Decision Making With Spreadsheet
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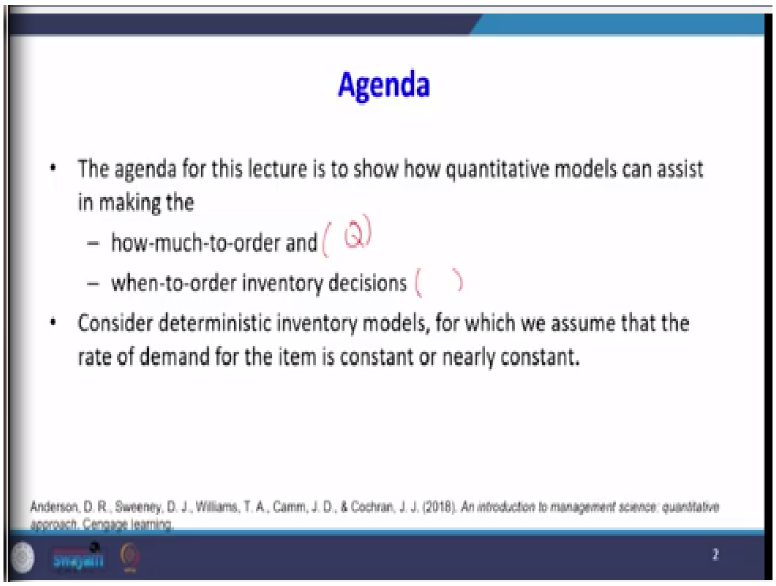
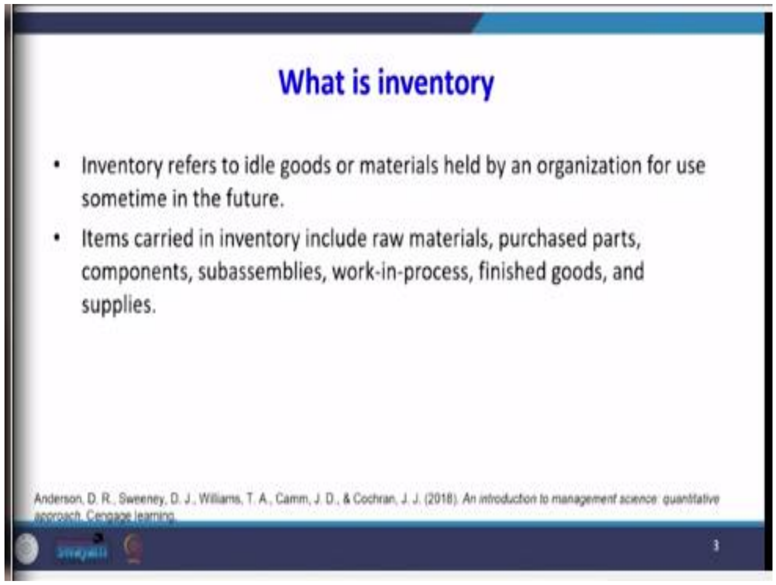
Lecture-33
Inventory Models: Economic Order Quantity.
(EOQ) Model-I

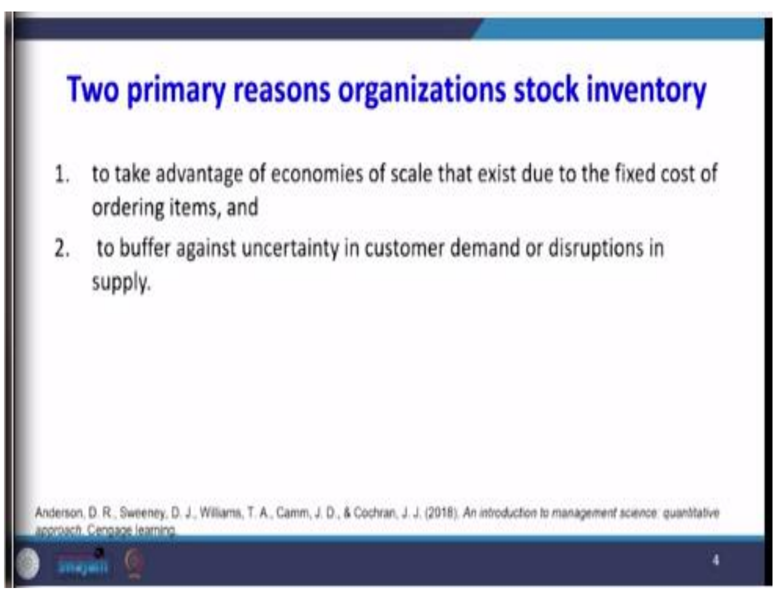
Dear students, In the previous class, I discussed project scheduling, in which I discussed project crashing. Today another important topic is inventory models. So, in this lecture, I am going to explain how to use Excel for making inventory decisions.



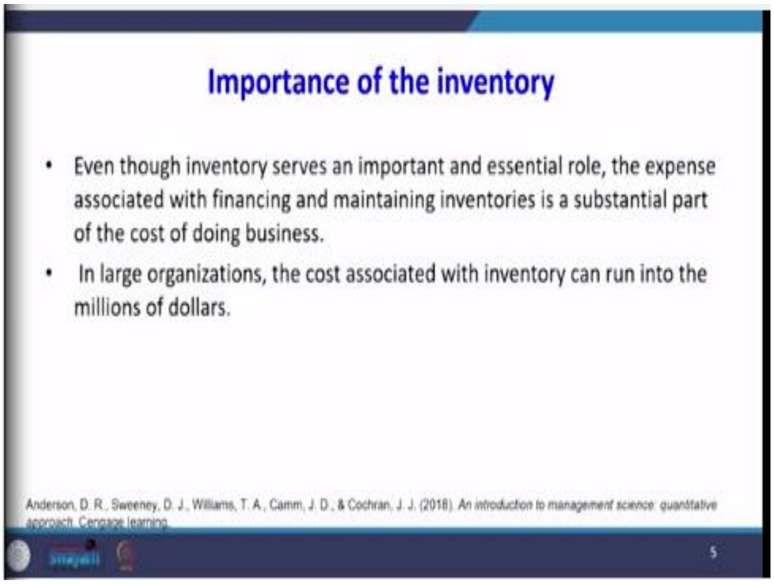
So, the agenda for this lecture is to show how quantitative models can assist in making how-much-to-order and when-to-order inventory decisions. In inventory management, these important questions are how much order is called Q, and the timing when it should be ordered. We are going to answer these questions. So, in this lecture, the assumption is deterministic inventory models; what is the meaning of deterministic inventory models? We assume that the rate of demand for the item is constant or nearly constant, and it will not change. So, that is the meaning of deterministic inventory models: the demand is known to us.



The reference for this lecture is from the book by Anderson et al., First, What is Inventory? So, inventory refers to idle goods or materials held by an organization for use sometime in the future; it is nothing but stocks. So, items carried in inventory include raw materials also, type of inventory, purchased parts inventory, components, subassemblies, work-in-process, finished goods, and the supplies which we have received from our suppliers. These are examples of inventory.

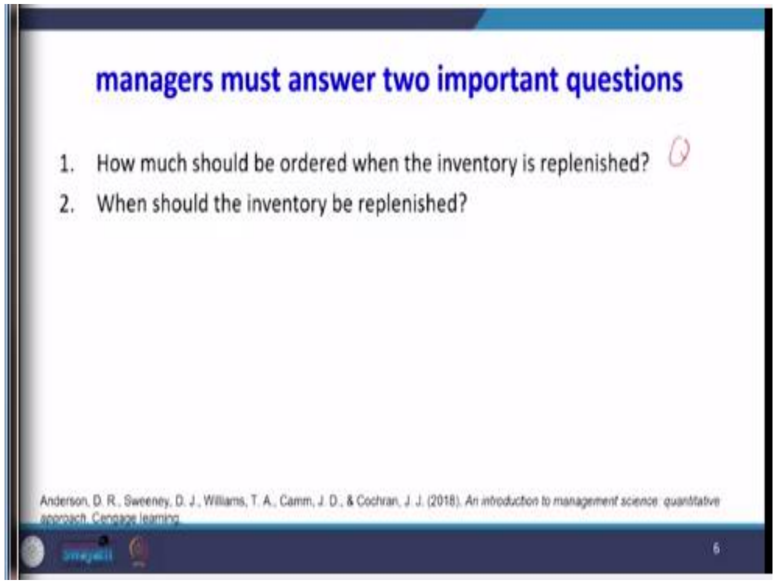


Two primary reasons organizations stock inventory: why is inventory required? The first reason is to take advantage of economies of scale that exist due to the fixed cost of ordering items, what is the meaning of this economies of scale? When you buy in larger quantities, the cost of ordering will be less. That is the meaning of economies of scale. The second one is to buffer against uncertainty in customer demand or disruption in supply. Whenever there is uncertainty in the demand or disruptions, it is better to hold some inventory so that there will not be any stockouts.

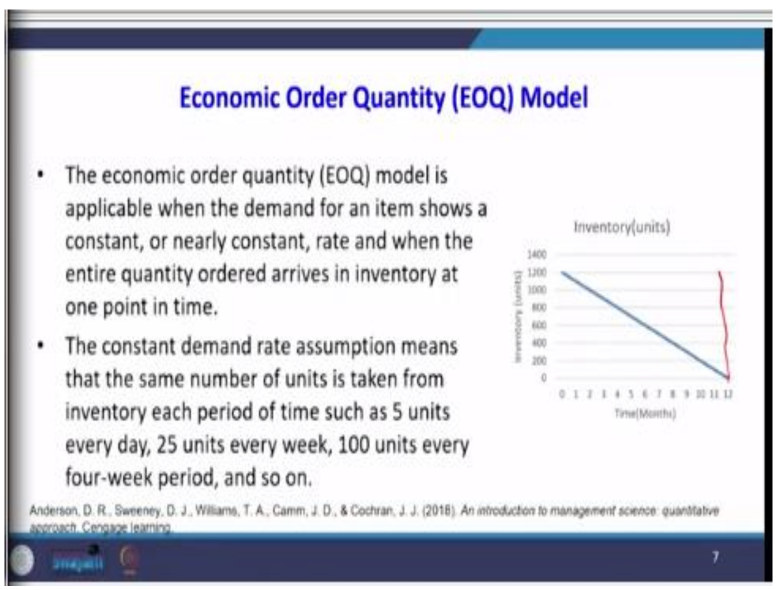


What is the importance of this inventory? Even though inventory serves an important and essential role, the expense associated with financing and maintaining inventory is a substantial part of the cost of doing business. So, when you say business indirectly, it is nothing but inventory management. Some professors used to say what we call supply chain management is nothing, but it is integrated inventory management.

Because inventory management is what is happening, the supply and demand are matched with the help of this inventory. In large organizations, the cost associated with inventory can run into the millions of dollars. So, if you are able to reduce the inventory cost, if you are able to have the right inventory policy, that will save a lot of our money.

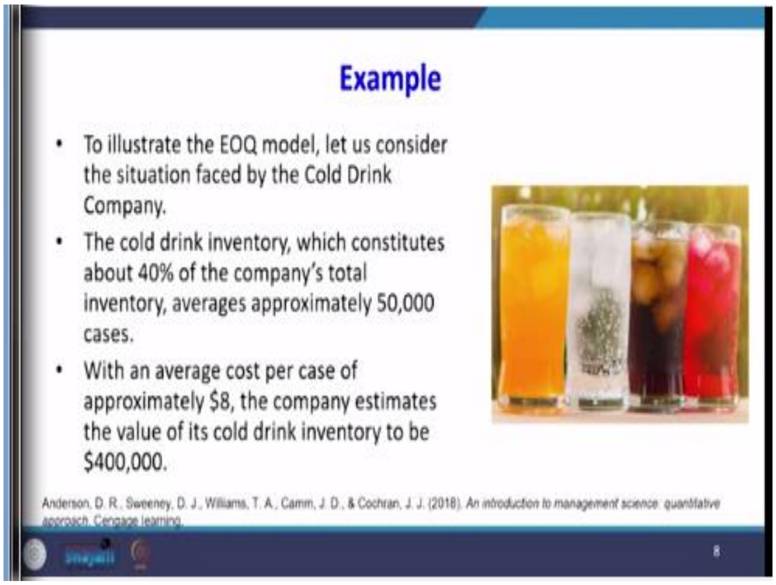


So, managers must answer two important questions while designing the inventory policy. One is how much should be ordered when the inventory is replenished. So, that is the Q generally; the notation we are using is how much has to be ordered. Another one is when it should be the inventory should be replenished, when we have to order, how much, and when. So, these questions have to be answered, and in this lecture, we are going to answer them.

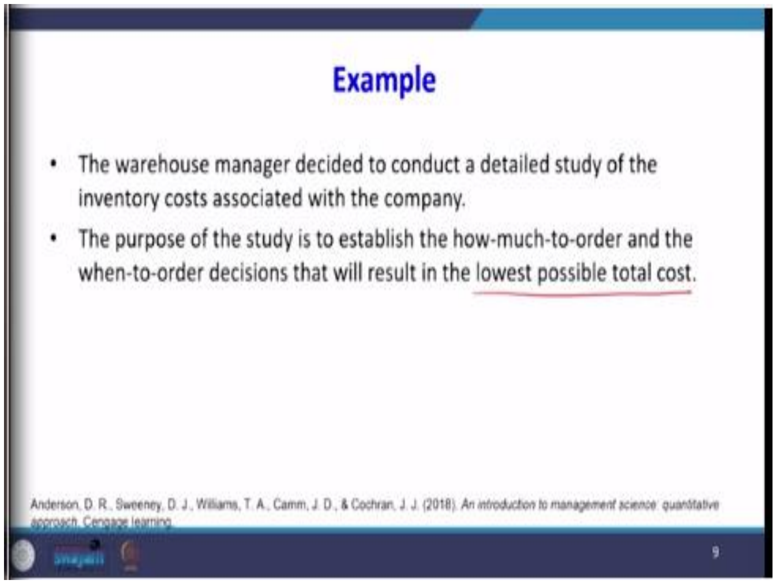


The economic order quantity model is applicable when the demand for an item shows a constant or nearly constant rate. And when the entire quantity ordered arrives in inventory at one point in time. For example, look at this figure here: when it was 0, I ordered 1200 units, and you see that the demand is constant, and the slope is constant. And another point is that when I replenish here right immediately I will get another 1200 units.

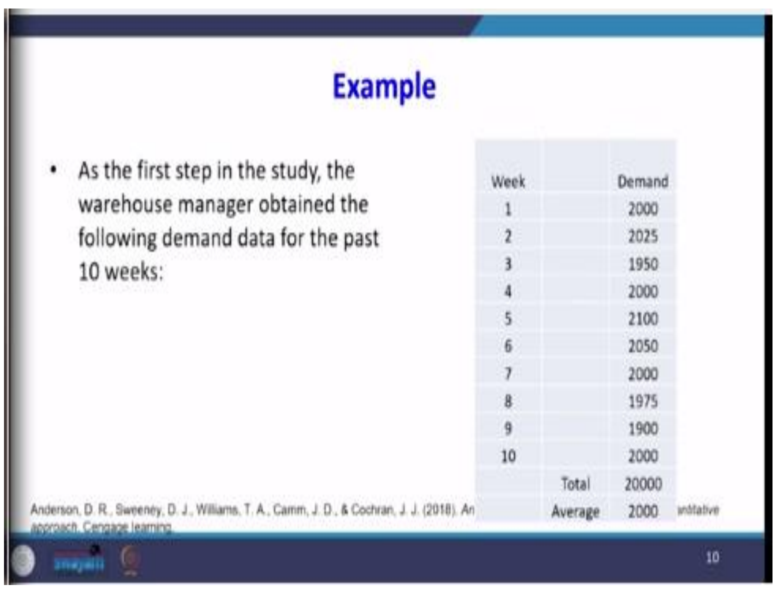
These 1200 units I will get at a time; it is not that I will get 1000, then 200, so these are the assumptions we will study in detail about the assumptions in coming lectures. The constant demand rate assumption means that the same number of units is taken from inventory each period of time, such as, say, 5 units every day. For a week, if there are 5 days, there will be 25 units, 100 units every 4-week period because 4 into 25 = 100, so the demand rate is fixed.



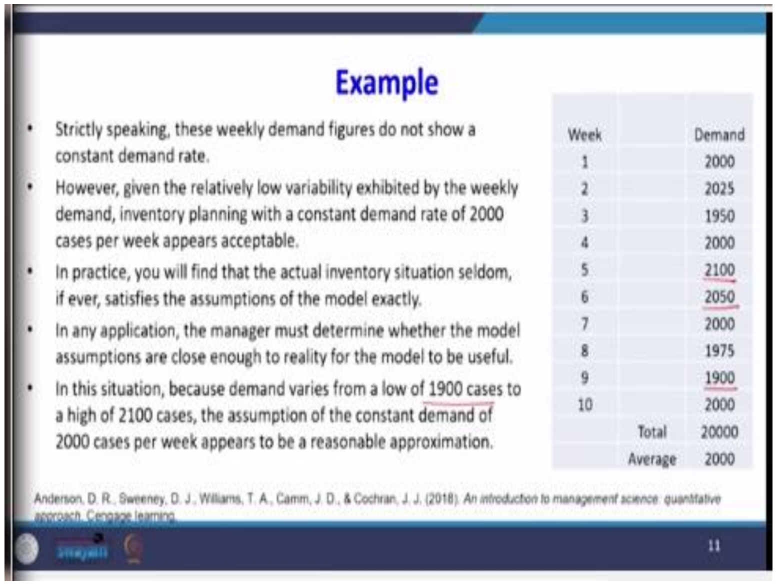
Now I am going to explain the concept of this all inventory management principles with the help of an example. What is this example? To illustrate this EOQ model let us consider the situation faced by a cold drink company. The cold drink inventory which constitutes about 40% of the company's total inventory, averages approximately 50,000 cases. With an average cost per case of approximately 8 dollars, the company estimates the value of it is cold drink inventory to be 400,000 dollars because 50,000 multiplied by 8, so the inventory cost is 400,000.



The warehouse managers decided to conduct a detailed study of the inventory cost associated with the company. The company is thinking about how we can reduce the inventory cost. So, the purpose of the study is to establish how much-to-order and when-to-order decisions will result in the lowest possible total inventory cost, which is what the company is planning.



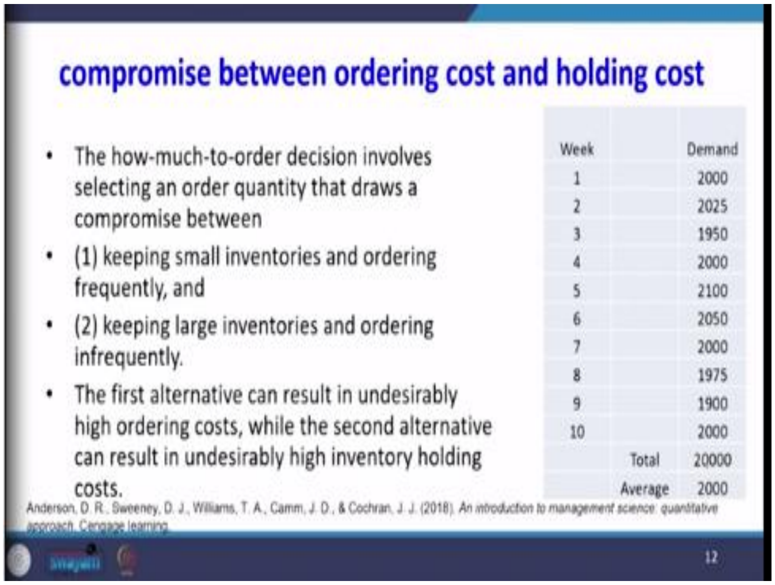
As the first step in the study, the warehouse manager obtained the following demand data for the past 10 weeks. So, he knows the demand from the customers for the 10 weeks. For example, week 1, 2000 units, week 2, 2025 like this, for example, 10th week it is 2000. So, the total demand is 20,000, so 20,000 upon 10, so the average demand is 2,000; every week, the average demand is 2,000.



Strictly speaking, these weekly demand figures do not show a constant demand rate; you see that sometimes it is not a straight line, sometimes 2000, sometimes 2025, sometimes below 2000, sometimes even see even 1900, and the lowest value and highest value is 2050. However, given the relatively low variability exhibited by the weekly demand, inventory planning with a constant demand rate of 2000 cases per week appears acceptable.

So, we are going to do this analysis, assuming that the average demand is 2000 cases per week. In practice, you will find that the actual inventory situation seldom, if ever, satisfies this assumption of model exactly. Many times, the demand will not be constant, but we are assuming it is a constant demand. In any application, the manager must determine whether the model assumptions are close enough to reality for the model to be useful.

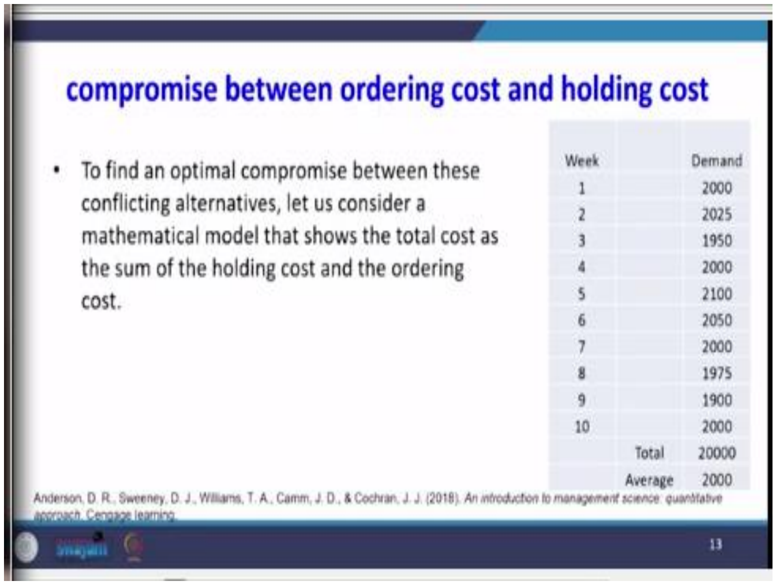
So, in this situation, the problem we are discussing with the demand varies from a low of 1900 cases to a high one of 2100, not 2050, where there are 2100 cases. So, the variability is not that much, so the assumption of the constant demand of 2000 cases per week appears to be a reasonable approximation, reasonably correct even though there is a slight variation.



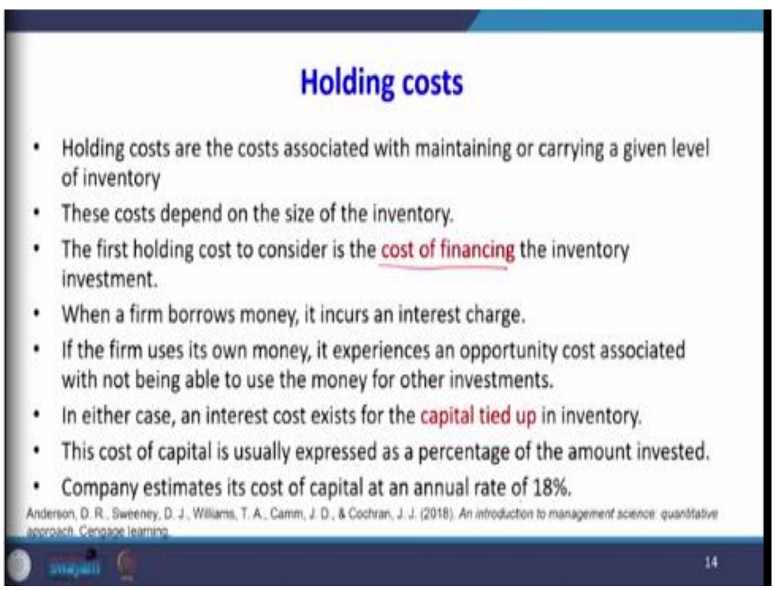
So, now we have to compromise between ordering and holding costs. I have not explained so far what is this ordering cost and holding cost; I will explain in detail. So, the how-much-to-order decision involves selecting an order quantity that draws the compromise between keeping small inventories and ordering frequently. So, what will happen when you keep a very small inventory? Obviously, you have to order frequently; what will happen?

Here the ordering cost will increase. So, you see that the total demand is, for example, say, 20,000, so what can we order? We can order maybe 100, so when you order 100, what do you have to do? You have to order 200 times, so what will happen? Every time, the ordering cost will increase. On the contrary, if you keep large inventories and order infrequently, there will be more ordering costs.

So, the first alternative can result in undesirably high ordering costs, which are when you keep a small inventory, while the second alternative can result in undesirably high inventory holding costs. So, we need to compromise between ordering cost and holding cost; that is what we are going to do now.

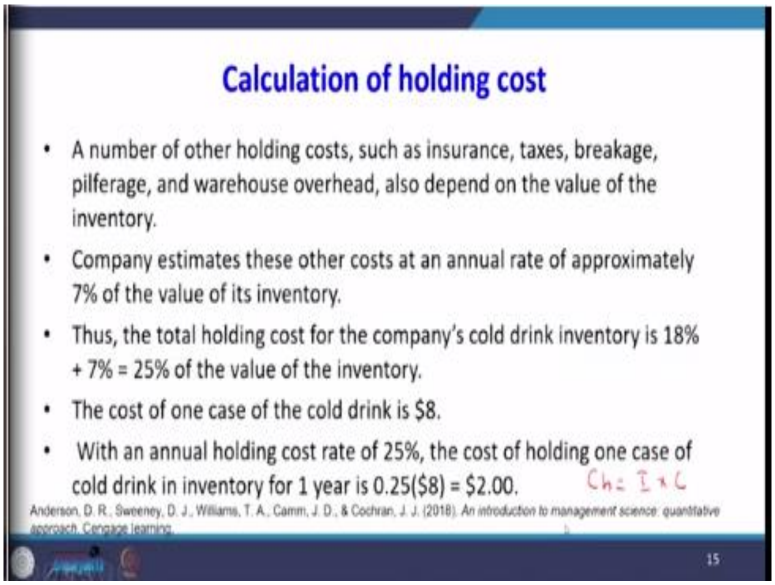


So, to find an optimal compromise between these conflicting alternatives, what are the conflicting alternatives? High ordering cost and high holding cost. Let us consider the mathematical model that shows the total cost as the sum of the holding cost and ordering cost; total cost is the total inventory cost.



First, we will explain what is this holding cost. The holding costs are the costs associated with maintaining or carrying a given level of inventory. This cost depends on the size of the inventory; when the size of the inventory is larger, the holding cost will be higher. The first holding cost to consider is the cost of financing the inventory investment, which is the interest rate. So, when a firm borrows money, it incurs an interest charge, which is your 1 example of holding cost.

If the firm uses its own money, it experiences an opportunity cost associated with not being able to use the money for other investments because your money is blocked. In either case, the interest cost exists for the capital tied up in inventory. The cost of capital is usually expressed as a percentage of the amount invested. So, for this company, it estimates the cost of capital at an annual rate of 18%.

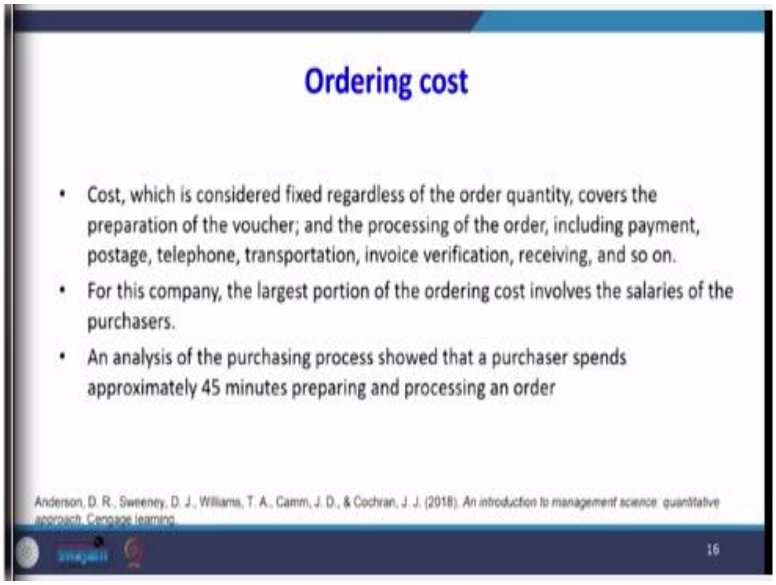


Apart from this cost of capital, a number of other holding cost, such as insurance, taxes, breakage, pilferage, and warehouse overhead, also depends on the value of your inventory. So, the company estimates this cost, other than the interest cost, at an annual rate of approximately 7% of its value in inventory. So, the total holding cost for the company's inventory is 18% for interest and 7% for other expenses, so 25% of it is the value of the inventory.

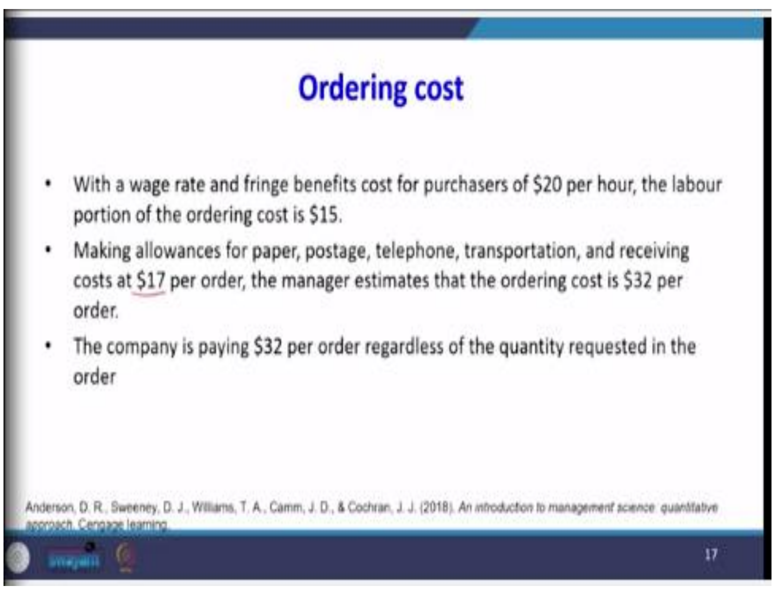
So, we know that the cost of 1 case of the cold drink is, say, 8 dollars with the annual holding cost rate of 25%, the cost of holding 1 case of cold drink in inventory for 1 year, so 0.25 multiplied by 8 dollars is a 2 dollar. So, the cost of holding cost holding a 1 unit for 1 year is 2 dollars, so how do we get this one? Cost of holding = I multiplied by C,

$$C_h = I \times C$$

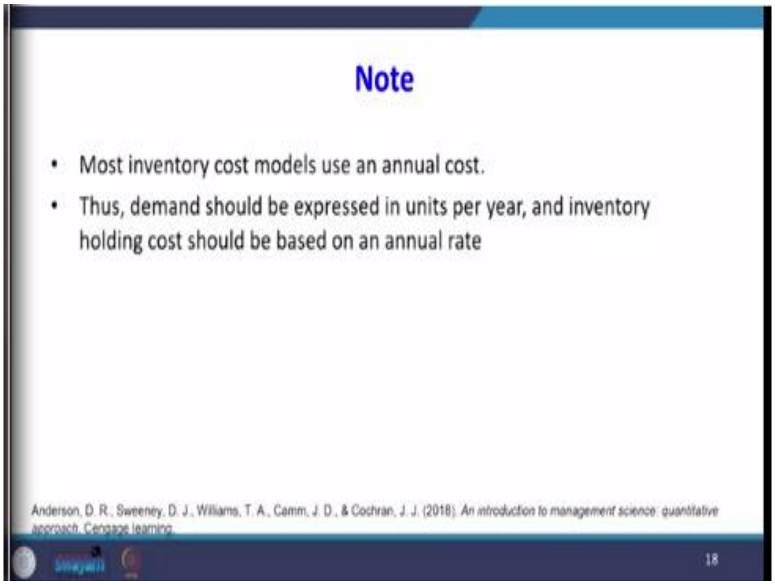
I is percentage, it may be interest rate or other, for example, here the cost of insurance, taxes, breakage. So, this has to be added, so that your interest rate will be multiplied by the unit cost, so that will be cost of holding cost per unit per year.



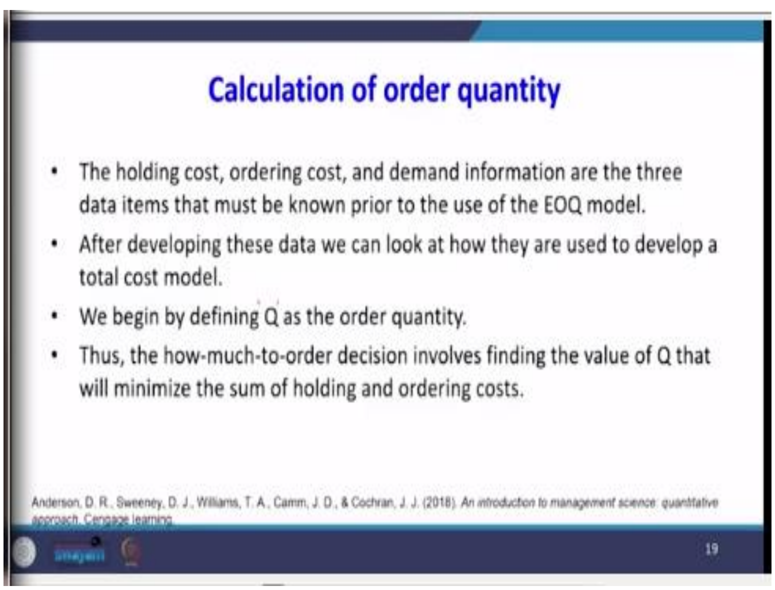
The next important cost is the ordering cost. The cost, which is considered fixed regardless of the order quantity, covers the preparation of vouchers and the processing of orders, including payment, postage, telephone, transportation, invoice verification, receiving, and so on. So, for this company, the largest portion of the ordering cost involves the salaries of the purchasers. An analysis of the purchasing process showed that the purchaser spends approximately 45 minutes preparing and processing an order.



With a wage rate and fringe benefits cost for purchase of 20 dollars per hour, the labor portion of the ordering cost is 15 dollars. So, the labor portion of this ordering cost is 15 dollars. Making allowances for paper, postage, telephone, transportation, and receiving costs is 17 dollars per order; the manager estimates that the ordering cost is 32 dollars per order; how? 17 dollars + 15 dollars, so the company is paying 32 dollars per order regardless of the quantity requested in the order. We know that the holding cost is 2 dollars and the ordering cost is 32 dollars. So, the holding cost is directly proportional to the inventory on hand, which is a quantity, but the ordering cost is independent of the order quantity.

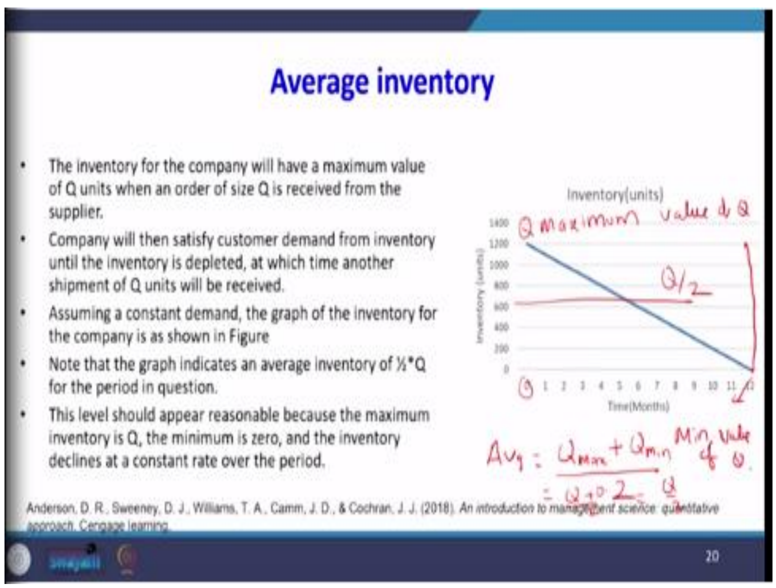


Another point you should note most inventory cost models use annual cost, whether it may be the holding cost, because the interest rate is generally for annual interest. So, when the holding cost is annual, the demand should also be expressed in units per year. So, the inventory holding cost should be based on the annual rate because all the cost and demand should be in the same unit.



Calculation of order quantity: so, before knowing the ordering quantity, we should know the information about the holding cost, ordering cost, and demand information. We know that the holding cost is 2 dollars, the ordering cost is 32 dollars, and the demand is also 2000 units per week. After developing these data, we can look at how they are used to develop the total cost model because why are we concerned about the total cost model?

We have to find out the order quantity that will minimize the total cost. So, we begin by defining Q as the ordering quantity, so this is the Q we are going to find out. So, how much order decision involves finding the value of Q that will minimize the sum of holding and ordering cost, that is the logic behind this. So, we have to find the value of Q that will minimize the sum of the holding cost and ordering cost.

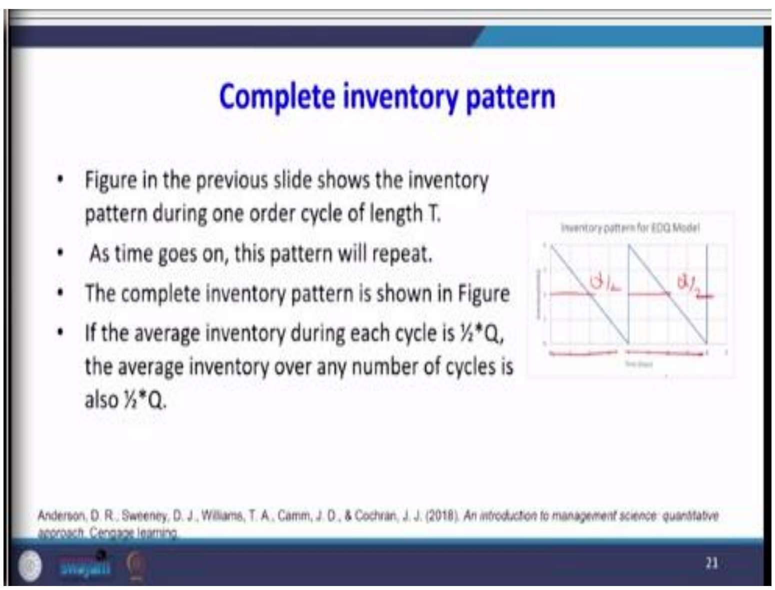


So, before finding this holding cost, first, we will explain what the meaning of this average inventory is. The inventory for the company will have a maximum value of Q units when an order of size Q is received from the supplier. For example, you see, say, 0th month, so here, say 1200 units, so this is your Q; what will happen? Suppose I have 1200 units in my hand; assume that this is monthly demand, so every month, I consume 100 units.

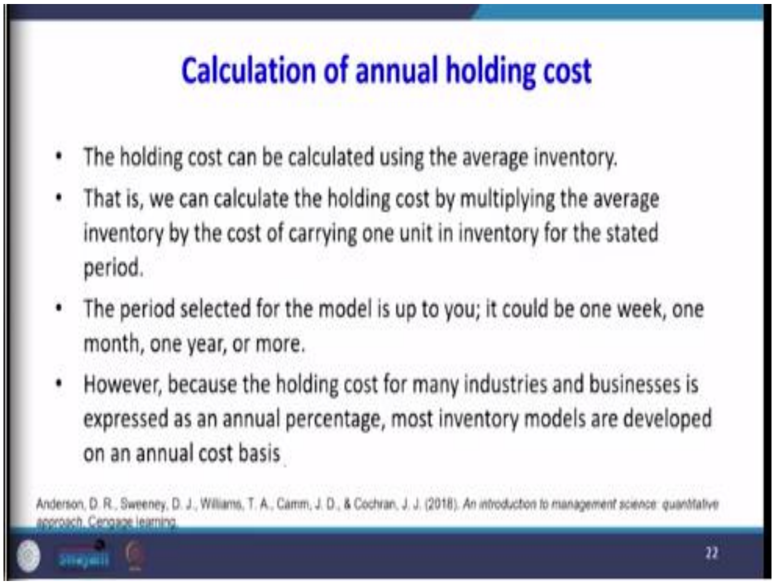
So, I will start from 1200, 1100, 1000, 900, 800 up to 0. So, here, we should know what the average inventory is, so this is the maximum inventory value of Q; here, this is the minimum value of Q. So, when we know the average inventory, (Q maximum+Q minimum) / 2, that is an average value. So, what we should learn from this graph? So, the company receives Q units, then the company will then satisfy the customer demand from it is inventory until the inventory is depleted, at which time another shipment of Q units will be received; what will happen?

So, I am starting from 1200 units; it will come to 0, so again, I will get 1200 units, which is the meaning. Assuming constant demand the graph of the inventory for the company is shown in this figure like this. Here, what is the average inventory, as I told you? So, here, Q maximum is Q only, and Q minimum is 0, so upon 2, the average inventory is nothing but Q by 2, so Q by 2 will be here, so this is my Q by 2.

Note that the graph indicates an average inventory is Q by 2 for the period in question. This level should appear reasonable because the maximum inventory is Q, the minimum is 0, and the inventory declines at a constant rate over the period. So, what is the formula for average? Q minimum is 0, so is only Q by 2.

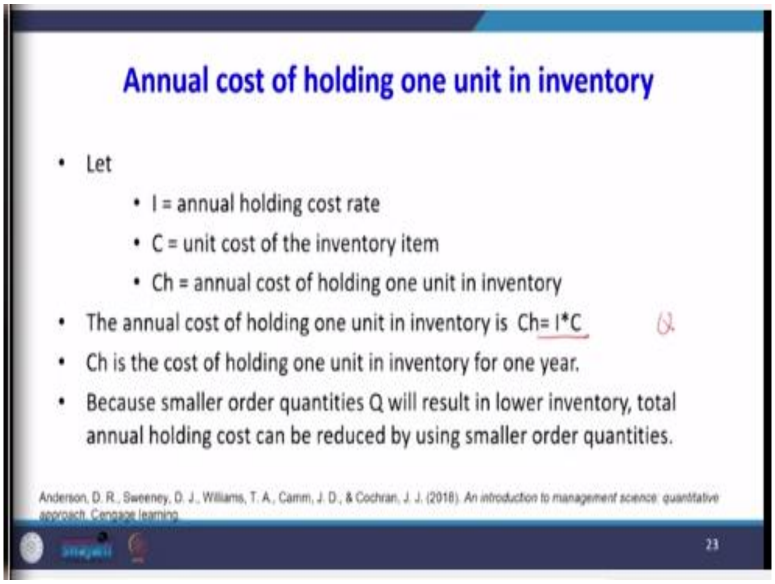


Now, we will discuss the complete inventory pattern; previously, it was done for only one cycle. The figure in the previous slide shows the inventory pattern during the 1-order cycle of the length T. As time goes on, this pattern will repeat. You see this: this is pattern 1 and pattern 2. So, this figure shows the complete inventory pattern. If the average inventory during each cycle is Q by 2, the average inventory over any number of cycles is Q by 2, so here it is Q by 2; for this cycle 2 also, it is Q by 2; in this, there are 2 cycles.



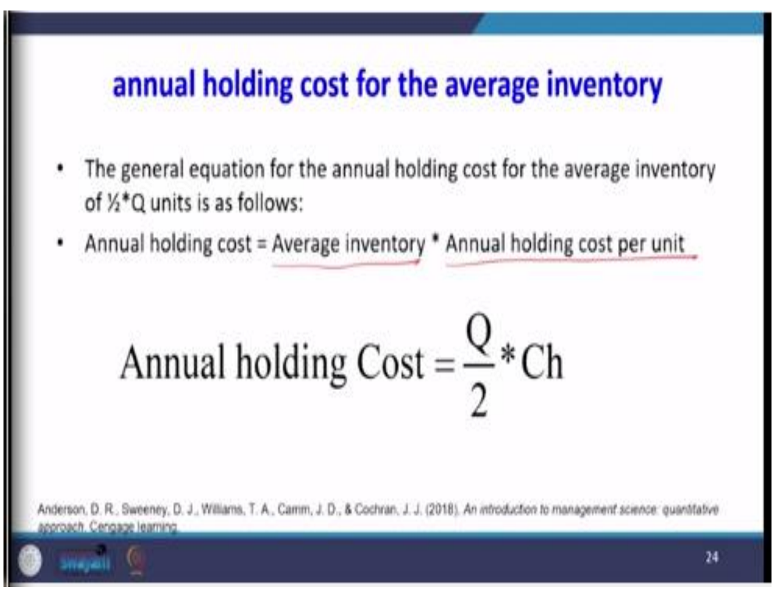
Now we have seen the average inventory, and now we will calculate the annual holding cost. The holding cost can be calculated using the average inventory. That is, we can calculate the holding cost by multiplying the average inventory by the cost of carrying 1 unit in inventory for the stated period. The period selected for the model is up to you; it could be 1 week, it could be a month, 1 year, or more than that.

However, the holding cost for many industries and businesses is expressed as an annual percentage because interest is generally expressed annually. Most inventory models are developed on an annual cost basis.



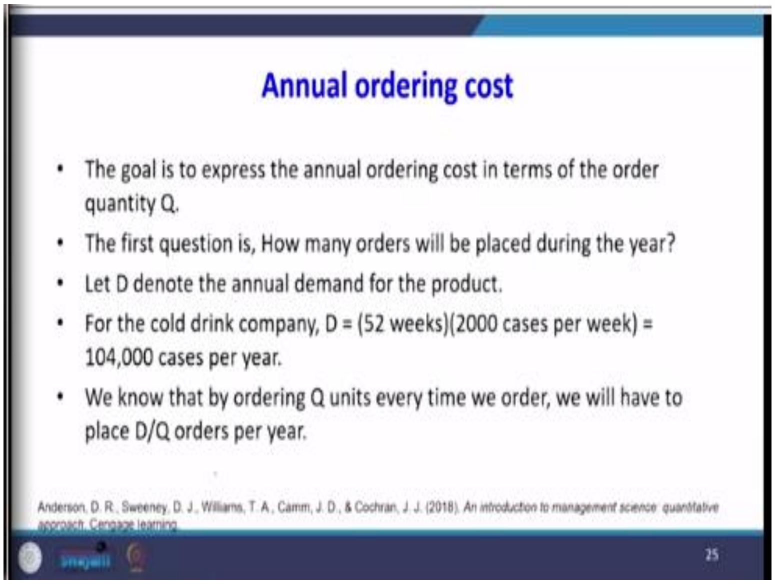
So, the annual cost of holding 1 unit in inventory, let I be the annual holding cost rate, C be the unit cost of the inventory item, and C_h is the annual cost of holding 1 unit in inventory. So, the annual cost of holding 1 unit in inventory that is $C_h = I * C$. Here, the C_h is the cost of holding 1 unit inventory for 1 year because smaller order quantities Q will result in lower inventory. The total annual holding cost can be reduced by using smaller order quantities.

If the value of Q is small numbers the inventory holding cost will be less, but what is the problem? We must frequently order, so what will happen? Then, there will be a number of orders, so ordering costs will increase.



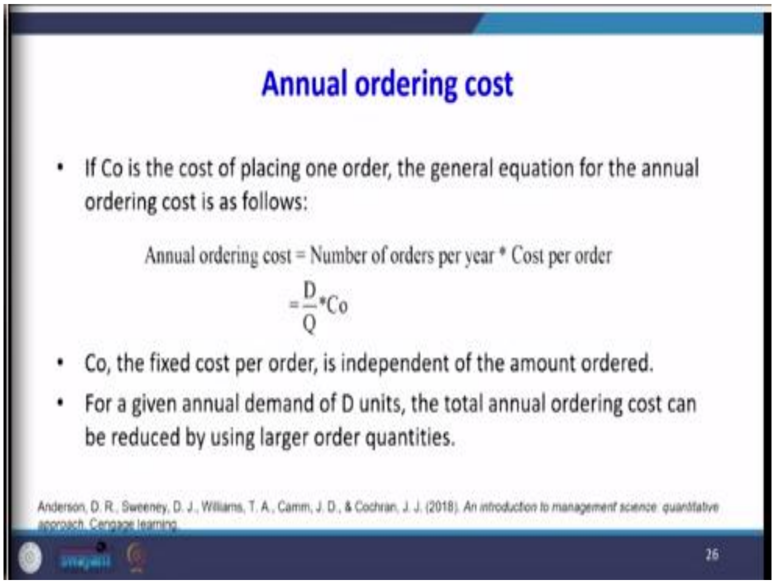
The general equation for the annual holding cost for the average inventory of Q by 2 units is as follows, what is that? The annual holding cost = average inventory multiplied by the annual holding cost per unit per year. So,

$$\text{Annual holding Cost} = \frac{Q}{2} * Ch$$



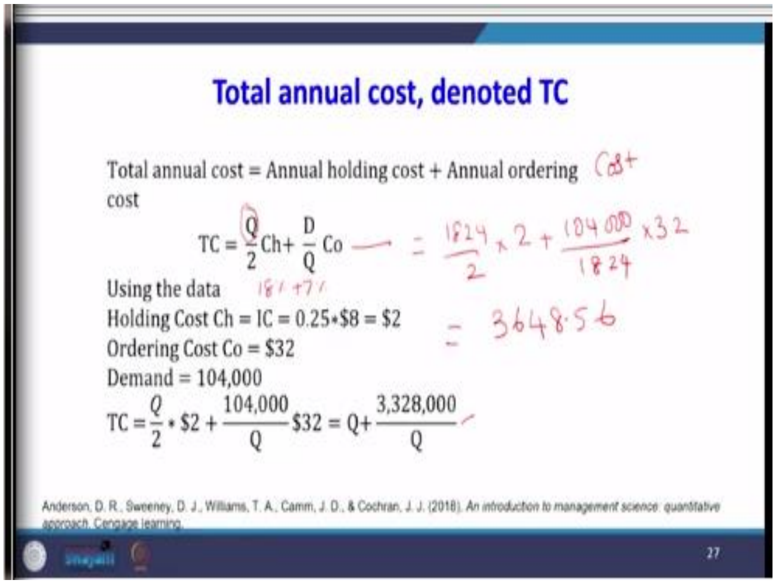
Now we can calculate the annual ordering cost. The goal is to express the annual ordering cost in terms of the order quantity Q. The first question is how many orders will be placed during the year. So, how many orders? That means the number of orders multiplied by the ordering cost that will give you the annual ordering cost. So, we should know how many orders, generally how will you find out how many orders?

Let D denote the annual demand for the period. For the cold drink company that we are discussing, the demand is equal to annually 52 weeks per week, and the demand is 2000; the total is 1,04,000 cases per year. We know that by ordering Q units every time we order, we will have to place D by Q orders, total demand delivered by a number of units order that will be the quantity Q that will give you the number of orders.



If Co is the cost of placing 1 order, the general equation for the annual ordering cost is, so the annual ordering cost is the number of orders per year multiplied by the cost per order. What is the number of orders? D upon Q. What is the D? Annual demand, Q is quantity ordered, Co is cost per order. For a given annual demand of D units, the total annual ordering cost can be reduced by using larger order quantities.

When you order large quantities, the number of orders will decrease; when the Q is bigger, the number of orders will decrease so that the annual ordering cost will be less. The Co that is ordering costs the fixed cost per order. It is independent of the amount ordered. Whether you order 1 quantity or 50 quantities, that is independent of the amount of orders; it is only based on the number of orders.



The total annual cost is the summation of the annual holding cost plus the annual ordering cost. So, Q by 2 is the average inventory multiplied by holding cost plus the number of orders multiplied by the cost per order Co. In our problem, I represent 18% for interest + 7% for some other expenses, so the value of I is 25%, so 0.25 is the unit cost, and C is the unit cost of 8 dollars, so it will be 2 dollars, what is a 2 dollar? Holding cost, ordering cost is 32 dollars, and the demand is 1,04,000.

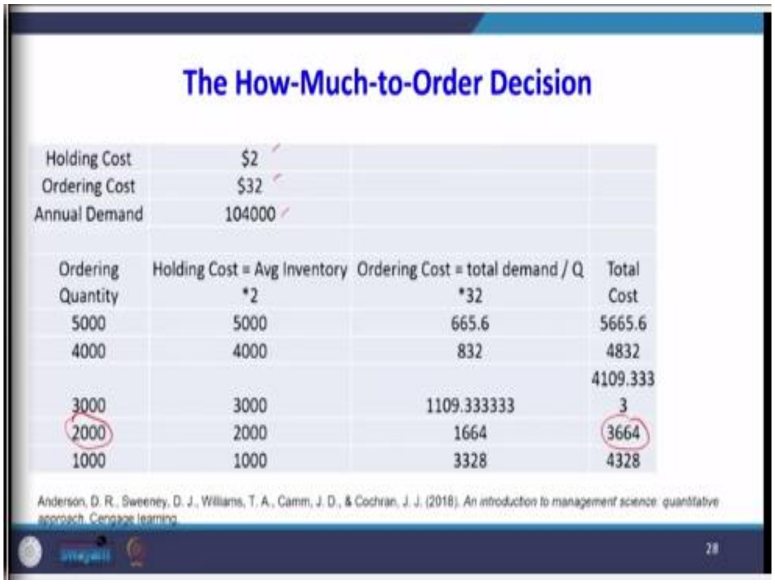
So, the total cost is I am substituting these values into this equations, so Q upon 2 multiplied by 2 demand divided by Q multiplied by 32, I am getting this expression Q + 33,28,000. So, this is the expression for our total cost.

Total annual cost = Annual holding cost + Annual ordering cost

$$TC = \frac{Q}{2}Ch + \frac{D}{Q}Co$$

Using the data
Holding Cost Ch = IC = 0.25*$8 = $2
Ordering Cost Co = $32
Demand = 104,000

$$TC = \frac{Q}{2} * \$2 + \frac{104{,}000}{Q}\$32 = Q + \frac{3{,}328{,}000}{Q}$$



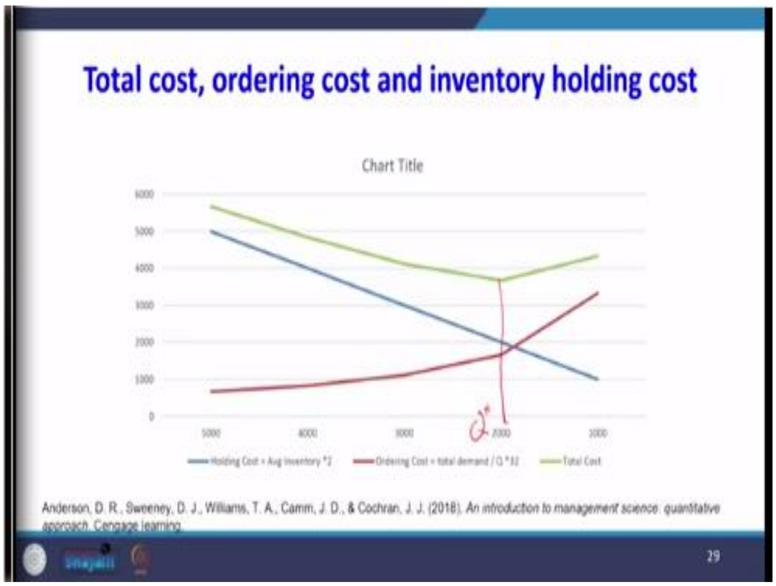
Now, I am going to use Excel to find out how much to order. So, I have taken a holding cost of 2 dollars, an ordering cost of 32 dollars, and an annual demand of 1,04,000, so now I will open Excel. Now you see D2 is my holding cost, D3 is my ordering cost, D4 is annual demand, and now the ordering quantity from C7 to C18, C7 to C12 I am going to substitute randomly 5000, 4000, 3000, 2000, 1000.

Here I am going to find out, suppose if I order 5000 units, what will be my holding cost? So, the D7 you see that formula in D7, average inventory, so C7 multiplied by holding cost, 2 dollars. Like that, I have dragged for up to, say, ordering quantity 1824, similarly ordering cost. If I look at the formula in E7, what is the formula for ordering cost? Total demand, total demand is D4 upon C7 ordering quantity that will be number of orders multiplied by ordering cost 32, so it is 665.6.

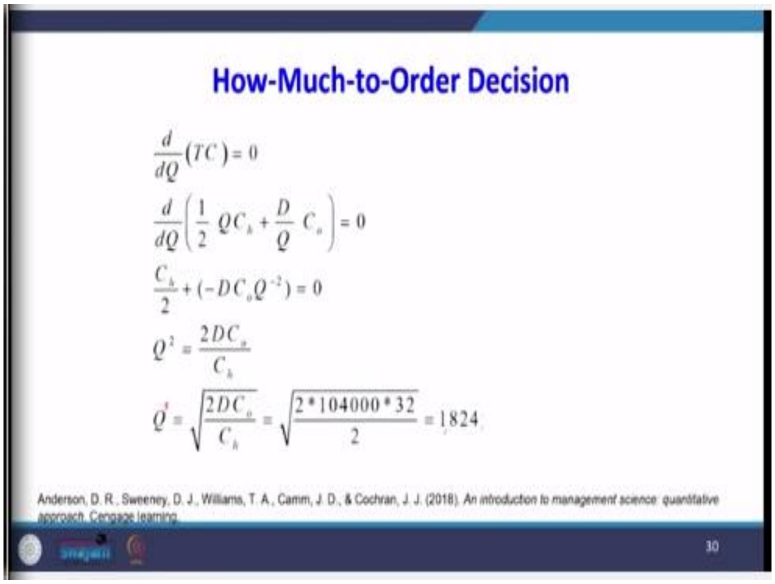
So, the total cost is the sum of the holding cost and ordering cost. Here, randomly, I can substitute the value of Q. So, when you look at this, when I substitute randomly, say 1824, for example, the ordering cost is minimum. How much? 3648, so the optimal ordering quantity is 1824. You should remember the ordering quantity of the values in the C column I have randomly substituted.

But since I am doing trial and error, this is not the right way, so what am I going to do? I am going to derive an expression for finding the optimal ordering quantity. So, not only that, you see the picture when I plot the blue one says the holding cost, the orange one says ordering cost, and the gray color shows the total cost. You see that the total cost starts from here, the total cost is minimum at this point approximately 2000.

I do not know the exact value because the exact value is 1824, so the optimal ordering quantity is 1824. So, this 1824 I am going to derive by using the concept of minima principle, so I will go back to my presentation. So, this is to see that suppose if you are not considering 1824, you see the minimum total cost is here 3664. So, if you order 2000 units, the total ordering cost will be minimal.



Look at the relationship between total cost, ordering cost, and inventory holding cost. The holding cost is high when you go for a higher quantity. When you go for a higher quantity, you see the ordering cost, it is ordering cost is much less. So, we need to have the trade-off between holding cost and ordering cost, so that is the total cost; the sum of this, the green color says the total cost approximately here we are getting this Q* approximately 2000 units.



So, we can derive the value of Q; we can call it a Q star from the expression of total cost. So, what do we have to do? The total cost expression we have to differentiate with respect to Q and equate it to 0.

So, the total cost expression is

$$\left(\frac{1}{2} QC_h + \frac{D}{Q} C_o\right) = 0$$

When you differentiate w.r.t. Q,

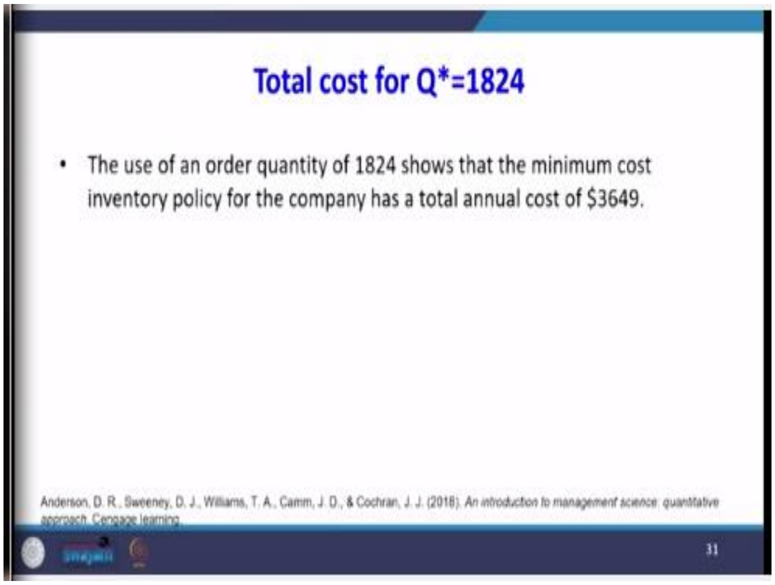
$$\frac{d}{dQ}\left(\frac{1}{2} QC_h + \frac{D}{Q} C_o\right) = 0$$

So, when I simplify this,

$$\frac{C_h}{2} + (-DC_oQ^{-2}) = 0$$

$$Q^2 = \frac{2DC_o}{C_h}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2*104000*32}{2}} = 1824$$



So, the total cost for Q*, that is, when you order 1824, is what we can do? If I go back to my Excel, I see that for the last C12, when I wrote 1824, the total cost was 3648. So, how can I get this 3648? I will go back to my presentations. So, when I substitute the value of Q = 1824, I get the total annual cost of 3648, so we have verified this value with the help of Excel.

So, what we conclude is that the optimal order quantity is 1824 units. In this lecture, I have discussed the importance of inventory. Then, I have explained how to calculate the holding cost and ordering cost, and then I have explained how to find the total cost. From the total cost expression, I have derived the formula for economic order quantity that is a Q star, so that will answer this question how-much-to-order.

The same how-much-to-order I have explained with the help of excels also by changing different order quantity. Then I have shown you how the total cost varies. In the next class, I will answer this question: when to order inventory decisions? So far, we have found how much to order, so the answer for when-to-order, I will do in the next session; thank you very much.