Decision Making with Spreadsheet
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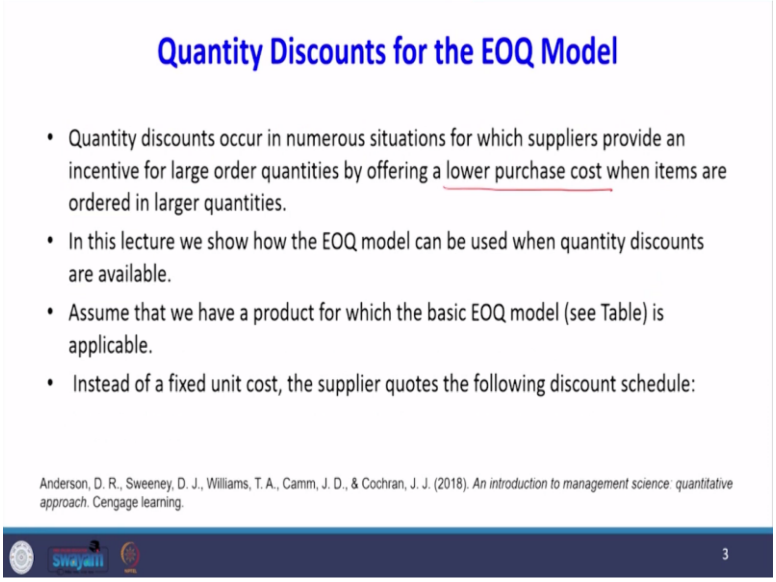
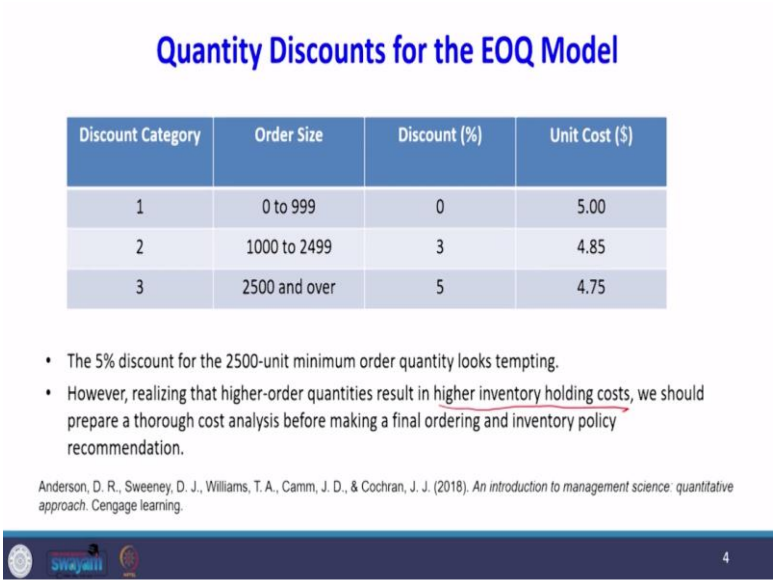
Lecture – 37
Inventory Model-III

Dear students. In this lecture, we are going to see another inventory model, which is the quantity discounts for EOQ. What will happen when we buy in large quantities? The seller used to give some discounts on their price. So, when they give their price we are tempted to buy more. When we buy more there is a possibility our holding cost may increase. So, we need to have the tradeoff between our holding cost and whether we have to avail of that offer or not. So, that kind of inventory model we are going to see in this lecture.



So, the agenda for this lecture is quantity discounts for the EOQ model. Quantity discounts occur in numerous situations for which suppliers provide an incentive for large order quantities by offering a lower purchase cost when the items are ordered in large quantities. So, whenever we buy larger quantities, these suppliers reduce the purchase cost. They give a discount so that you will buy more, but we have to be very careful whether we can take that offer or not.

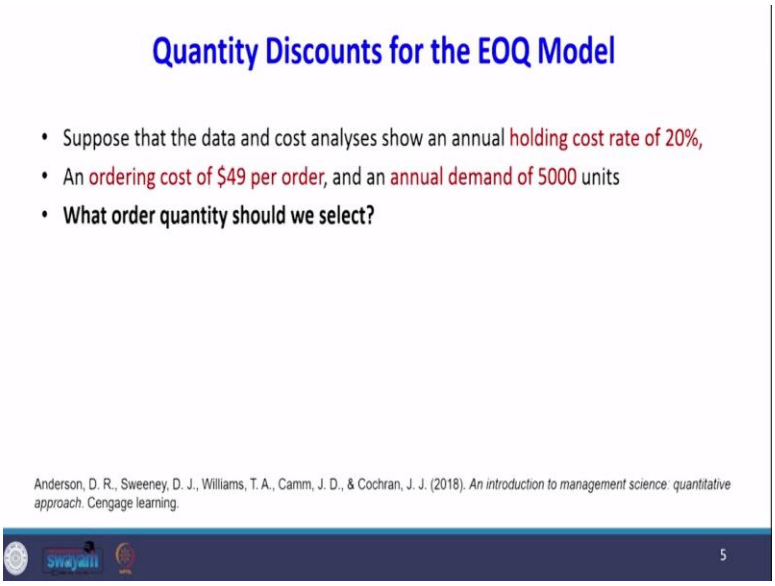
So, in this lecture, we show how the EOQ model can be used when quantity discounts are available. Assume we have a product for which the basic EOQ model applies. I will show you the table. Instead of a fixed unit cost, the supplier quotes the following discount schedule.



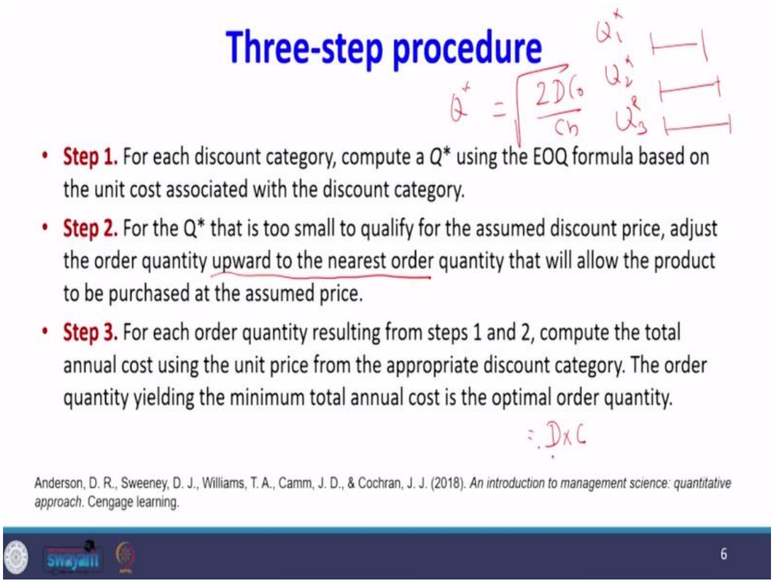
So, this table shows that if you buy, you see discount categories 1, 2, and 3 order size discount percentage and unit cost dollar. Suppose you buy a quantity between 0 to 999, there would not be any discount. So, when your order size is higher, it goes between 1,000 and 2,499, and there is a 3 percent discount on the unit cost. There will be 15 percent, so there will be 5 – 0.15 will be 4.85.

If you buy above 2,500 and above, you see they are giving 5 percent, so the purchasing price is only dollar 4.75. So, the 5 percent discount for the 2,500-unit minimum order quantity looks tempting. So, we used to buy more because the selling price was low. However, realizing that higher order quantities result in higher inventory holding costs, we should be very careful about this.

When you buy more, we have to maintain that inventory so that the inventory holding cost will increase. We should prepare a thorough cost analysis before making a final ordering and inventory policy recommendation. That is what we are going to do. We are going to answer whether we should go for this discount or not, whether offering this discount, and whether taking this discount from the supplier is saving our cost or not.



Suppose that the data on cost analysis show the annual holding cost rate is 20 percent, the ordering cost is dollar 49 per order, and the annual demand is 5,000 units. So, the question we should answer is, what order quantity should we select if they give the quantity discounts? What should be the order quantity?



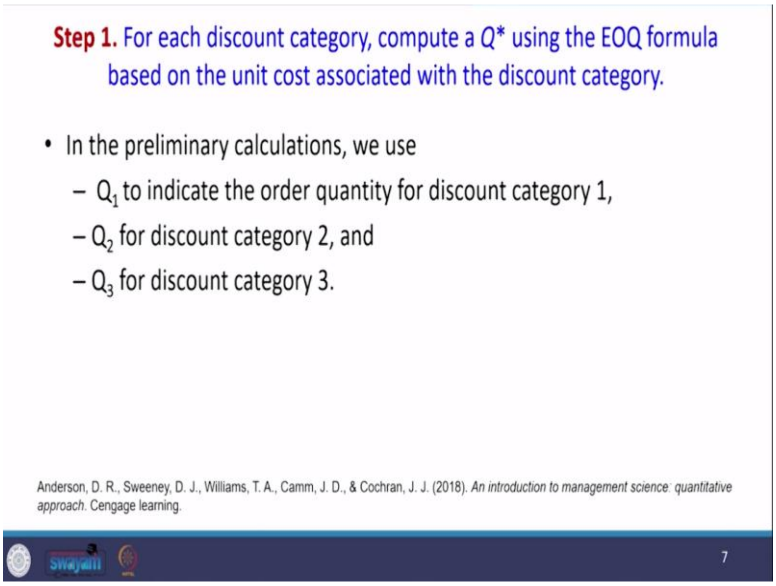
There are three steps procedure is there to solve this kind of problem. Step one for each discount category computes a Q* using our traditional EOQ model. What is the EOQ model? 2 DCo/Ch square root is our Q*. So, for each discount category, compute a Q* using the EOQ formula based on the unit cost associated with the discount category. Step 2 for the Q* that is too small to qualify the assumed discount price.

Adjust the order quantity upward to the nearest order quantity that will allow the product to be purchased at the assumed price. See, for example, say there are three intervals there. So, we have to find out, say, for example, Q1*, Q2*, and Q3*. The first step we are finding Q1, Q2, Q3. Look at step 2 for the Q* that is too small. For example, the Q* is very small. If it is very small, we cannot avail of this discount price.

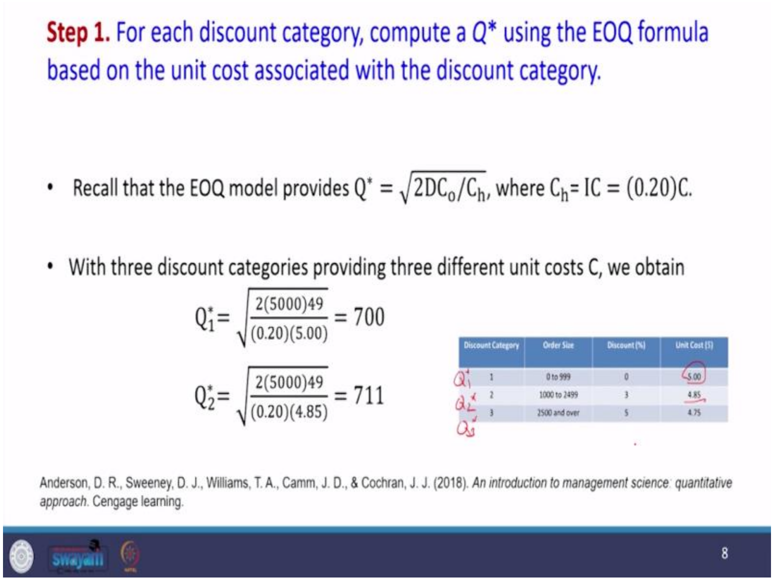
So, if that is what you have to do, we have to order the quantity upward to the nearest order quantity. So, we have to look at the right-hand side of the upper limit for which quantity we can get the discount. So, that will allow the product to be purchased at the assumed price. The third step for each order quantity resulting from steps 1 and 2 computes the total annual cost using the unit price from the appropriate discount category.

The order quantity yielding the minimum total annual cost is the optimal order quantity. One important point I wanted to make is that so far, we have not considered the actual purchasing cost, but here, we are going to consider that demand and multiply it by the unit cost. So, here we are going to consider the actual purchasing cost apart from your ordering cost and holding cost because this quantity will decide.

So, the quantity how much quantity you are going to buy will decide whether the cost is minimal or not.



So, what is step 1? For each discount category compute a Q star using the EOQ formula based on the unit cost associated with the discount category. So, in the preliminary calculations, there are three categories given, so Q1 indicates the order quantity for discount category 1. Q2 for discount category 2 and Q3 for discount category 3.



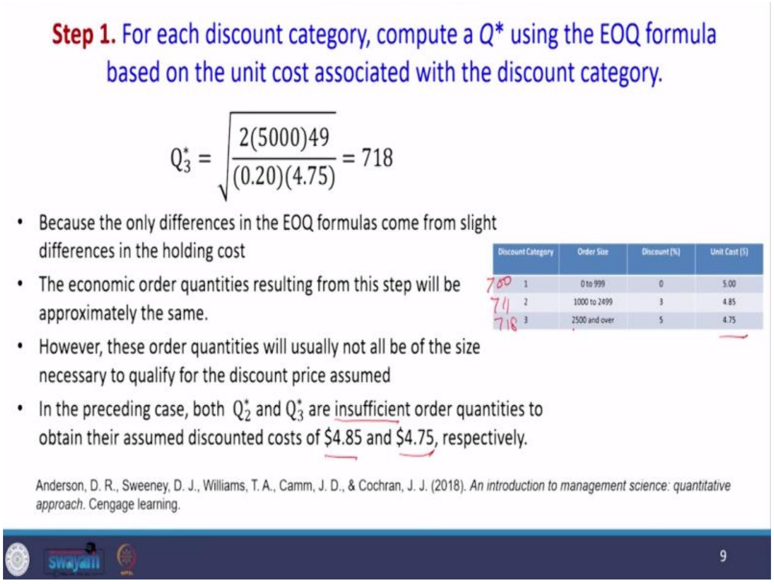
So, look at this table here for each one we are finding Q1, Q2, Q3. In fact it is a Q1*, Q2* and Q3*. So, we know the formula Q* is 2DCo/Ch. So, holding cost is the percentage of interest rate multiplied by unit cost. Here, there is no difference in ordering cost, but the holding cost will change because for different intervals, there are different costs, and the value of C is different. For example, Q1 demand is 5,000 units, and the ordering cost is dollar 49.

The inventory holding cost is 0.2 multiplied by 5. See this 5. For Q2*= 2DCo 0.2 multiplied by 4.85. For Q3*, it is 4.75, Q3* = 2 DCo/ Ch. So, we got Q1*, Q2*, and Q3*.

$$Q_1^* = \sqrt{\frac{2(5000)49}{(0.20)(5.00)}} = 700$$

$$Q_2^* = \sqrt{\frac{2(5000)49}{(0.20)(4.85)}} = 711$$

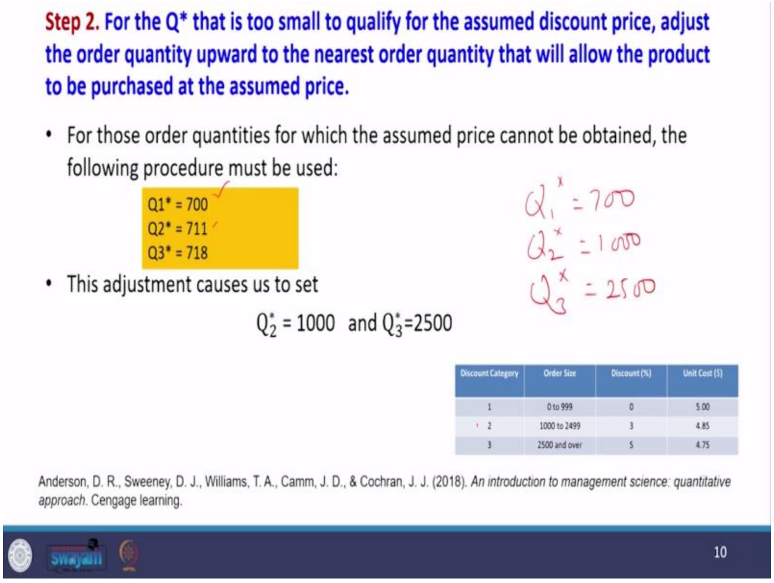
$$Q_3^* = \sqrt{\frac{2(5000)49}{(0.20)(4.75)}} = 718$$



Because the only difference in the EOQ formulas comes from the slight difference in the holding cost. The ordering cost is the same. The economic order quantities resulting from this step will be approximately the same. However, the order quantities will usually not all be of the size necessary to qualify the discount price assumed. In the preceding case, both Q2* and Q3* have insufficient order quantities to obtain their assumed discount cost of 4.85 and dollar 4.75, respectively, because what is the value of Q1*?

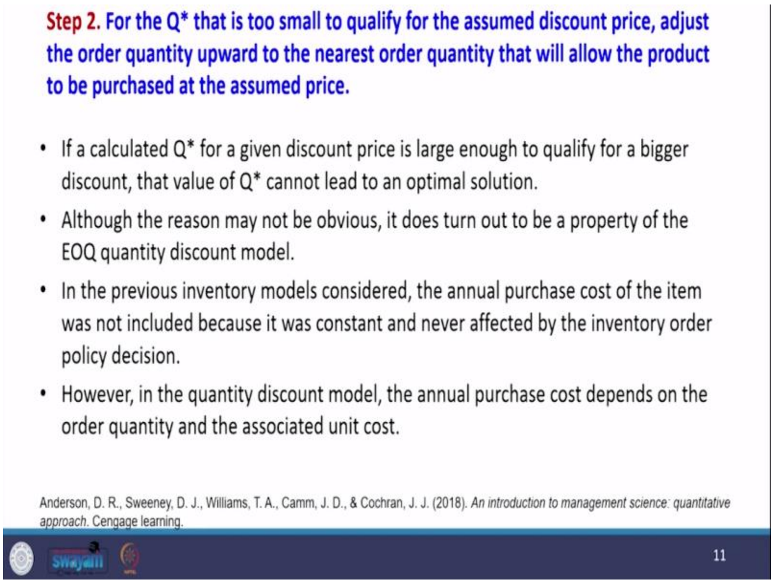
The value of Q1* is 700. Yes, the 70 is within this interval when you see the Q2* is 711. We need to have a minimum of 1,000 units, so the Q2* is nonqualifying. So, if it is not qualifying, which is the higher side of the 1,000 units, we have to order 1,000 units. For that, we have to find out what is the total price, and total annual cost.

Q3* also has 718 units not qualifying, so we have to find out the ordering cost of having 2,500 units.

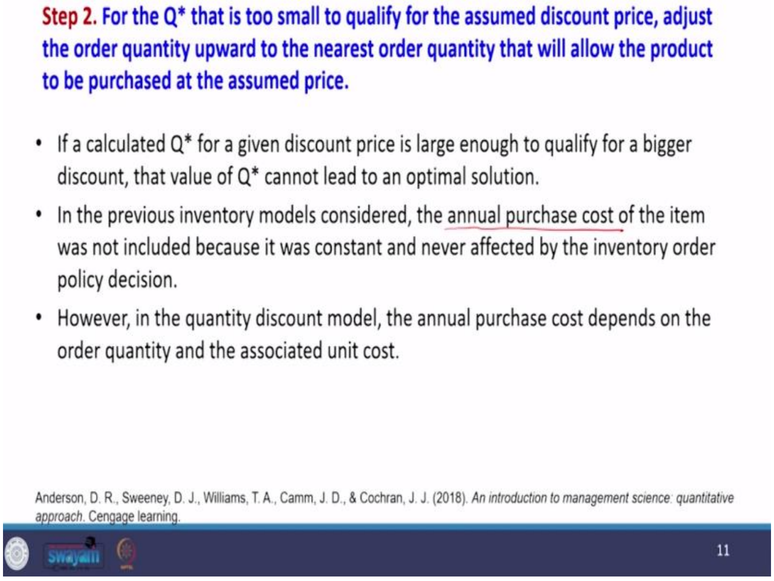


In step 2, for the Q* that is too small to qualify the assumed discount price, adjust the order quantity upward to the nearest order quantity that will allow the product to be purchased at the assumed price. For those order quantities for which the assumed price cannot be obtained, the following procedure must be used. For example, we got Q1* is 700, this is eligible, but you see Q2* = 711 it is not eligible.

Similarly, Q3*= 708 is also not eligible. So, what you have to make for this adjustment is to set Q2* as 1,000 units, Q2*, and Q3* as 2,500 units. So, what we are going to do in step 3 if Q1* is 700 units, if Q2* is 1,000 units, if Q3* is 2,5 00 units, we are going to find out the total cost, whichever the minimum quantity needs to be ordered.

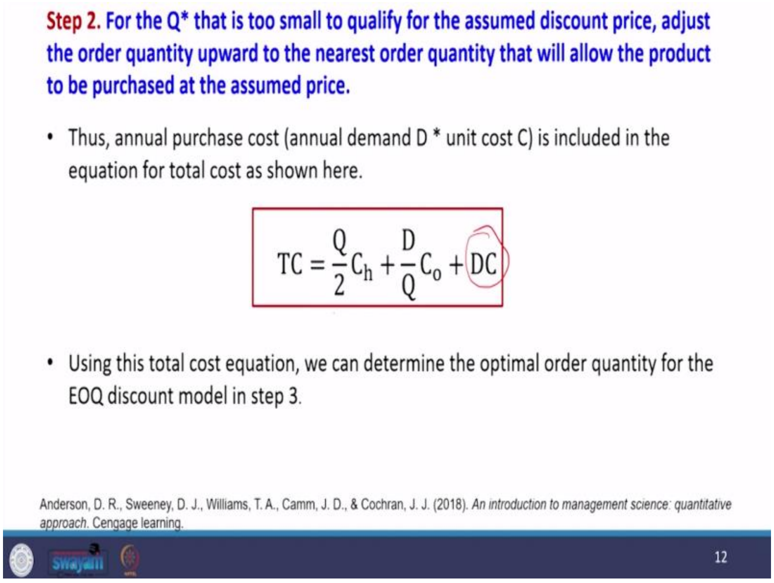


If a calculated Q* for a given discount price is large enough to qualify for a bigger discount that value of Q* cannot lead to the optimal solution. What is the meaning? For example, we got a Q1* is 700. Suppose you are getting the value of Q1*, say 1,200, so that we cannot consider option one because the Q1* exceeds the first interval. It goes beyond 999. So that category offer is not optimal. So, we have to omit that the first option, then we have to do only for the second option.

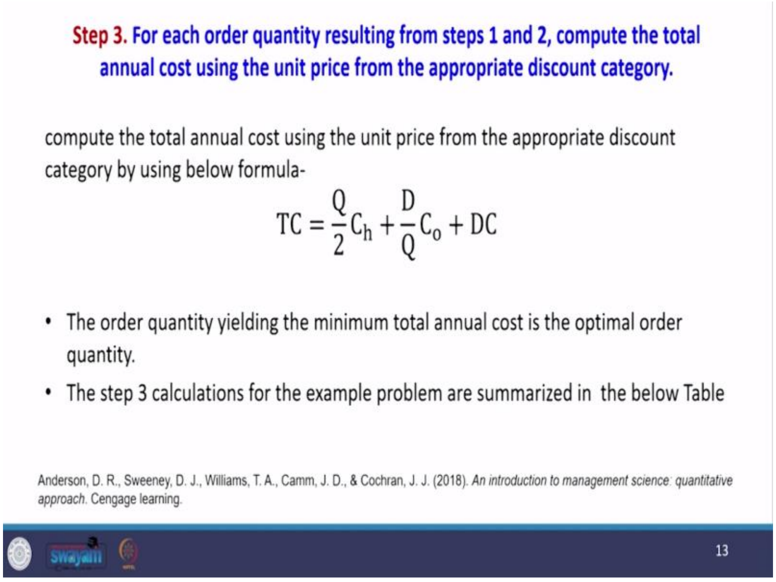


Step 2 for the Q* that is too small to qualify for the assumed discount price adjust the order quantity upward to the nearest order quantity that will allow the product to be purchased at the assumed price. So, if a calculated Q* for a given discount price is large enough to qualify for a bigger discount, that value of Q* cannot lead to the optimal solution. In the previous inventory model, the annual purchase cost of the item was not included because it was constant and never affected by the inventory policy decision.

So, for the previous inventory models, we did not consider annual purchasing cost; however, in the price discount model, in the quantity discount model, the annual purchase cost depends on the order quantity and the associated unit cost. So, in this method, we have to consider the annual purchasing cost also.

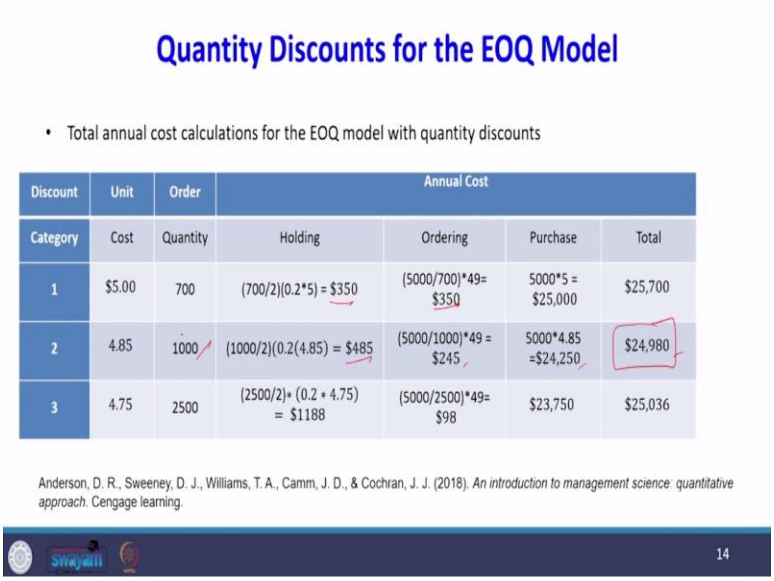


So, the annual purchasing cost is annual demand multiplied by the unit cost, which is included in the equation for the total cost. So, there are three elements is there what are they annual holding cost, annual ordering cost, and annual purchase cost because the value of C differs. So, we also need to consider this element. So, using this total cost equation, we can determine the optimal order quantity for the EOQ discount model in step 3.



So, in step 3, what do we have to do for each order quantity resulting from steps 1 and 2? How do we compute the total annual cost using the unit price from the appropriate discount category? So, what we need to do is compute the total annual cost using this formula, which is annual holding cost, annual ordering cost, plus annual purchasing cost. The order quantity yielding the minimum total annual cost is optimal.

So, step 3 calculations, for example, the problem, are summarized in the table below. So, we will take one sample example and explain all the steps.



The total annual cost calculations for the EOQ model with quantity discounts. So, we got the Q1* of 700 for that. What is the annual holding cost? We know the formula for annual holding cost is Q/2 average inventory multiplied by holding cost. Ordering cost is the number of orders of 5,000 / 700 multiplied by the ordering cost of 350.

Then the purchase cost of the demand is 5,000 units. If you go for 700, the purchasing cost will be dollar 5, so it will be dollar 25,000. The total is 25,700 dollars.

Discount	Unit	Order	Annual Cost			
Category	Cost	Quantity	Holding	Ordering	Purchase	Total
1	$5.00	700	(700/2)(0.2*5) = $350	(5000/700)*49= $350	5000*5 = $25,000	$25,700

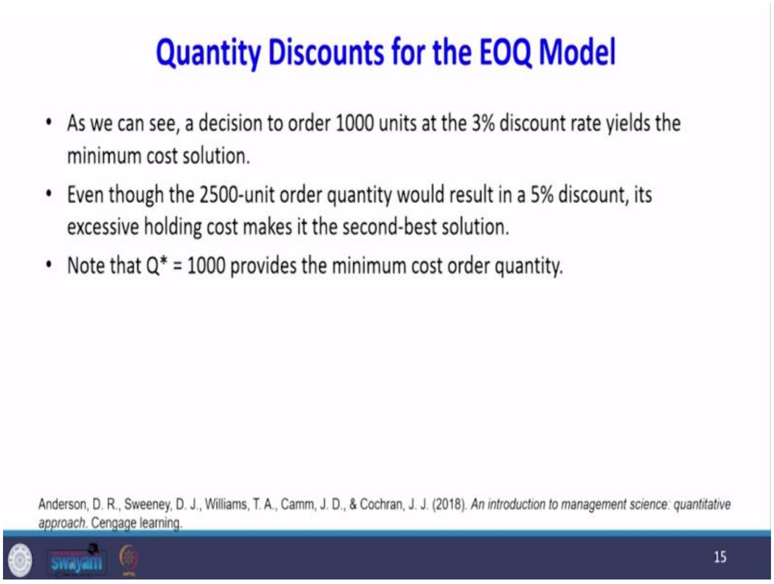
In the second interval, if you go for 700 units, the cost is 4.85. Then what will the holding cost be? Q/2 multiplied by 0.2 into 4.85, so 485? What is the ordering cost D/Q 5,000 divided by 1,000 multiplied by ordering cost 249, then purchasing cost 5,000 multiplied by 4.85? You see that previously 5 it is 4.85, so 24,250.

Discount	Unit	Order	Annual Cost			
Category	Cost	Quantity	Holding	Ordering	Purchase	Total
1	$5.00	700	(700/2)(0.2*5) = $350	(5000/700)*49= $350	5000*5 = $25,000	$25,700
2	4.85	1000	(1000/2)(0.2(4.85) = $485	(5000/1000)*49 = $245	5000*4.85 =$24,250	$24,980

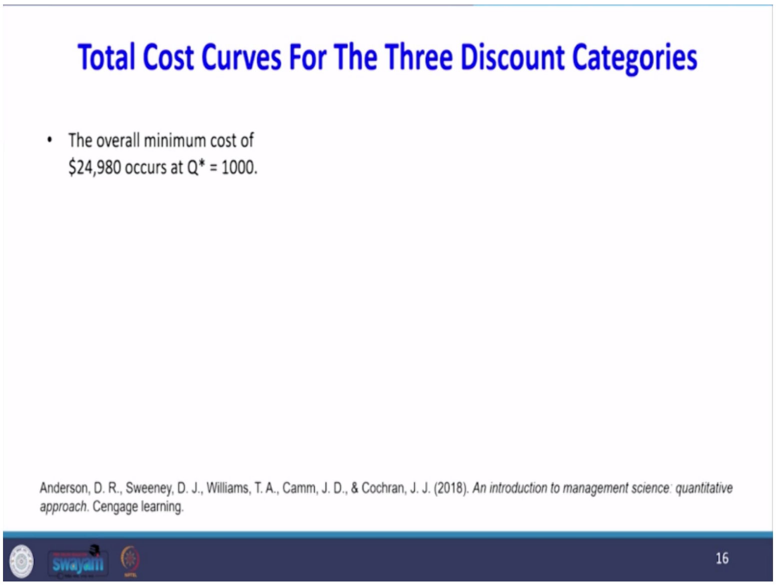
So, when you add it, it is 24,980. Suppose you are buying 2,500 units, the buying cost is 4.75, so the average inventory is 2,500 divided by 2, multiplied by the holding cost, which is 1,188 ordering cost, and the number of orders multiplied by this is 49, so dollars 98. So, when you add it to the purchasing price, what is the purchasing price of 5,000 multiplied by 4.75? It will give you 23,750.

Discount	Unit	Order	Annual Cost			
Category	Cost	Quantity	Holding	Ordering	Purchase	Total
1	$5.00	700	(700/2)(0.2*5) = $350	(5000/700)*49= $350	5000*5 = $25,000	$25,700
2	4.85	1000	(1000/2)(0.2(4.85) = $485	(5000/1000)*49 = $245	5000*4.85 =$24,250	$24,980
3	4.75	2500	(2500/2)* (0.2 * 4.75) = $1188	(5000/2500)*49= $98	$23,750	$25,036

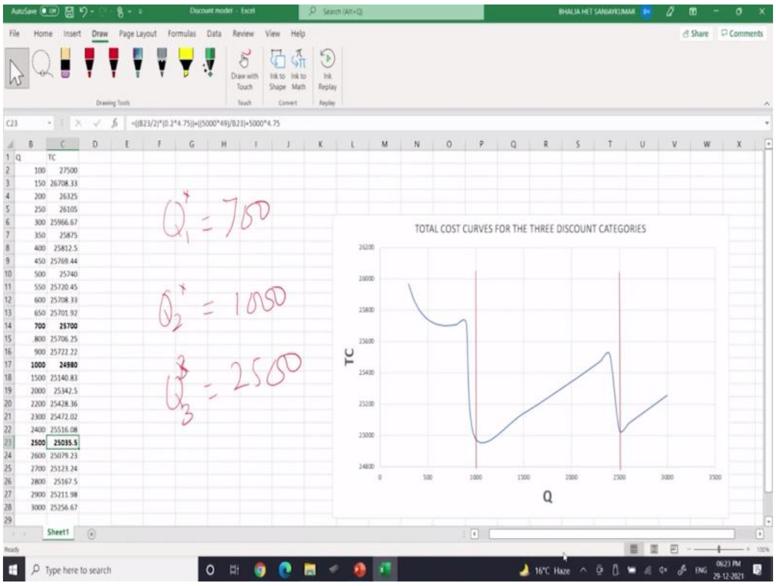
So, the total cost is dollar 25,036. So, out of these three, which is the minimum? So, this one is minimal. So, according to our inventory policies, we should go for ordering 1,000 units.



As we can see, the decision to order 1,000 units at the rate of a 3 percent discount rate yields the minimum cost solution. Even though a 2,500-unit order quantity would result in a 5 percent discount, its high holding cost makes it the second best solution. So, you see that Q* = 1,000 provides the minimum order quantity.



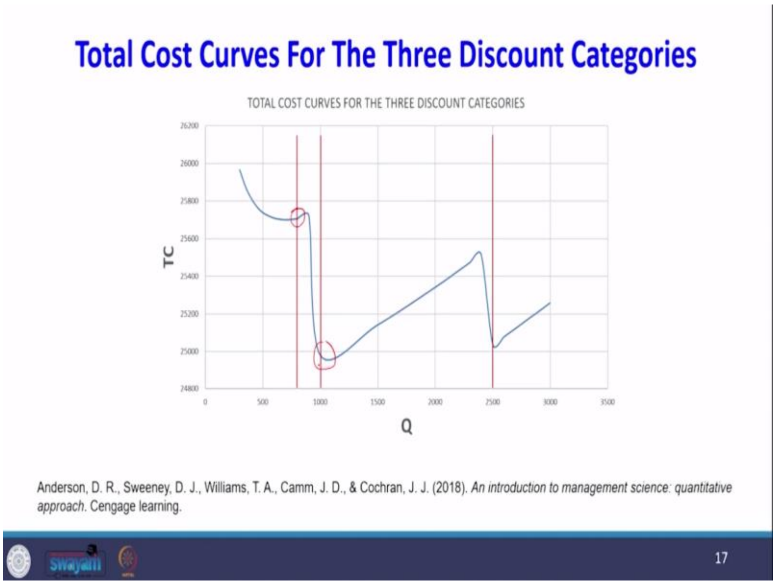
Now, with the help of Excel, I am going to explain how to plot these values.



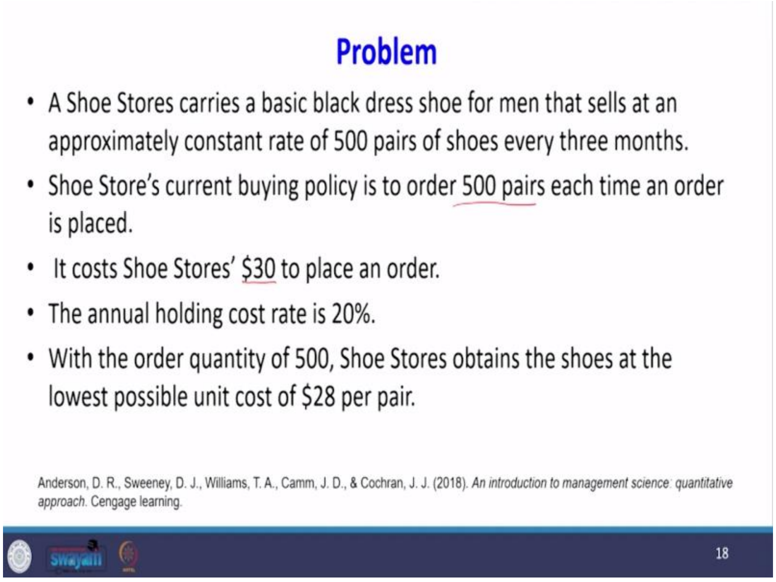
Now, I am going to explain the total cost principles with the help of this Excel model. So, I have taken the value of Q* and the corresponding total cost. For example, say B2. See that B2 is 100 units. What will be the total cost? So click on C2 to see the total cost is average inventory multiplied by inventory holding cost plus D/Q multiplied by ordering cost plus purchasing cost. You see, up to 700, we got the value of Q1*.

So, in our model Q1* is 700. So, I have kept up to 700, and I found the total cost. The Q2* was 1,000 units. So, when I increase the ordering quantity, then correspondingly, for example, when you click on C15. For C15 you see the total cost we got this much. So, the Q3 * was 2,500, so if you see the C23 formula, as usual, we have written the annual holding cost plus ordering cost plus annual purchasing cost.

So, when I plot that on the right-hand side, you can see the figure the total annual cost is less if you order 1,000 units, not exactly 1,000 units, but you see when you go for 2,500 units, our total cost is above 25,000. By looking at this figure, we are concluding that so ordering 1,000 units is the most optimal ordering quantity for us. Now, I will go back to the presentation.

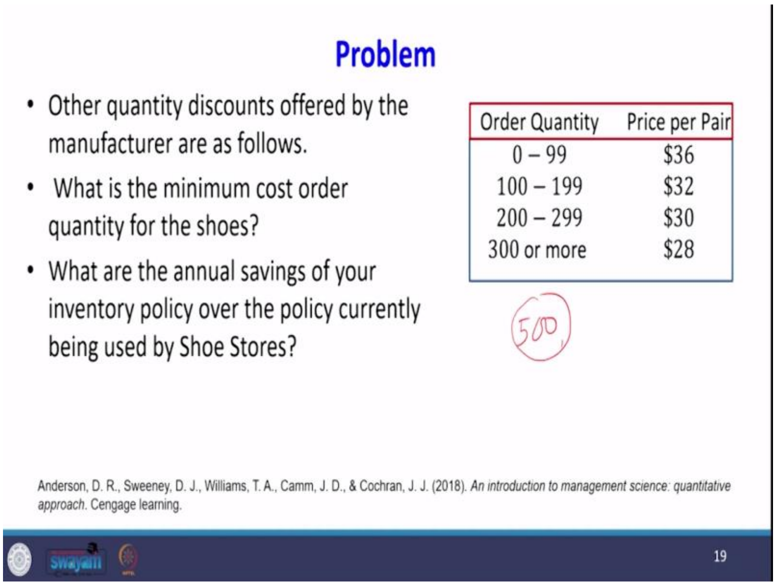


Now look at this here, here it is the most optimal quantity. You see that here also we are getting a lesser quantity, but when we go for these 1,000 units, we are getting the least total cost. So, our conclusion here is we should go for ordering 1,000 units.



Now I will take another problem to explain this concept with the help of another problem. What the problem is that a shoe store carries a basic black dress shoe for men that sells at an approximately constant rate of 5,000 pairs of shoes every 3 months. The shoe stores current buying policy is to order 500 pairs each time an order is placed. So, what are the cost details? It costs the shoe store dollar 30 to place an order.

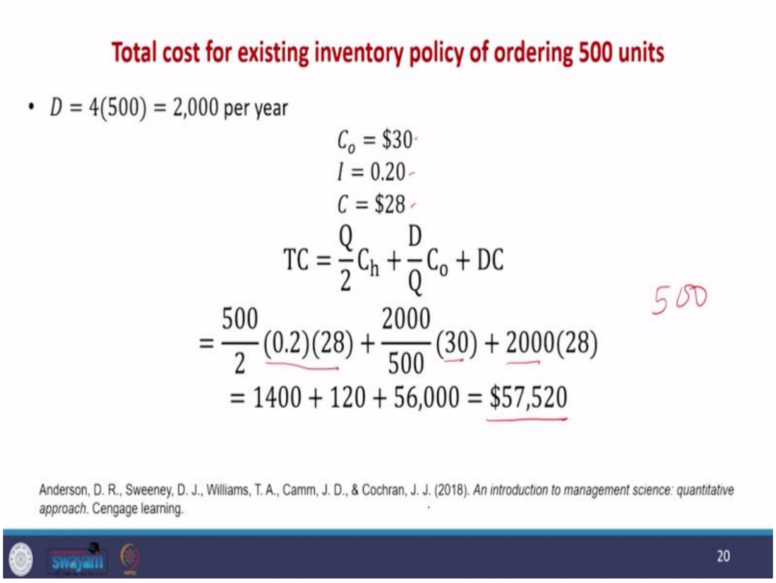
So, the ordering cost is dollar 30. The annual cost of holding is 20 percent. With the quantity with the ordering quantity of 500 units, the shoe store obtains the shoe at the lowest possible unit cost of dollar 28 per pair.



Now, there is an offer for them, whether they should buy 500 units. Now, the seller is giving the offer. What that offer says is that if they buy 0 to 99, it is a dollar 36 buying price. If it is 100 to 199, it is a dollar 32. If it is 200 to 299, it is a dollar 30. If the company buys more than 300, which is dollars 28. So, now the question is, what is the minimum cost order quantity for the shoes? What is the annual savings of your inventory policy over the policy currently being used by the shoe stores.

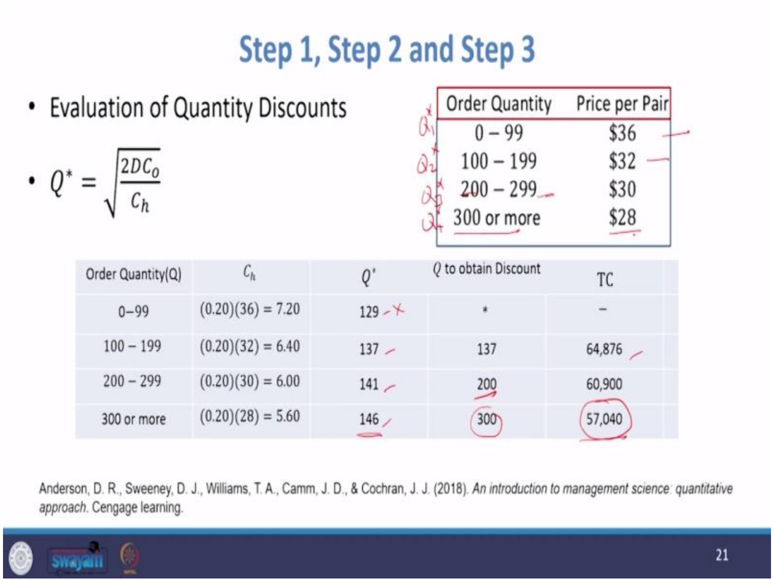
Order Quantity	Price per Pair
0 – 99	$36
100 – 199	$32
200 – 299	$30
300 or more	$28

What is the current policy? They order 500 units. So, the supplier is now offering a discount policy. So, we are going to compare whether I should continue with my current policy or I have to take this offer, which is most profitable for me.



So, what was the first step we took previously? The first step is to find Q1*, Q2*, Q3*, and Q4*. Before that, we will find out the total cost for the existing inventory policy of ordering 500 units. Now, the total cost is because they are ordering 500 units, so Q/2 is multiplied by the annual holding cost plus the number of orders and ordering cost, and the annual purchasing cost. So, if they go for this policy, ordering 500 units the total inventory cost is 57,520.

$$C_o = \$30$$
$$I = 0.20$$
$$C = \$28$$
$$\mathrm{TC} = \frac{\mathrm{Q}}{2}\mathrm{C_h} + \frac{\mathrm{D}}{\mathrm{Q}}\mathrm{C_o} + \mathrm{DC}$$
$$= \frac{500}{2}(0.2)(28) + \frac{2000}{500}(30) + 2000(28)$$
$$= 1400 + 120 + 56{,}000 = \$57{,}520$$

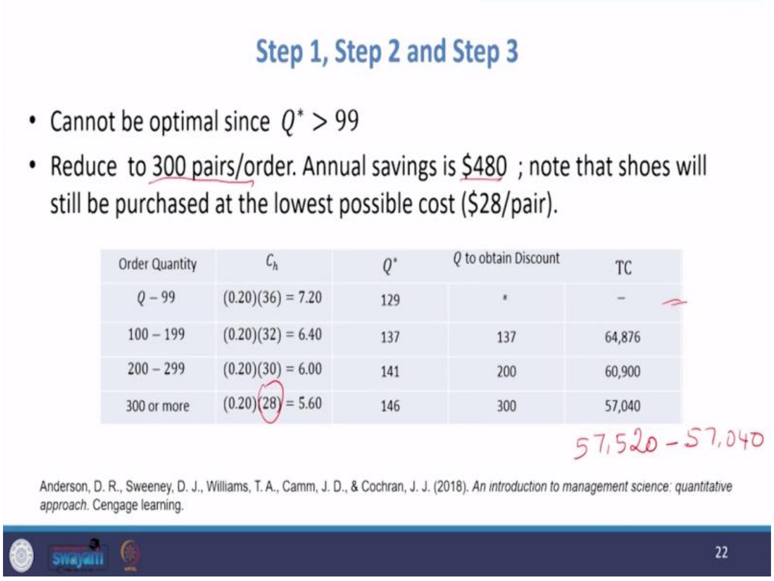


Now there is an offer by the supplier. Now we will see what will happen whether we have to go for this offer or not. So, the first step is the first step. We have to find out Q1*, Q2*, Q3*, and Q4* here. So, here I found this Q1*, Q2*, Q3*, and Q4*. Now look at Q1* it is 129, but you see that it is 0 to 99. So, it is going to the next level.

So, this is not valid, so Q2* obtaining this interval need not be considered. We may not find out the total cost also. The Q2* is 137. Yes, it is within this interval. If it is 137, we have to find out what the total cost is. So, what is the total cost here? There will be 137 units, an annual holding cost plus annual ordering cost plus purchasing cost. That is your 64,876. Looking at Q3* is 141, so 141 does not fall in this range of 200 to 299.

So, if it is 141, what is the next highest upward side? What is the highest value of 200? So, 200 should be the order quantity for this order quantity. We have to buy the total cost. Q3* is 146. The 146 is not in this interval because we can avail this dollar 28 price only if we buy more than 300, so instead of order quantity 146, we should consider 300 units. For these 300 units, we have to find out the annual holding cost plus ordering cost plus annual purchasing cost.

So, that amounts to 57,040. Now look at Q2*, Q3*, and Q4*. Which is the lowest one? The lowest one is 57,040. You remember previously if you buy 500 units, there was some price, but if you go for 300 units and if you avail this discount of dollar 28, you are getting a lesser annual total cost.



Step 1, Step 2 and Step 3

- Cannot be optimal since $Q^* > 99$
- Reduce to 300 pairs/order. Annual savings is $480 ; note that shoes will still be purchased at the lowest possible cost ($28/pair).

Order Quantity	C_h	Q^*	Q to obtain Discount	TC
Q – 99	(0.20)(36) = 7.20	129	*	–
100 – 199	(0.20)(32) = 6.40	137	137	64,876
200 – 299	(0.20)(30) = 6.00	141	200	60,900
300 or more	(0.20)(28) = 5.60	146	300	57,040

57,520 – 57,040

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., & Cochran, J. J. (2018). *An introduction to management science: quantitative approach*. Cengage learning.

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So, our conclusion is we should go for the inventory offer. So what happened is that the first category Q* is greater than 99. So, this column need not be calculated. Now, what happens is that if you go for this offer, instead of 500, if you order 300 pairs per order, the annual saving is 480. How did we get this annual saving of 480? So, previously, the price was 57,520, so 57,520 – 57,040. So, the difference is dollar 480, but note that this shoe will still be purchased at the lowest possible cost of dollar 28 see this dollar 28.

So, our conclusion is this shoe company should go for taking this offer if they take offer instead of ordering 500 units. If they order 300 units, they will save the inventory cost how much for dollar 480. So, dear students, in this lecture, I have explained the quantity discount model. In the quantity discount model, what you have learned there are three steps. What are the three steps?

Step one is for each interval we have to find out the optimal order quantity. For example, Q1*, Q2*, Q3* if there are three intervals. Step 2 is if any optimal order quantity is not lined with that interval, we should find out the next highest value for that value should be our ordering quantity. Step 3 is now where we have to find out the total annual cost for all three of these ordering quantities, whichever is the minimum quantity that needs to be ordered.

And one more thing, when compared to the previous inventory models here, you also have to consider the annual purchasing cost because that will influence your annual total cost. In the next class, we are going to talk about a very interesting model, the Newsboy model, that we will see in the next class. Thank you very much.