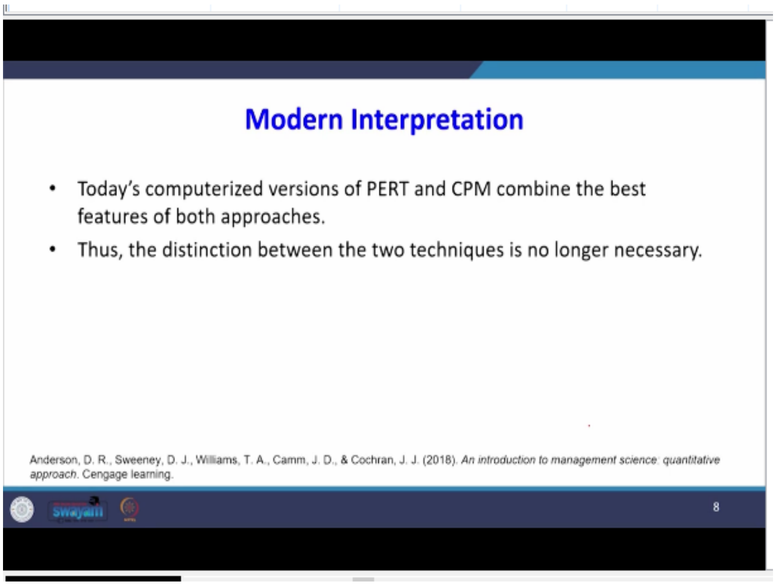
There may be some noncritical activities, and there will be some allowable time that activities can be delayed, but there is a limit to the delay. So, beyond a certain point, even the noncritical activity may affect your total project completion time. So, these questions will be answered with the help of this project network, which is called PERT and CPM.



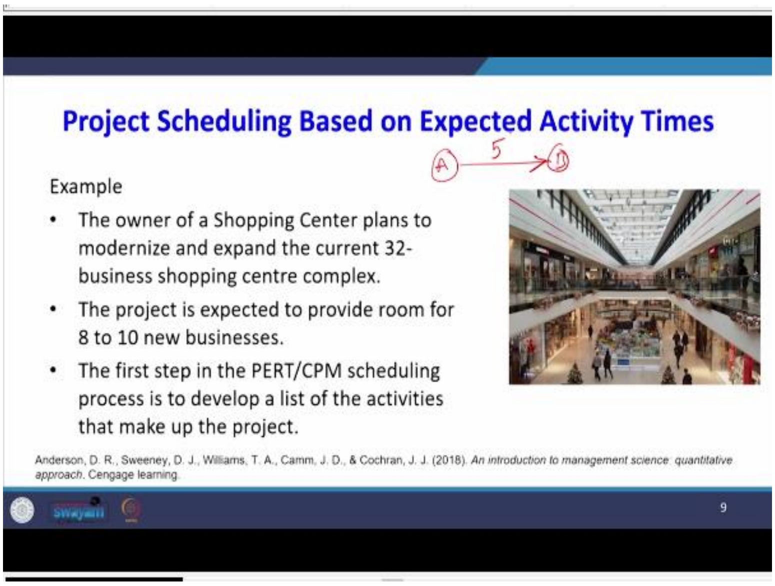
When you look at this origin and history although PERT and CPM have the same general purpose and utilize much of the same terminology, the techniques were developed independently. The Navy developed PERT in the late 1950s specifically for the Polaris missile project to handle uncertain activity times. This is a critical difference between PERT and CPM.

In the PERT, this activity duration is uncertain; whenever any new projects, very innovative projects, it is very difficult to exactly estimate the time taken to complete an activity. So, whenever there is uncertainty in the activity duration, we should go for PERT. CPM critical path method was developed originally by DuPont and Remington Rand primarily for industrial projects for which the activity times were certain. Here, it should be very important that activity times were certain for CPM.

The activity time is uncertain for the PERT, and when you look at the CPM, the activity time is certain, and the variability was not a concern. The critical path method offered the option of reducing activity times by adding more workers and or resources, usually at an increased cost. Later part of the time will be studying a concept called crashing, where I will be using the concept of linear programming; there, I will introduce the solver so that we will be studying after some time. Thus, the distinguishing feature of CPM was that it identified trade-offs between time and cost for various project activities.



When you look at the modern interpretation, today's computerized version of PERT and CPM combine the best features of both approaches. Now, the distinction between the two techniques is no longer necessary.



Now, I am going to take one example; the example I have taken from this book is called Anderson et al., Project Scheduling Based on Expected Activity Time. When I say expected activity time, we can say average activity time; suppose activity A, for example, event A and event B. So, this is an activity here. So, for example, 5, the average activity time is given. The owner of a shopping center plans to modernize and expand the current 32-business shopping center complex.

The project is expected to provide room for 8 to 10 new businesses. The first step in the PERT/CPM scheduling process is to develop a list of activities that make up the project. So, the first step is we have to identify the list of activities that are required for completion of the whole project.

Table – 1
List of activities

Activity	Activity Description	Immediate Predecessor	Expected Activity Time
A	Prepare Architectural Drawings	-	5
B	Identify potential new tenants	-	6
C	Develop prospectus for tenants	A	4
D	Select Contractor	A	3
E	Prepare building permits	A	1
F	Obtain approval for building permits	E	4
G	Perform construction	D, F	14
H	Finalize contracts with tenants	B, C	12
I	Tenants move in	G, H	2
		Total	**51**

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning.

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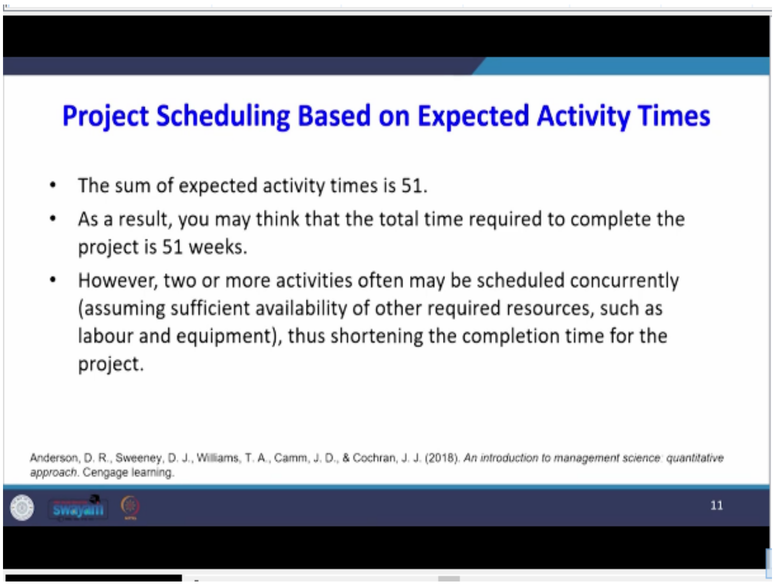
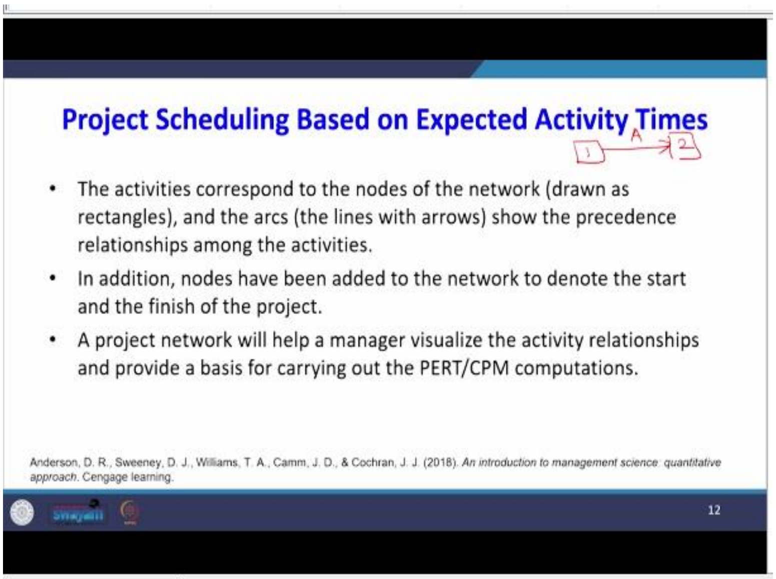
So, this table provides all the activities for the shopping mall project. There are 1, 2, 3, 4, 5, 6, 7, 8, and 9 activities there; each activity description is given. First activity A prepares architectural drawings; after that, the immediate predecessor. There is no immediate predecessor, and for that activity, the expected time is five units; it may be day or any units. Like this activity, B identifies potential new tenants. C, develop a prospectus for tenants.

So, for this, there is an immediate predecessor. So, activity C can be started after the completion of activity A. Similarly, for activity D, select a contractor that can be started after the completion of activity A because it is a predecessor. Similarly, for activity E, prepare a building permit that can also be started once you complete activity A. That is why these 3 activities, C, D, and E, have A as a predecessor. Similarly, activity F and activity G have had 2 predecessors, D and F, activity H has 2 predecessors, B and C, and activity I has another 2 predecessors.



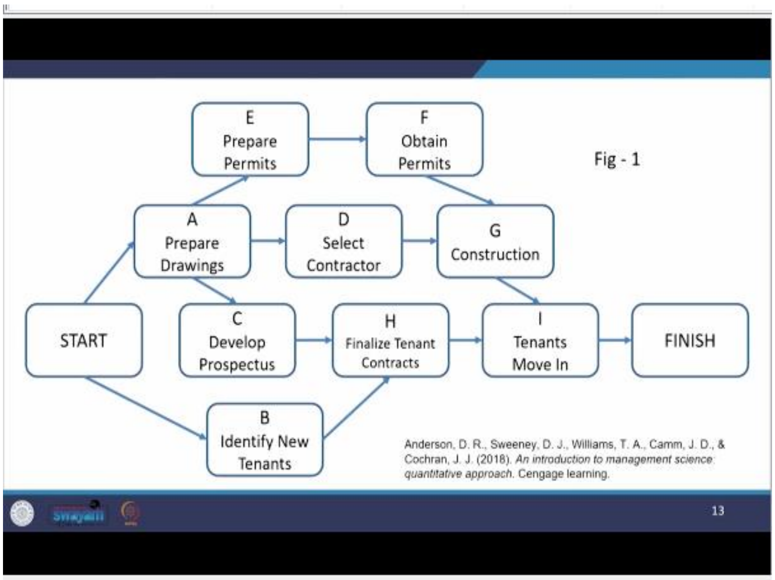
When you add and sum up all the project duration time, it is 51 say I think the unit is weeks. So, what will happen when you do all the activities one by one? It will take 51 days; that is, the sum of the expected activity time is 51. As a result, you may think that the total time required to complete the project is 51 weeks, but that is not correct.

Here is what we can do: two or more activities may often be scheduled concurrently, assuming sufficient availability of other required resources such as labor and equipment. Thus, shortening the completion time for the project is not necessary. It will take 51 weeks. Certain activities can be done parallelly so that they can be finished in less than 51 weeks.

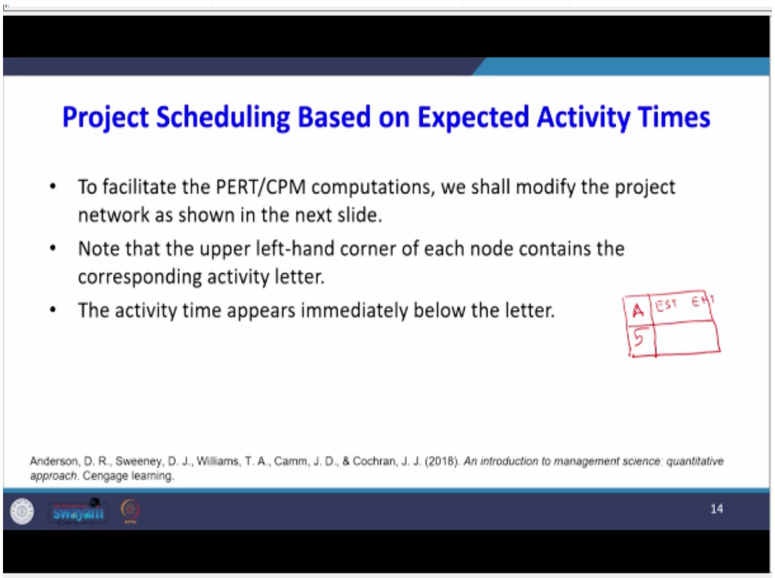


The activities correspond to the node of the network; here, we are the nodes we represent in the form of a rectangle and an arc. So, if I say this way like this and arc, this one shows the precedence relationship among the activities. In addition, nodes have been added to the network to denote the start and finish of the project. The node represents, for example, this activity. If I say A, this is the start of that activity and A; this is a node. Two is another node, the activity is A.

There is a distinction between node and activity. Activity will consume time and resources, but the node is just to indicate when this activity started or when the activity will be finished. A project network will help the manager visualize the activity relationship and provide the basis for carrying out PERT/CPM computations. So, all the activities can be represented pictorially, considering various precedence constraints. So, that picture is nothing but your project network.

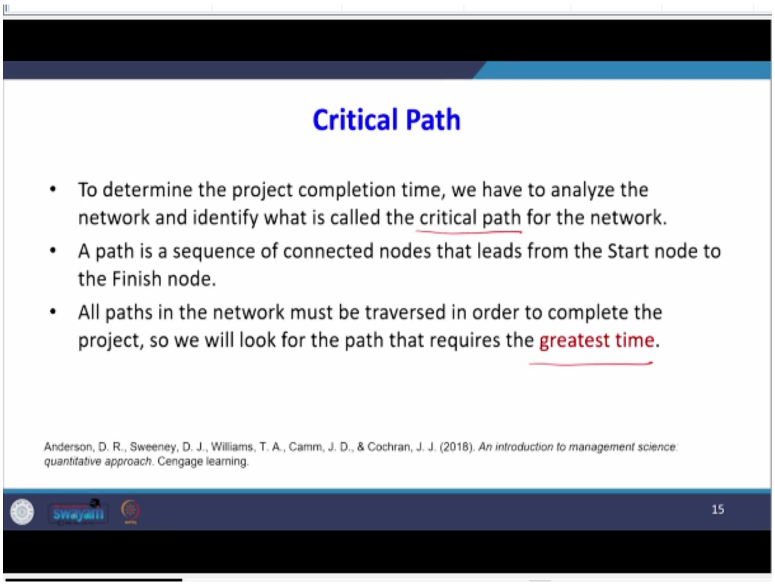


Look at this: this is the project network; there is a start. I have given, say, activities A, B, C, D, E, F, G, H, I at the end, there is a finish.



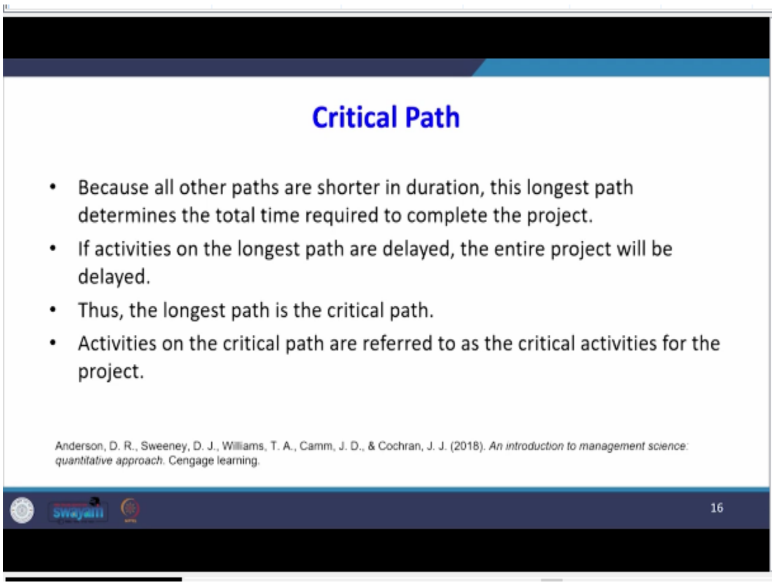
Project scheduling is based on expected activity times. To facilitate the PERT and CPM computations, we shall modify the project network as shown in the next slide. Note that the upper left corner of each node contains a corresponding activity letter. So, what we are going to write? We are going to represent like this. So, if I say A, this is the upper left-hand corner that denotes the activity letter. If I say some number, for example, here for activity A, how much time is it taken? Activity A is 5.

Say if I say five this is our activity duration. And the activity time appears immediately below the letter. Here is what I am going to write; here, I am going to write the earliest starting time and the earliest finishing time. Here, I am going to write the latest finishing time and start time. This is the notation which I am going to use.

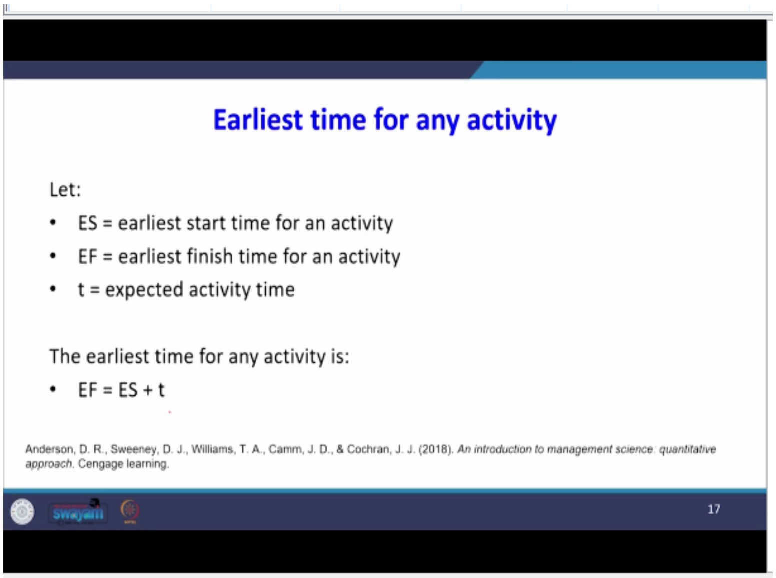


The first aspect is I need to know the critical path. So, to determine the project completion time, we have to analyze the network and identify what is called a critical path for the network. What is the meaning of critical path? The activities in the critical path cannot be delayed; if any activity along the critical path is delayed, that will extend, which will delay the whole project completion time.

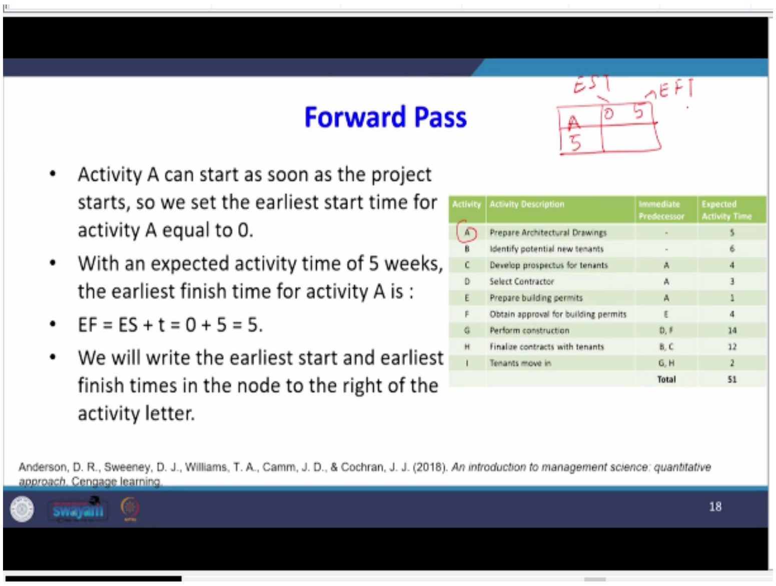
So, what is a critical path? A path, when I say path, is a sequence of connected nodes that leads from the start node to the finish node. That is a path; all paths in the network must be traversed in order to complete the project. So, we will look at the path that requires the greatest time. So, the path that requires the greatest time is called the critical path.



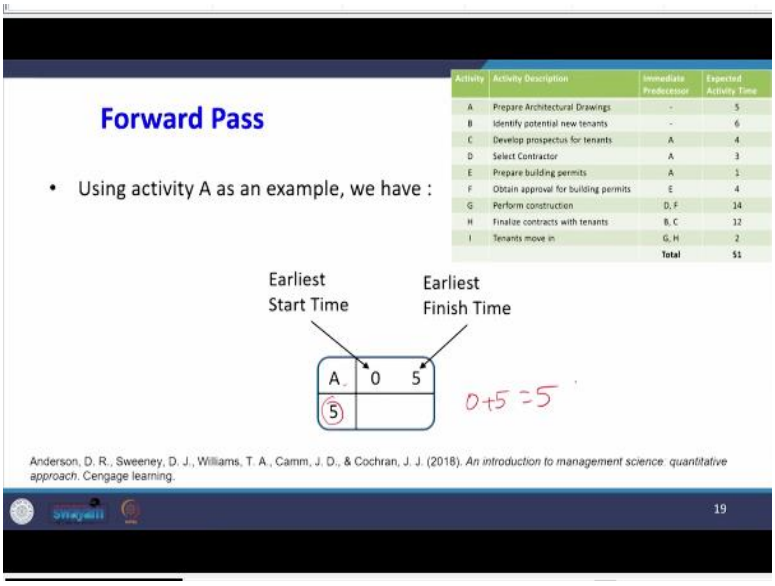
Because all other paths are shorter in duration, this longest path determines the total time required to complete the project. If activities on the longest path are delayed, the entire project will be delayed. So, the longest path is called the critical path. Activities on the critical path are referred to as critical activities for the project.



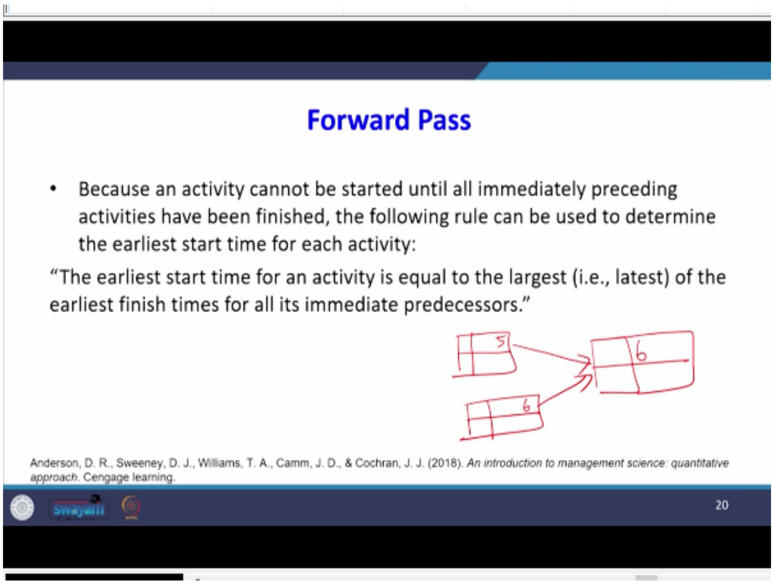
Now I am going to explain how to find the critical path; you should know the earliest time for any activity; it may be the earliest starting time, or it may be the earliest finishing time. The ES represents the earliest start time of an activity; EF represents the earliest finish time for an activity, and t is the expected activity time. So, the earliest finish time for any activity is the earliest start time plus the duration. That is the expected activity time.



Now I am going to explain what this forward pass is. Activity A: when you look at activity A, the expected activity time is 5 weeks. So, the activity A can start as soon as the project starts. So, we set the earliest start time for that activity as equal to 0. Suppose I write this way. So, the earliest start time is 0. I am writing this way, and the earliest finishing time is 5. How do I get 5? 0 + 5 = 5? So, we will write the earliest start and earliest finish time in the node to the right of the activity letter. So, I have written this way, this is the way we have to write. So, this represents the earliest start time; this represents the earliest finish time EFT.

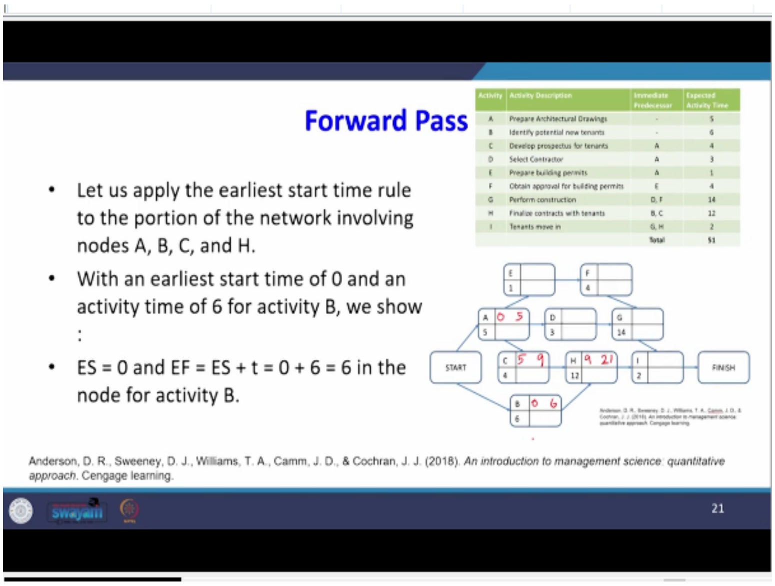


Using activity A as an example, you see A, the activity duration is 5 weeks. So, the earliest start time is 0, and the earliest finishing time is 5 because 0 + 5 = 5. Like this we have to write for all activities.

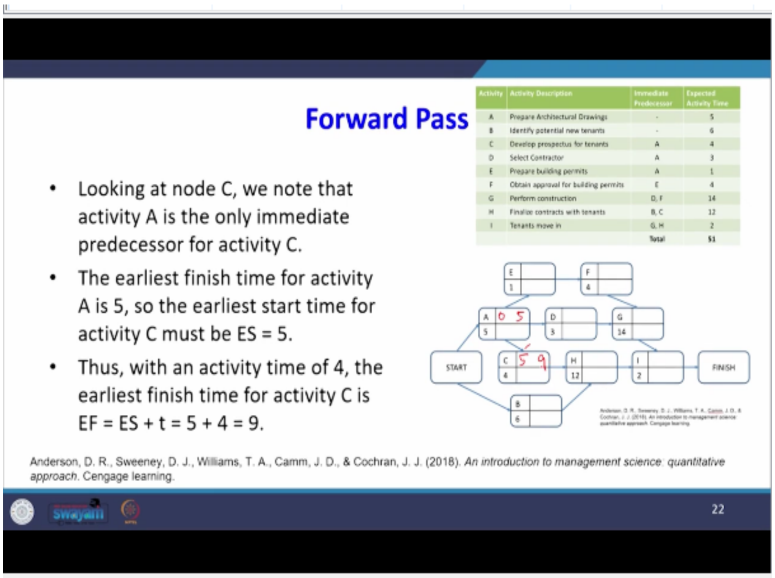


Now, I am going to explain what this forward pass is because an activity cannot be started until all immediately preceding activities have been finished. The following rule can be used to determine the earliest start time for each activity. What is that rule? The earliest start time for an activity is equal to the largest of the earliest finish time for all its immediate predecessors. So, what will happen if there is a situation like this? Suppose I am writing this way.

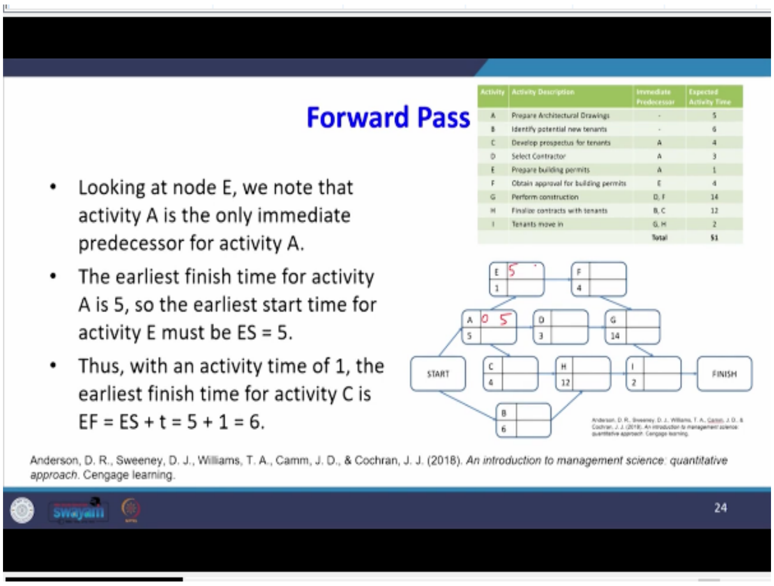
So, what is the meaning? Suppose here the earliest finishing is 5, here the earliest finishing time is 5, now suppose assume that these are predecessors, here the earliest start time is nothing but 5 and 6, which is larger? 6 is the larger. So, 6 means the earliest start time; that is the meaning. The earliest start time for an activity equals the largest of the earliest finish time for all its immediate predecessors. So, there are two predecessors. So, the largest of this earliest finish time is 6. So, these six should be the earliest start time for the succeeding activity. That is the meaning of this forward pass.



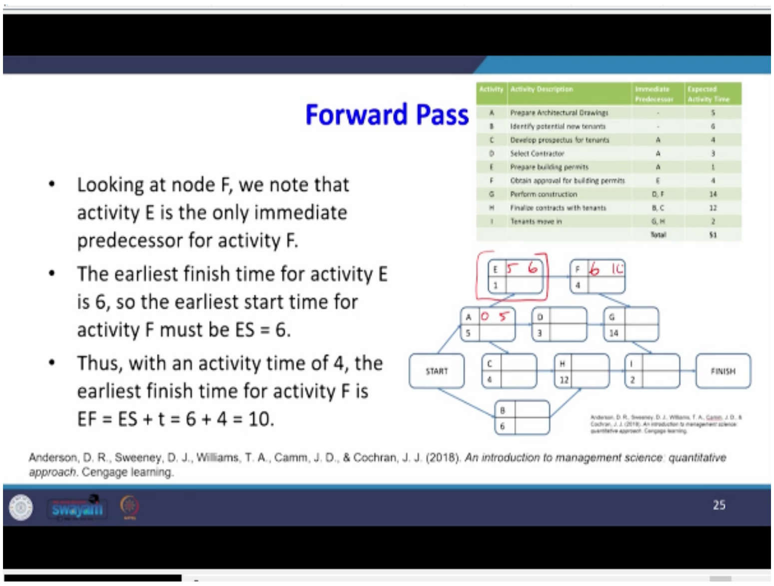
Let us apply the earliest start time rule to the portion of the network involving A, B, C, and H. We know that the earliest start time is 0, and the earliest finish time is 5; for B, the earliest start time timing is 0, and the earliest finish time is 6; for C, the earliest start time is 5, and the finish time is 9. Now, for Activity H, there are 2 earliest finish times; one is from Activity C and one from Activity B. So, for Activity C, the earliest finishing time is 9, and for Activity B, the earliest finishing time is 6. So, the earliest start time for activity H will be 9, 9 + 12 = 21. So, that is what I have written in the text. For B, I have written 6.



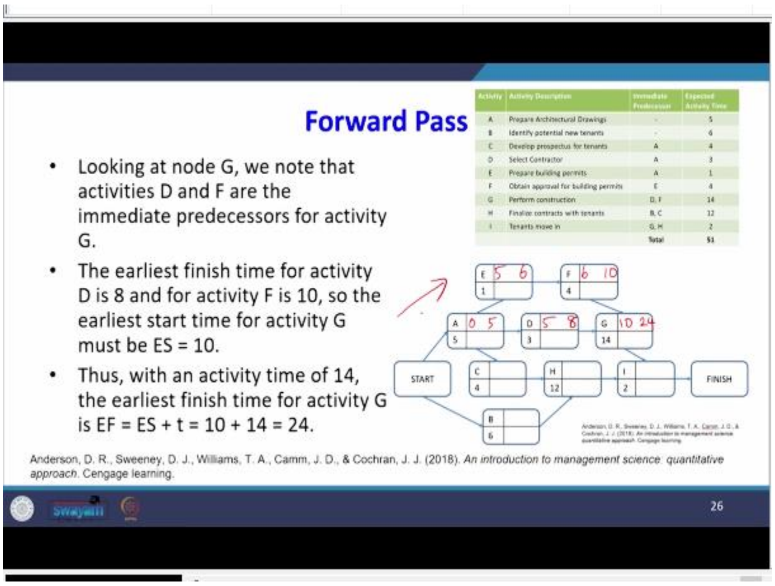
When you look at node C, we note that activity A is the only immediate predecessor for activity C. So, the earliest finish time for activity A is 5. So, the earliest start time for activity C must be 5; that is, I have written here 5. Thus, with an activity time of 4, the earliest finished time for activity C is 9. So, 5 + 4 is 9. So, for activity C the earliest finish time is 9.



Now we look at node D. So, node D, the earliest finish time of activity A is 5. So, the earliest start time for D must be 5. So, that duration is 3. So, 5 + 3 = 8. So, the earliest finishing time for activity D is 8. Now look at the node E, here it is 0, it is 5. We know that activity A is the only immediate predecessor for activity A. So, the earliest start time for the E is 5, and this duration is 1 is 5 + 1, which is 6.

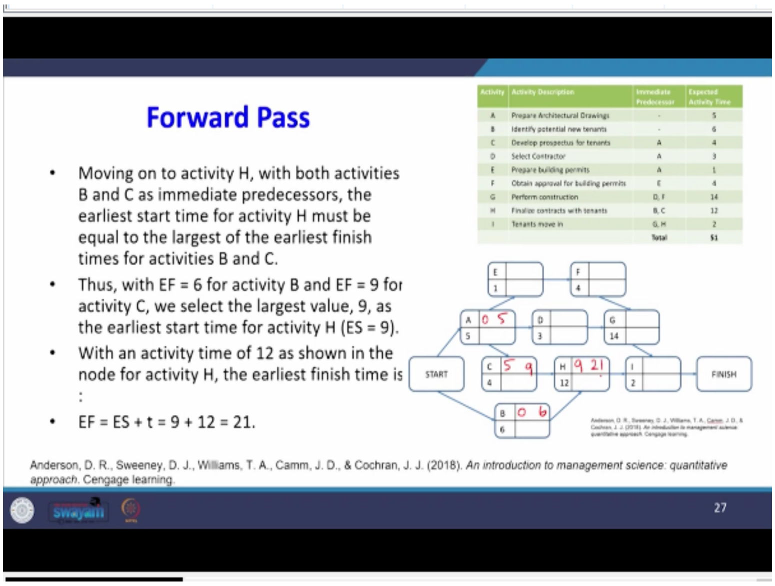


Looking at node F, we know that activity E this activity is the only predecessor for activity F; the earliest finish time for activity E is 6 this 6. So, the earliest start time for activity F must also be 6, and the duration of activity F is 4. So, the earliest finish time will be 6 + 4 = 10.

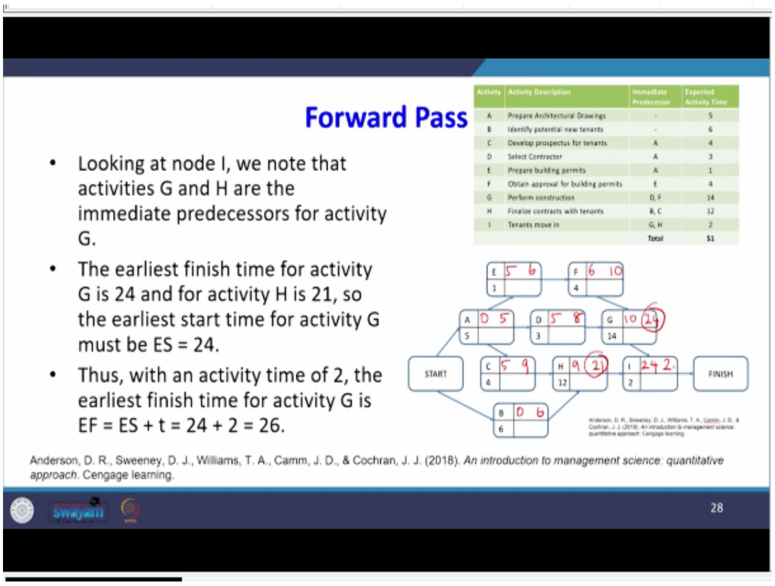


Now look at the node G; in the node G, we note that activities D and F are immediate predecessors for G. So, what are the earliest finishing times for activities F and D? So, here 0, 5, 5, 6, 6, 10, 5, 8. So, the earliest finish time for Activity D is 8, and for Activity F, it is 10. So, the earliest start time is the larger value of 8, and 10 is the 10. So, 10 will be your earliest start time for activity G.

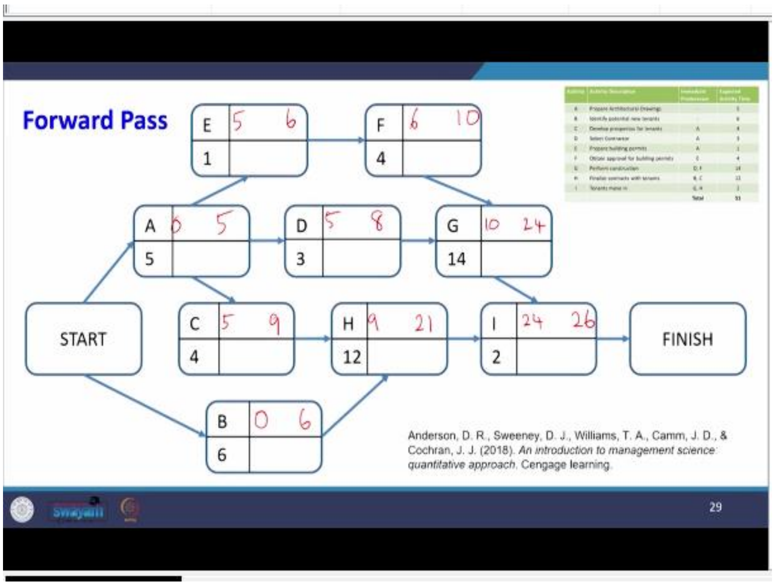
So, with the activity of 14. So, the earliest finish time will be 10 + 14; it is 24. Why I am saying forward pass see I am going this direction is called forward pass. When I go from left to right, whichever is the larger earliest finish time, that should be the earliest start time of the succeeding activity.



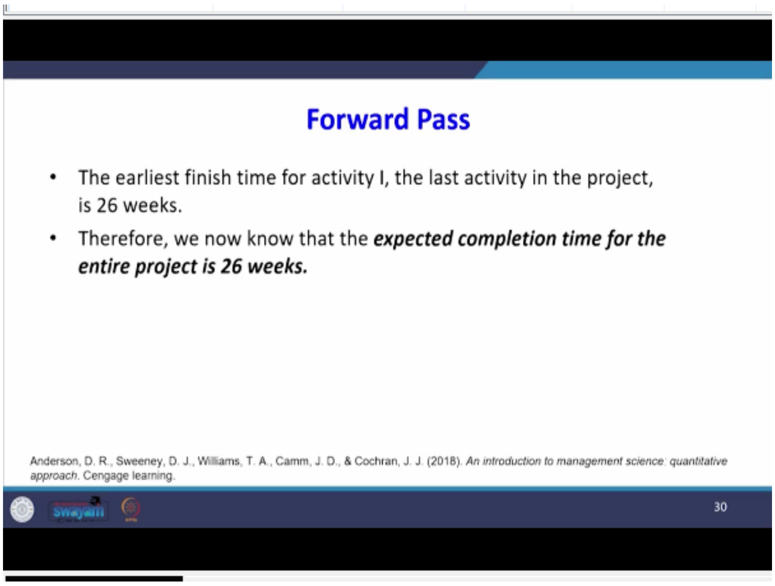
Now, moving on to activity H. In this activity, there are two predecessors, C and B. For B the earliest finishing time is 6. So, 9 is the earliest finish time for activity C. So, we select a larger value that is between 9 and 6. So, 9 is larger. So, 9 will be your earliest start time for activity H. So, 9 + 12 = 21 will be your earliest finish time.



Now, looking at node I, when looking at node I, there are two predecessors, H and G. Now we have to see which is the largest earliest finish time between H and G; what are they? So, the earliest finish time for activity G is 24, and for activity H, it is 21; you see this is 21, which is 24, which is the largest 24 larger. So, 24 will be the earliest start time for the activity I. The duration of this activity I is 2. So, the earliest finish time will be 26.



So, what I am going to do? Now, I am going to write all the earliest start times and earliest finish times at a stretch. So, this is A does not have any predecessor. So, to 0, here also it does not have any predecessor. It is 0. So, 0 + 5 is 5; here, 0 + 6 = 6. So, the earliest finish time of activity A will be the earliest start time for activity C. So, 5 + 4 9. So, here 5 + 1 = 6. So, 6 + 4 = 10, here 5 + 3 = 8. So, the larger one is 9 + 12 is 21. Here, the larger between F and D the larger one is 10 is 24. So, between 21 and 24, 24 is the larger one. So, 24 + 2 = 26. So, the total project completion time is 26.



So, the earliest finish time for activity I the last activity in the project, is 26 weeks. Therefore, we know that the expected completion time for the entire project is 26 weeks. We have arrived at this with the help of a technique called forward pass. So, in this lecture, I have explained how to find the earliest start and earliest finishing times using the technique called forward pass. What is that forward pass? If there are two predecessors for an activity so, the earliest start time for that activity is the largest finishing time of the previous two activities. So, that is the technique called forward pass. In the next lecture, I will discuss the backward pass; thank you.