Decision Making With Spreadsheet

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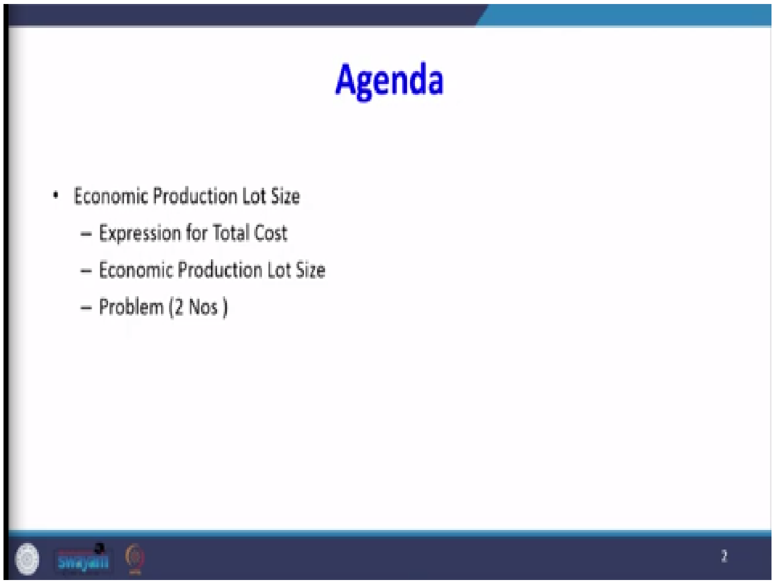
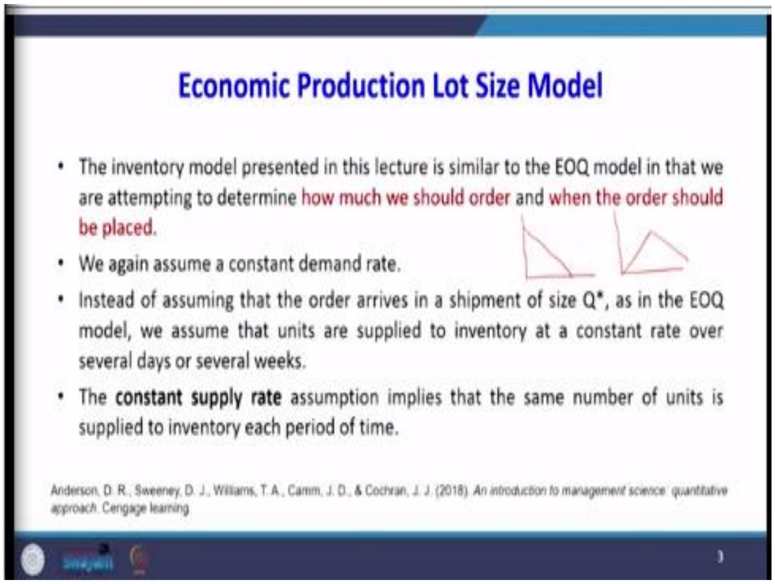
Lecture-35
Economic Production Lot Size Model

Dear students, in the previous lecture, I explained about economic order quantity. So, in this lecture, I am going to explain another inventory model that is called the economic production lot size model.

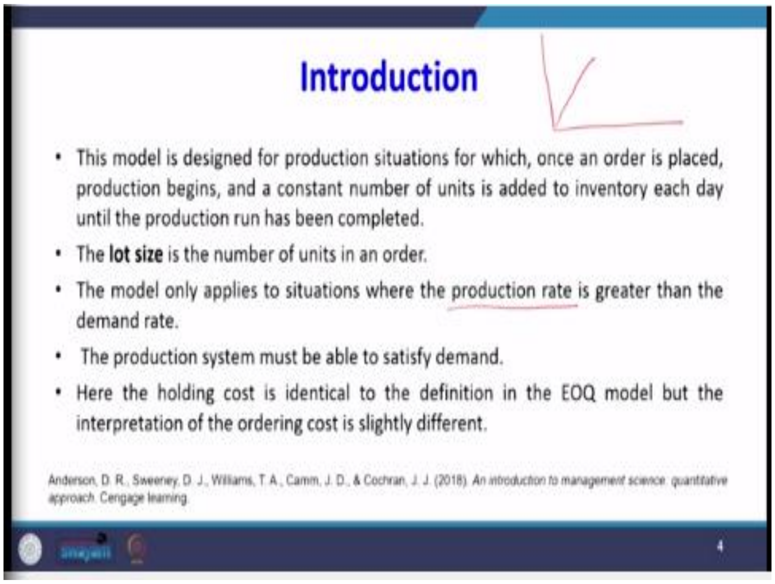


So, the agenda for this lecture is to explain what economic production lot size is, and then I am going to derive an expression for the total cost. So, from that total cost expression, I am going to find out the optimal, which is economic production quantity; lot size means your quantity. So, I have taken 2 sample problems, and then I will solve these two problems.



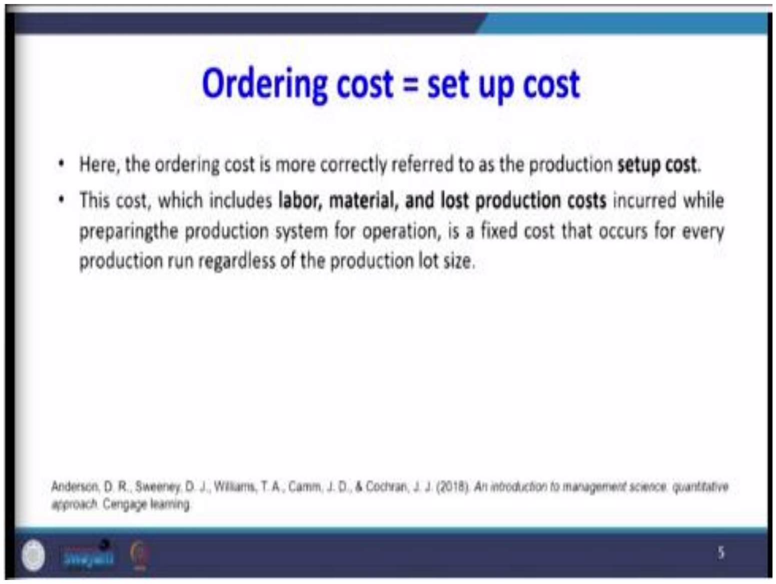
So, economic production lot size model. The inventory model presented in this lecture is similar to the EOQ model in that we are attempting to determine how much we should order and when the order should be placed. This problem is taken from the book Anderson et al. So, in this model, as the economic production lot size model, we are assuming a constant demand rate. So, instead of assuming that the order arrives in a shipment of size Q star as in the EOQ model, we assume that the units are supplied to inventory at a constant rate over several days or several weeks.

So, what is the meaning? In the previous EOQ model, we have assumed this way but now we are going to assume that it is not going to come at a time. So, first, we are going to build the inventory, and then after that, we are going to utilize that inventory. So, the constant supply rate, so after producing, we are going to supply; the constant supply rate assumption implies that a same number of units is supplied to inventory each period of time how we are able to supply after the production.



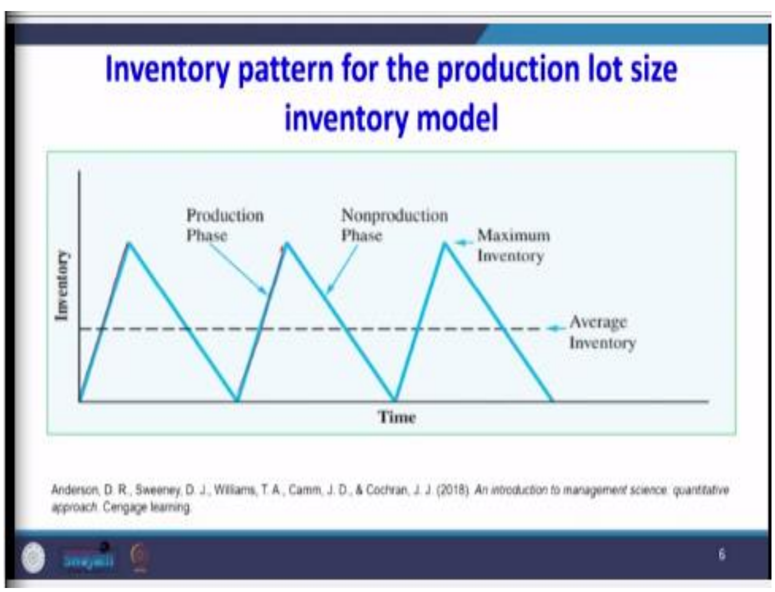
This model is designed for production situations for which, once an order is placed, production begins, and a constant number of units is added to inventory each day until the production run has been completed. The lot size is the number of units in an order, so it is like this: we received an order, so we keep on building inventory by production. So, here, the lot size is the number of units in an order. So, this model only applies to situations where the production rate is greater than the demand rate, which is one assumption.

What is that? So, we are producing more than the demand, so the production rate is greater than the demand rate. So, the production system must be able to satisfy the demand. That is why we have this assumption: that production is more than demand. Here, the holding cost is identical to the definition of the EOQ model, but the interpretation of the ordering cost is slightly different. Holding cost is similar to EOQ, but for the interpretation of ordering cost, we are going to use another name for this ordering cost.

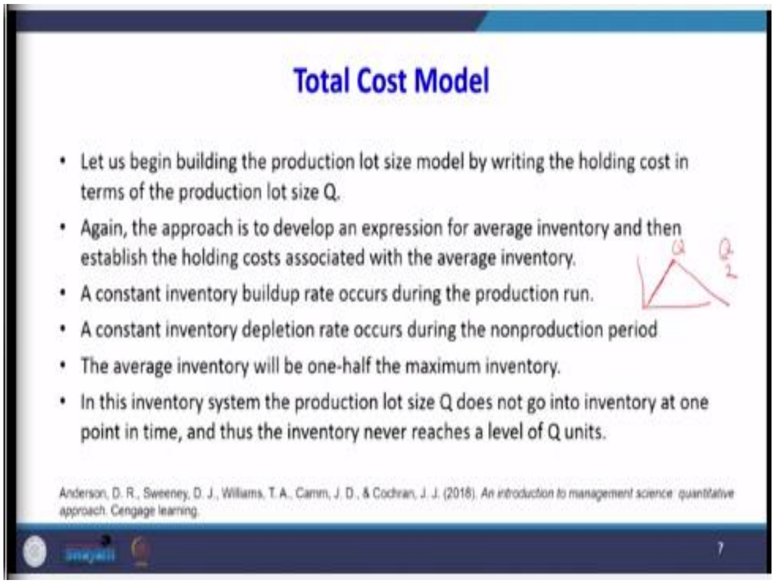


So, the ordering cost is nothing but the setup cost. Here the ordering cost is more correctly referred to as the production setup cost. So, every time we are preparing for production, there is a set of costs we have to incur, so that cost is equivalent to our ordering cost. What is the setup cost? This cost includes labor, materials, and the lost production cost incurred while preparing the production system; whenever we receive the order, we have to prepare the production system.

For that there a certain amount of cost is incurred, so that cost is set up cost. So, preparing the production system for operation is a fixed cost for every production run regardless of the production lot size.



So, the inventory pattern for the production lot size inventory model is like this. You see, on the axis, there is time; on the axis, we are building the inventory, so that is the production phase. Once it reaches the maximum level of inventory, we will stop production again. Consumption takes place in the nonproduction phase. So, the average inventory is the dotted line that says the average inventory.

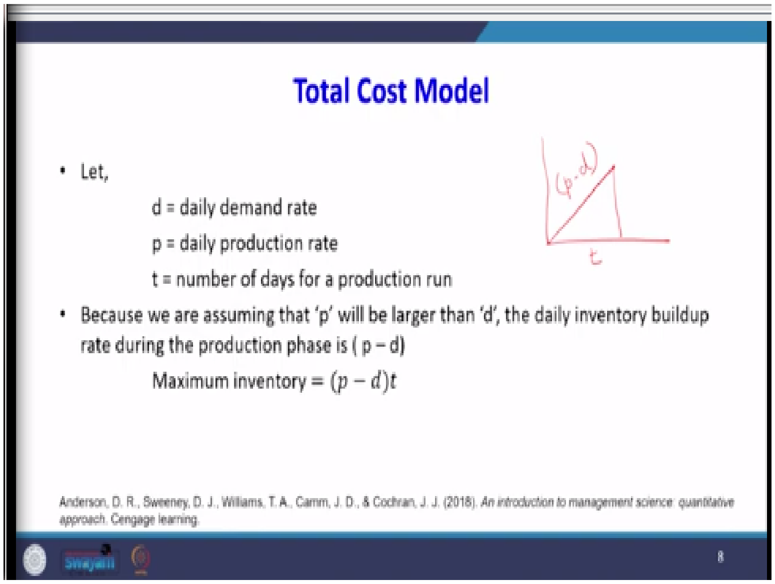


First, you will derive the total cost. Let us begin building the production lot size model by writing the holding cost in terms of production lot size Q. Again; the approach is to develop an expression for an average inventory and then establish the holding cost associated with average inventory. So, first, we are going to find out what the average inventory is, and then we are going to multiply it by the holding cost so that we will get the average inventory holding cost.

The constant inventory buildup rate occurs during the production run. So, see that during production time, inventory buildup takes place. A constant depletion rate occurs during the nonproduction period. So, during nonproduction time, we consume the inventory. So, the average inventory will be one-half of the maximum inventory. Suppose this is the maximum inventory, so half of this is similar to what we have done in the EOQ model.

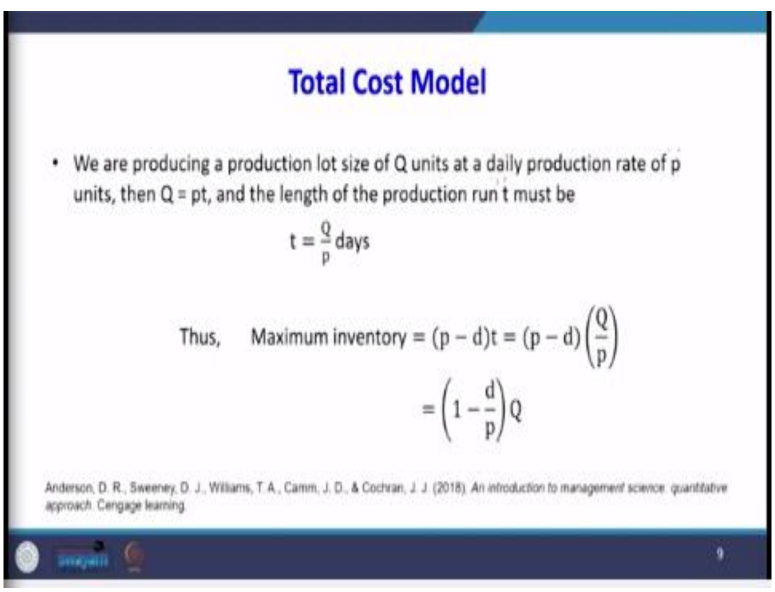
So, one-half of the maximum inventory is the average inventory. So, in this inventory system, the production lot size Q does not go into the inventory at one point in time. You see that it is not the vertical line; it keeps on increasing, and thus, the inventory never reaches the level of Q units.

Since the conception also takes place simultaneously, the inventory level never reaches the level of Q units.

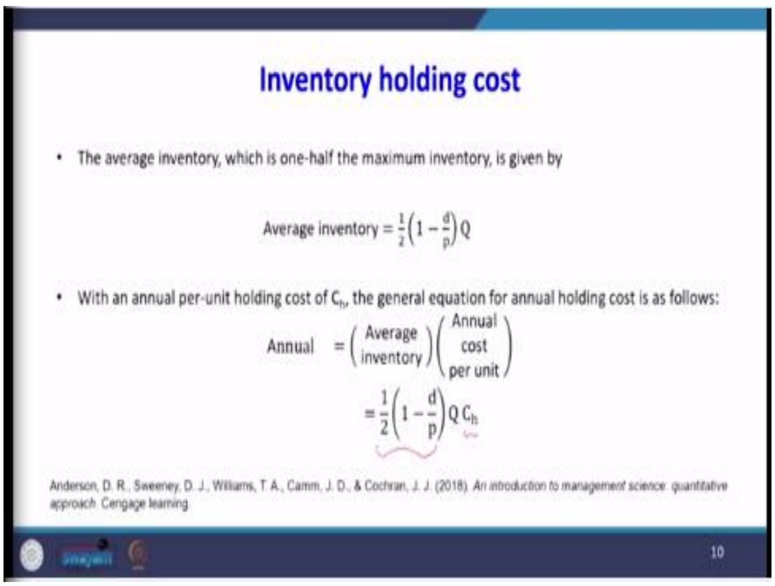


Now we will deal with the expression for total cost. Let d represent the daily demand rate, p represents the daily production rate, and t represents the number of days for a production run. Because we are assuming that the daily production rate p will be larger than the daily demand rate, the daily inventory buildup rate during the production phase is p - d. So, during the production phase, the inventory buildup is (p – d).

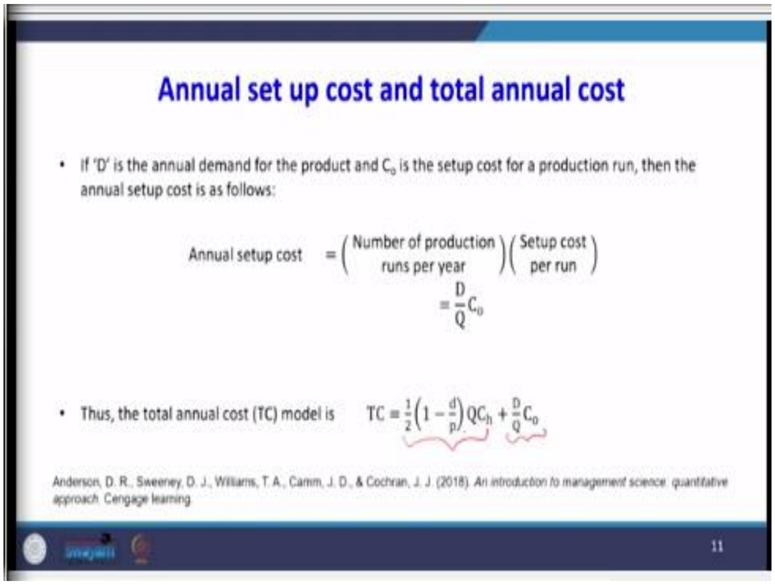
So, when we can reach when the inventory becomes maximum, so, the maximum inventory is this t, so duration is multiplied by t, so the maximum inventory is (p – d) multiplied by t.



Now we are going to get an expression for the t; how? We are producing a production lot size of Q units at a daily production rate of p units. Then, the lot size Q is equal to per day production multiplied by the number of days. Then, the length of the production run t must be, so Q upon p, what is a Q? Q is the lot size; p is the per-day production rate. So, we are going to substitute the value of t into our maximum inventory formula. So, what is the maximum inventory formula? p - d multiplied by t, instead of t, we are substituting Q upon p. So, when we simplify this, you will get (1 – d) by p and multiply by Q.

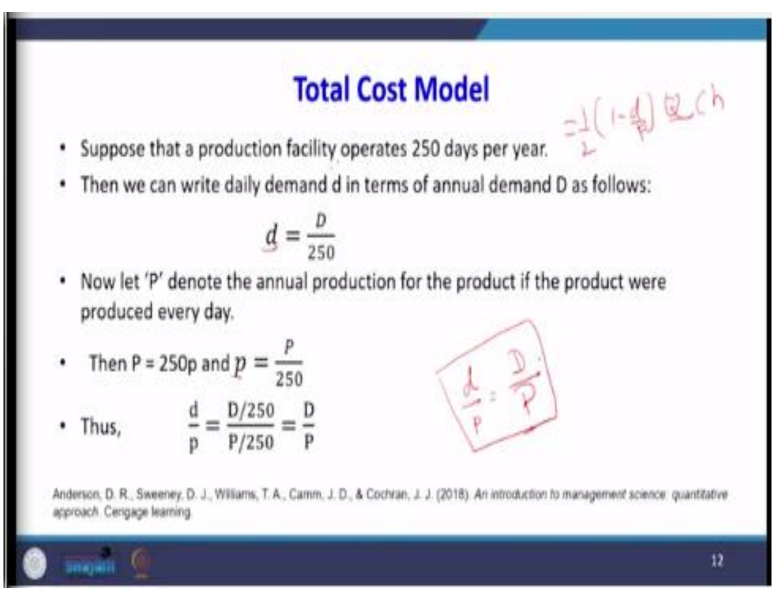


Previously, we got the expression for maximum inventory; now, the average inventory is one-half of the maximum inventory. So, the previous expression we are going to divided by 2, so 1 by 2 multiplied by (1 – d) by p into Q. So, when you multiply this average inventory by annual inventory holding cost, so we will be getting annual inventory holding cost. What is that average inventory? This expression is average inventory then C_h is annual inventory holding cost per unit.



Now, we will find the annual setup cost and the annual total cost. If D is the annual demand for the product and Co is the setup cost. See previously, we have used this notation Co for the ordering cost; here, Co is the setup cost for a production run. Then the annual setup cost is as follows: how? The number of production runs per year is multiplied by setup cost. So, what will be the number of orders?

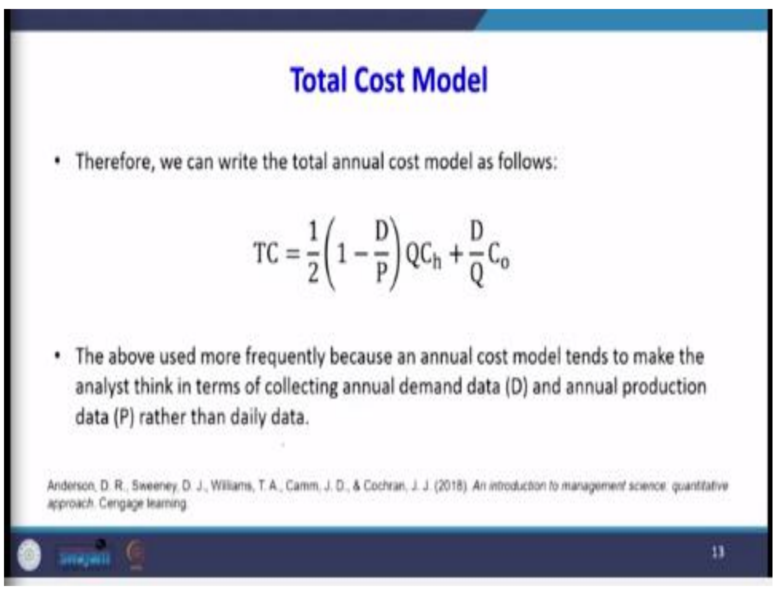
It is similar to the number of orders demanded divided by lot size, which will be the number of production runs multiplied by setup cost per production run. So, now we have the expression for this one for the inventory holding cost, which is for our setup cost. So, here, we have to find the value of Q that minimizes the total cost.



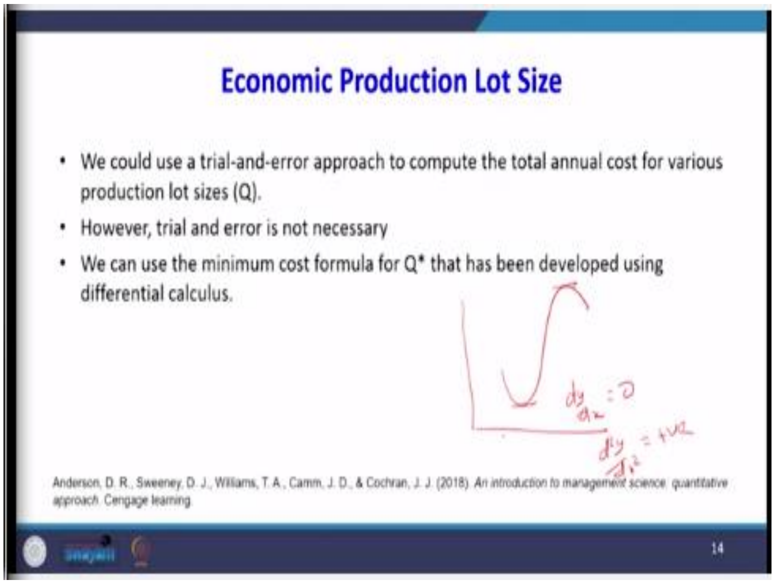
Suppose the production facility operates 250 days per year, and then we can write the daily demand d in terms of annual demand D as follows. In fact, what are we trying to do? We are going to because we have written (1 – d) by p, so this is Q and Ch for example, here C_h. Here the d is the per day demand, p is per day production, so we are going to get the annual demand here, so we are going to replace the d upon p by D upon P that is what we are going to do.

So, d is per day demand is annual demand by number of working days. Let P denote the annual production for the product if the production were produced every day, so what would be the annual production? Will be 250 multiplied by p, and there will be annual production. So, from this expression, we can get the per day production, so we got per day demand per day production. So, instead of d by p, when you substitute, we are getting the expression. So, these 250 working days are not important.

So, what I am saying is that the ratio of per day demand and per day production is equal to annual demand upon annual production, P, so this ratio is the same. So, in our total cost formula, we are going to use this annual demand and annual production.

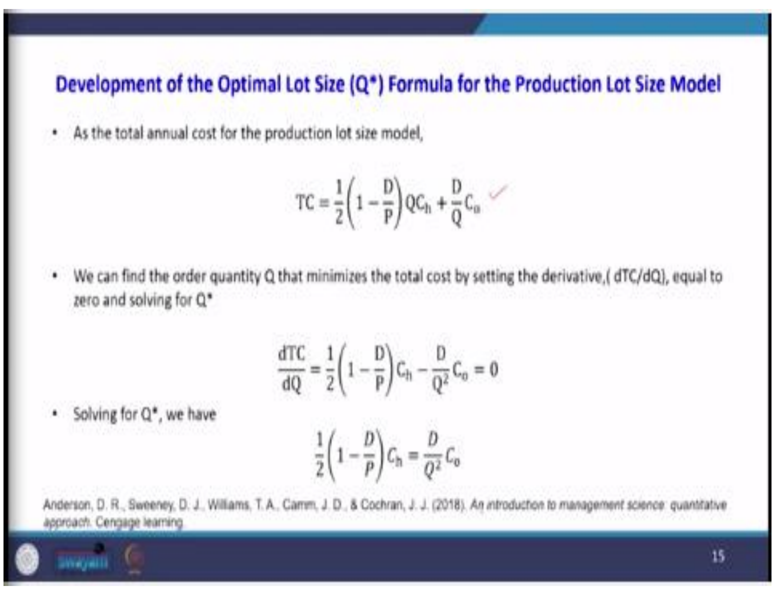


So, what have we done? So, we have rewritten it so that instead of d by p, we have written D by P. The above formula is more frequently used because the annual cost model tends to make the analyst think in terms of collecting annual demand data and annual production data P rather than daily data.



Now we have to get the value of Q. So; we could use the trial and error approach to compute the total annual cost for various production lot sizes Q. However, trial and error is not necessary, so we can use the minimum cost formula Q star that has been developed using differential calculus. We know what the conditions for minimum and maximum I have already explained also.

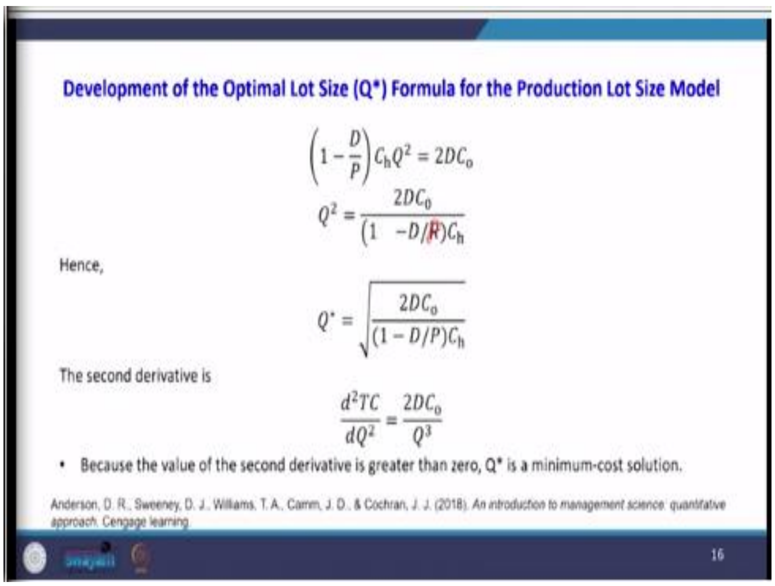
So, if you want to know, say this is the minimum, so what is the condition? (dy/dx) = 0, and you will get the minimum point; that is what we are going to do. But here we have the expression in terms of Q.



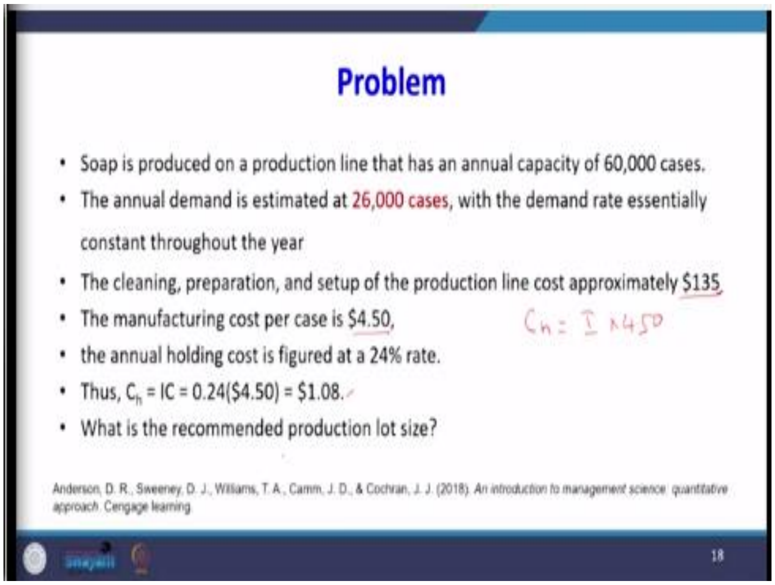
So, the optimal lot size Q star formula for the production lot size model is developed. We got the total cost expression.

$$TC = \frac{1}{2}\left(1 - \frac{D}{P}\right)QC_h + \frac{D}{Q}C_o$$

We can solve this by using a nonlinear method, but we can also derive it using the minima principle. So, we can find the order quantity Q that minimizes the total cost by setting the derivatives. So, what are we going to do? This expression, we are going to differentiate with respect to Q and equate it to 0, then we are going to get the value of Q.

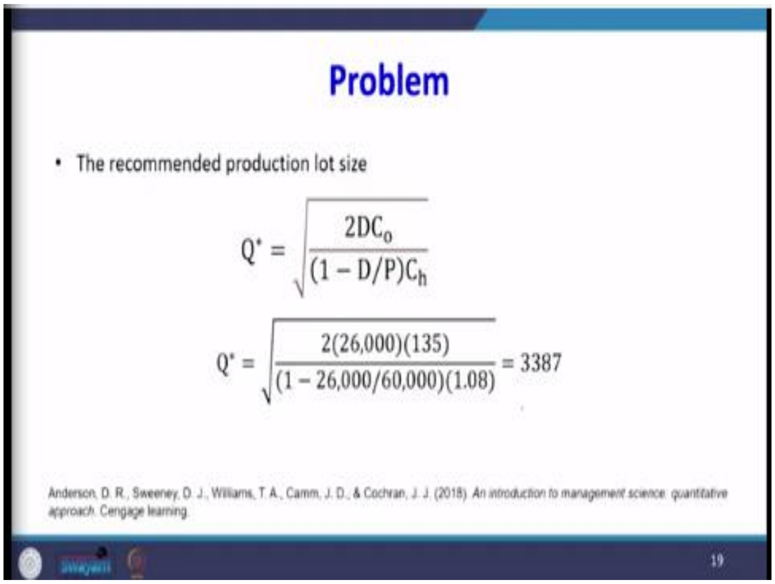


So, this ratio becomes 0 then it will become $2DC_0$ by C_h. So, when the production rate approaches infinity, the formula for economic order quantity and economic production quantity are the same.

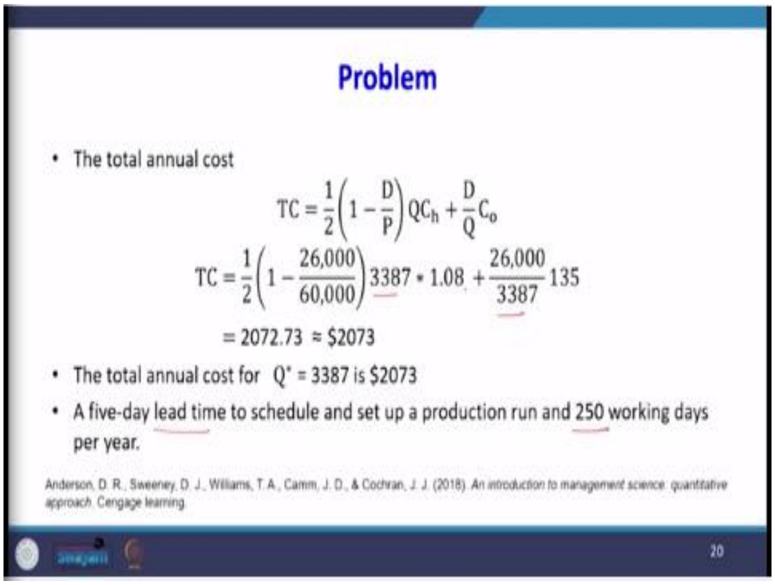


Now, we will take 1 sample problem; then we can find the economic production lot size. So, as a company produces soap, soap is produced on a production line that has an annual capacity of 60,000 cases. The annual demand is estimated at 26,000, with the demand rate essentially constant throughout the year. The cleaning, preparation, and setup for the production line cost approximately 135 dollars, so this 135 dollars is our setup cost.

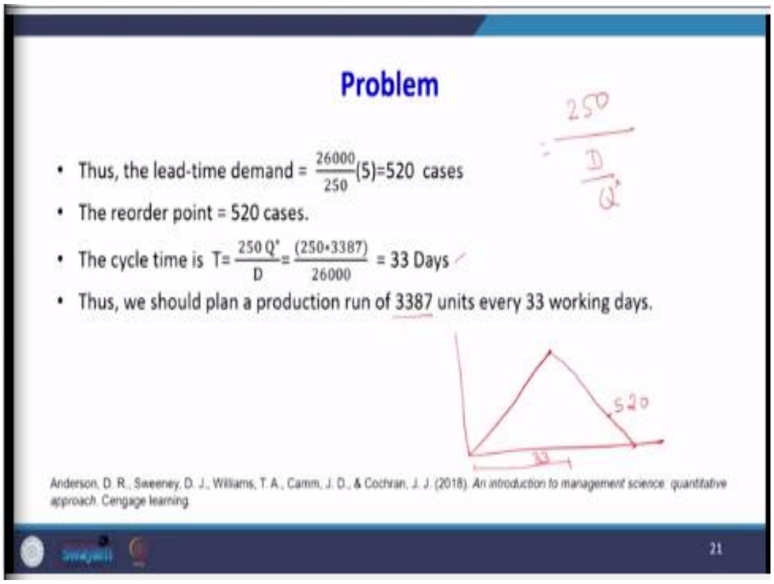
So, the manufacturing cost per case is 4.5 dollars, and the annual holding cost is 24%, so the holding cost is the interest rate multiplied by the unit. So, what will happen? 4.5 dollars multiplied by 0.24, so the annual holding cost is 1.08 dollars. Now what is the recommended production lot size?



So, the demand is given 26,000, the ordering cost is given 135 dollars, and the production capacity is also given 60,000; all the values are given here. So, when you substitute, we are getting Q star, which is 3387, which is our economic lot size; the other way is production lot size.

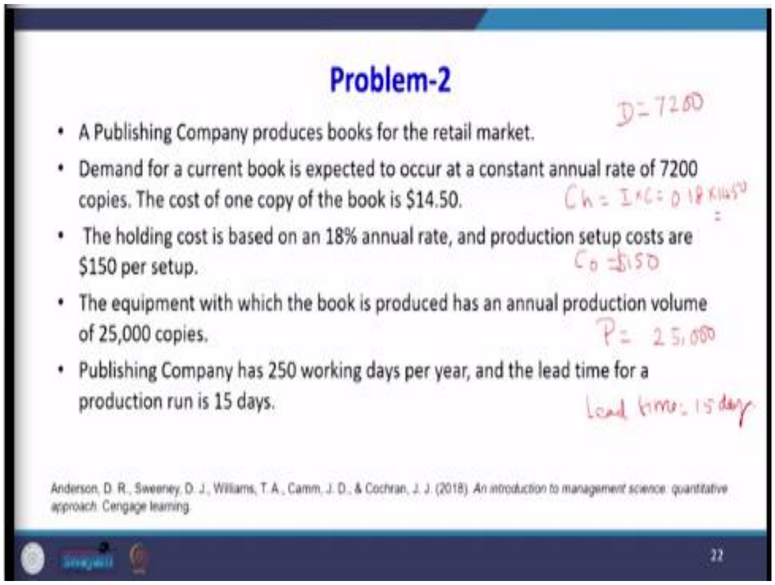


So, now we get the Q value when you substitute into the total cost expressions we are getting, so Q I have substituted 3387 here also, here also, so I am getting 2073. See the total annual cost. If you follow this lot size 3387, the annual cost is 2073 dollars. Suppose some more information is given; what are they? A 5-day lead time to schedule and set up a production run and 250 working days per year, so this information is also given; what are they? Lead time is given, and the number of working days per year is also given. So, by using this information, we are going to find out the reorder point.



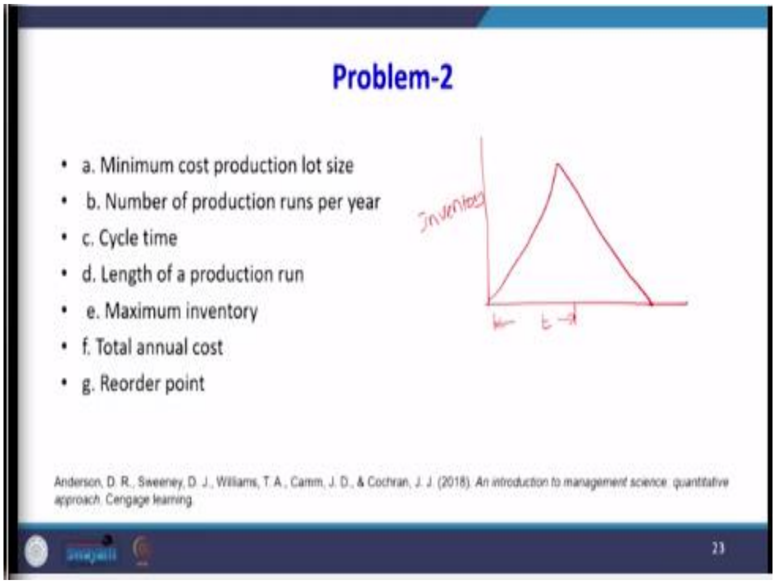
So, first, we will find out the lead time demand. What is the lead time demand? Per day demand multiplied by the lead time, lead time is 5, annual demand is 26,000, and the number of working days is 250, so you will get the per day demand. So, when you multiply the number of we will get to 520 cases, so the reorder point is 520 cases. Now, we are going to find out the cycle time; it is similar to what we have done in terms of economic order quantity. So, number of setups, how will we get the number of setups? A number of set setups is D upon Q star.

So, for this value, we have to know what is the total number of working days, 250, so when you divide 250 by the number of productions, that is D upon Q star, so it will become 250 Q star upon D, so it is 33 days. So, what is 33 days is? We should plan a production run of 3387 units every 33 working days. So, this duration is 33, so the Q value, the Q that is optimal production quantity, is 3387. So, the reorder point is when it reaches 520 units; what is the meaning of the reorder point? When it reaches the inventory level of 520 units, you should get ready for production.

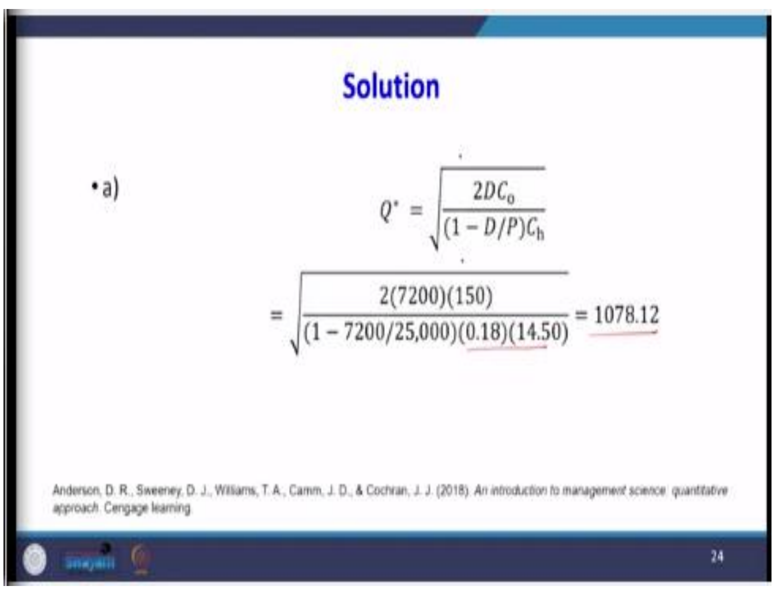


We can take another problem to understand this concept. So, a publishing company produces books for the retail market, so the demand for the book is expected to occur at a constant annual rate of 7200 copies, so here the 7200 is your D. The cost of 1 copy of the book is 14.5 dollars; the holding cost is based on 18% annual rate. So, with this information, we can find out the annual holding cost; what is that?

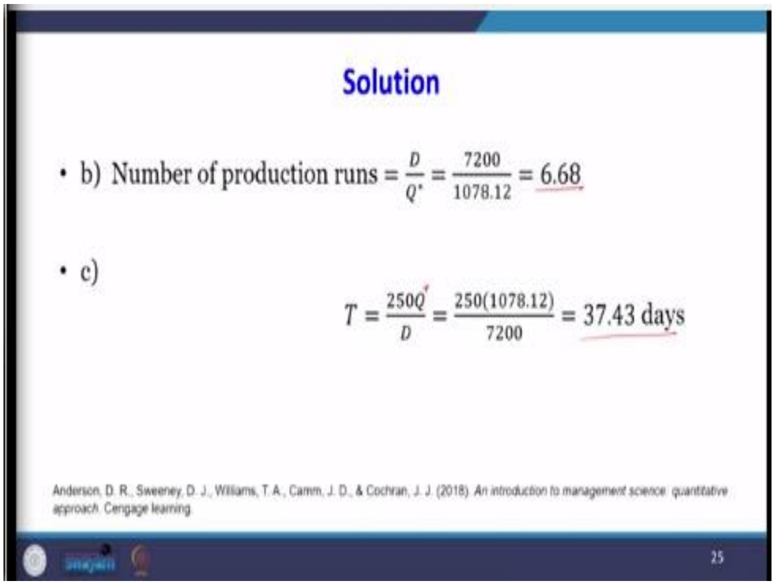
What is the interest rate multiplied by the unit cost? 0.18% multiplied by 14.50 will be the annual inventory holding cost. The setup cost, setup cost is 150 dollars, and the equipment with which the book is produced has an annual production volume of 25,000 copies. So, this P is annual production is 25,000 copies. That publishing company has 250 working days per year and the lead time for the production run is 15 days. So, lead time is given, so a number of working days is also given.



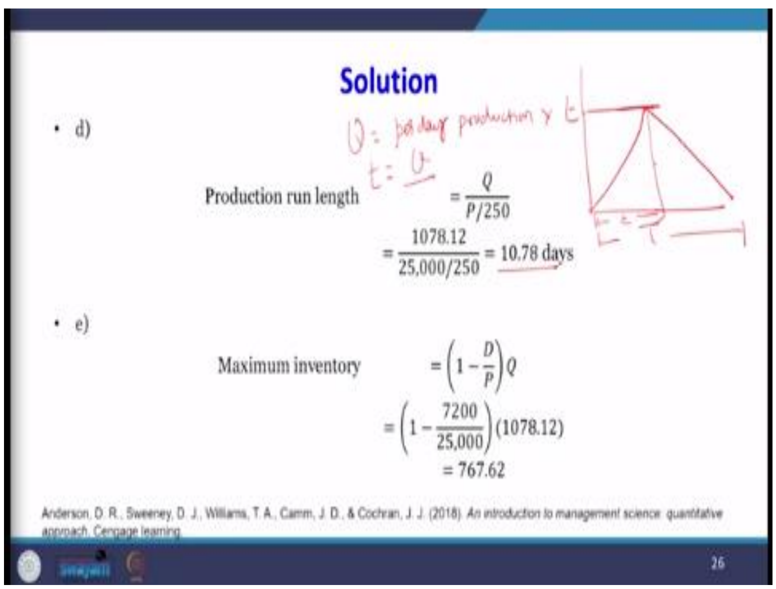
So, what do we have to find out? For minimum cost production lot size Q star, we have to find out the number of production runs per year and cycle time, length of the production run, maximum inventory, and total annual cost and reorder point. So, in our lot size model, the y-axis is inventory, so this is where our production run takes place after the consumption takes place. So, we have to find out Q, Q* the number of production runs, cycle time, length of production, and so on.



First, we will find out the production run; see Q* is 2DC0/ ((1 – D)/P) multiplied by C_h, the annual demand is given 7200, ordering cost is also given, and the production capacity is also given. So, 7200 upon 25,000 then you see that we are finding the annual holding cost. So, this 1078.12 is the optimal production quantity otherwise optimal lot size.

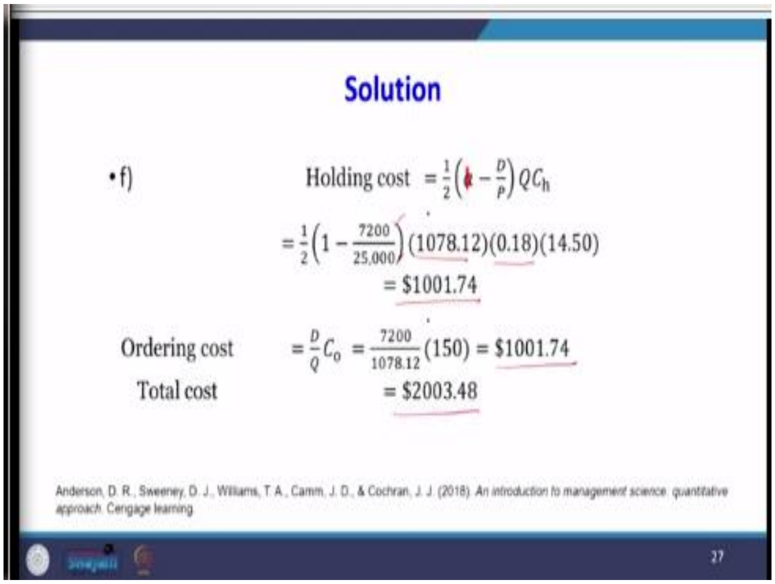


The second question is the number of production runs. So, we know that the annual demand is 7200, so when you divide that one by a number of production runs, there should be 6.68 production runs. And what is the cycle time? The cycle time is the number of working days divided by the number of production runs, that is, D by Q. So when you simplify, it will be 250Q star by D, so this is 37.43, which is our cycle time.

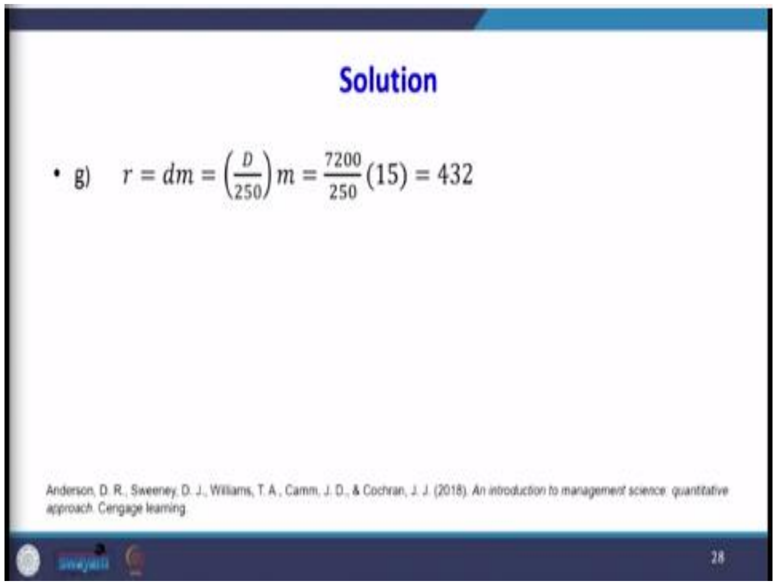


Then the production run length, you see the cycle time is this much distance, so this much distance is our cycle time, says T. So, in this, the t is this much distance where the production run is taking place, that is a production run length. So, how do we find the production run length? So, we know this already: we have derived an expression Q is equal to per day production multiplied by t, and this t is our production length.

So, the Q is given, Q is 1078, and the annual production rate is given. So, when you divide this annual production rate by the number of working days, you will get the per-day production run. So, from this expression, if you want to know the t, it is Q upon per day production run, so that is 10.78 days. What is the maximum inventoryso this point is maximum inventory, this point, so that is 767.



So, with annual demand, annual production rate Q, and the holding cost per unit annual holding cost, you will get 1001.74 dollars as the annual holding cost. So, here, the ordering cost is equivalent to the total setup cost. So, the number of setups D by Q is multiplied by the set of costs, so the ordering cost is 1001. So, the holding cost is 1001.74, and the ordering cost is 1001.74, so the total cost is 2003.48 dollars.



Next, we are going to find out the reorder point. So, the reorder point is the per-day demand multiplied by lead time. So, per day demand is 7200 upon 250, so lead time is 15 days, so when you multiply you will be getting 4032. So, in this lecture, I have explained the economic production lot size model. I have derived the formula for the economic production lot size model, and then I have derived an expression for the total cost. Then I took 2 problems, and I solved that problem to understand the concepts; thank you very much.