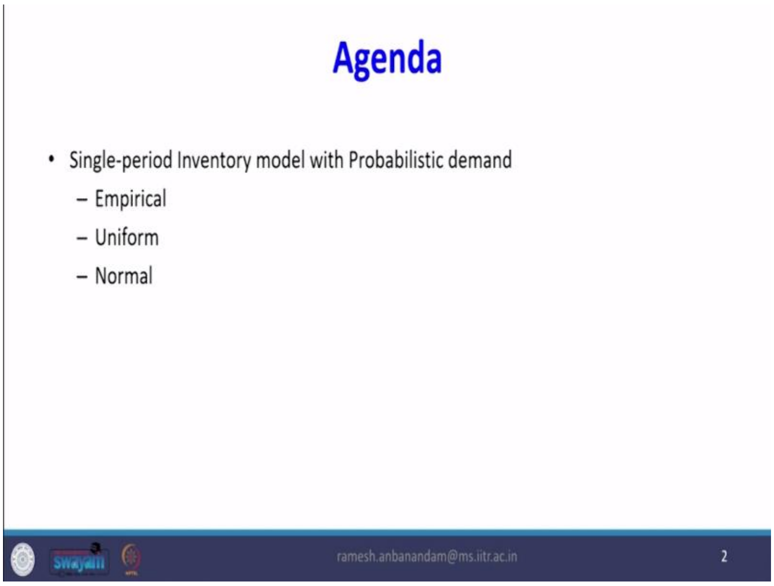
Decision Making with Spreadsheet
Prof. Ramesh Anbanandam
Department of Management Studies
Indian Institute of Technology – Roorkee

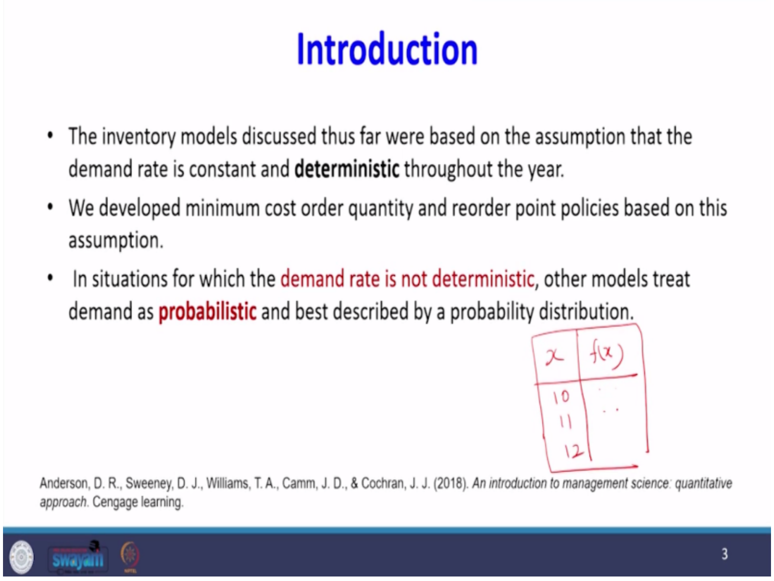
Lecture – 38
Singe Period Inventory Model with Probabilistic Demand

Dear students, we are discussing inventory models. So far, we have discussed inventory models where the demand is deterministic. What the meaning of deterministic demand is known to you. Hereafter, we are going to discuss a new inventory model where the demand is not deterministic. It is probabilistic.



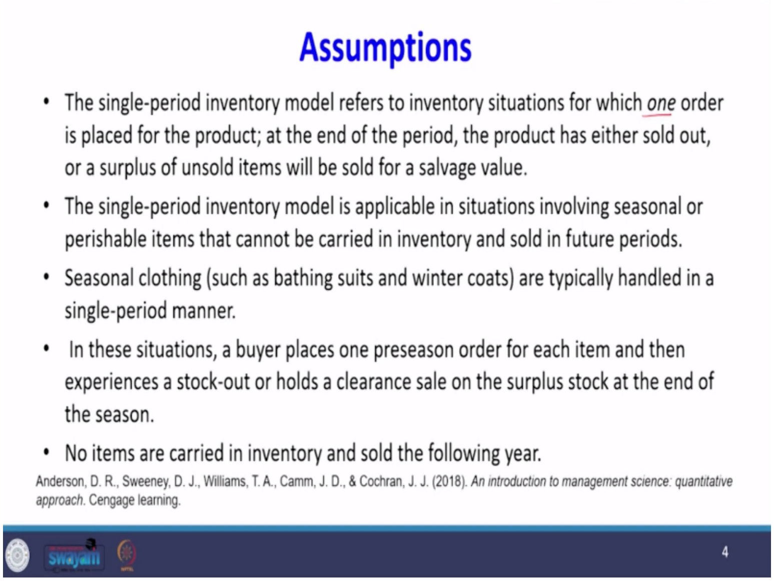
So, the agenda for this lecture is it is a single-period inventory model with probabilistic demand. Here, the period is only one period, so this kind of inventory model is applicable for perishable products. The very famous problem is called the Newsboy problem. So, in this class, we are going to discuss the Newsboy problem. We are going to consider three types of demand patterns.

Sometimes, the demand may follow an empirical distribution, it may follow a uniform distribution, or it may follow a normal distribution. If, for a single period, the demand follows any of these three distributions, how do we find out the inventory models that are going to be the objective for this lecture?



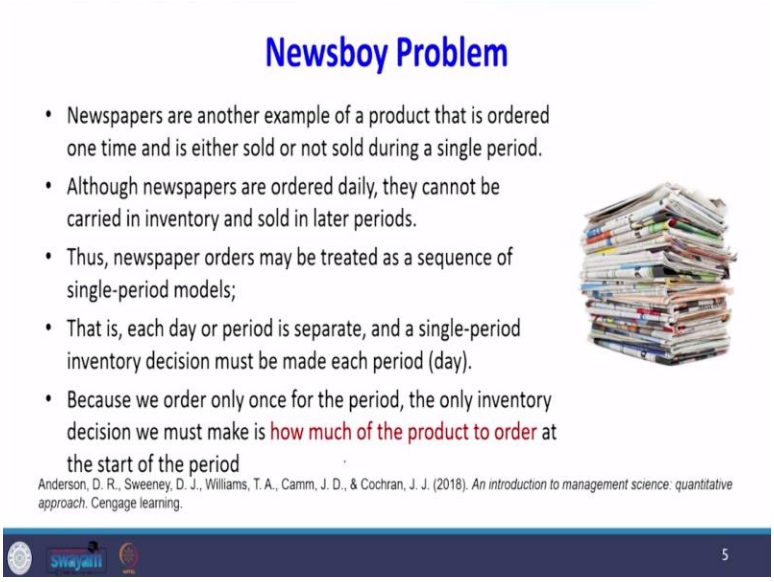
The inventory models discussed so far were based on the assumption that the demand rate is constant and deterministic throughout the year. For the deterministic demand, we developed minimum cost order quantity and reorder point policies based on this assumption. In a situation for which the demand is not deterministic, other models treat demand as probabilistic, and that is best described by a probability distribution.

What is the probability distribution? There will be x also will be there, there will be f of x also will be there here f of x is a probability. So, for each demand, a probability is attached; for example, say 10, 11, 12, there will be some probability values that are the meaning of this probability distribution.



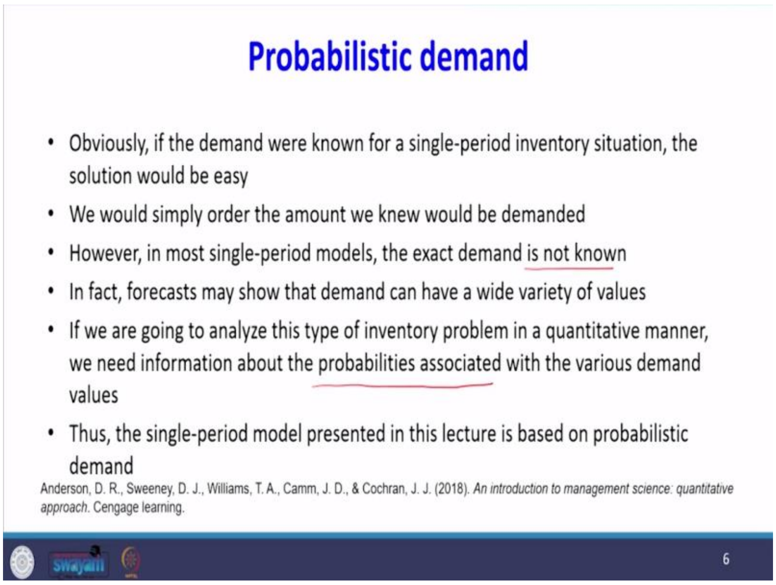
Now, we can discuss the assumptions for this model. The single-period inventory model refers to the inventory situation for which one order is placed for the product at the end of the period the product has either sold out or a surplus of unsold items will be sold for salvage value. Here, there is no concept of holding the inventory because it is only one period. If the product is not sold, then that product is based.

A single-period inventory model is applicable in situations involving seasonal or perishable items that cannot be carried in inventory and sold in future periods. So, for perishable items we cannot carry the inventory for the next period. Seasonal clothing such as bathing suits and winter coats are typically handled in a single-period manner. In these situations, a buyer places one preseason order for each item and then experiences a stock out or holds a clearance sale on the surplus stock at the end of the season. No items are carried in inventory and sold the following year because it is a perishable item.



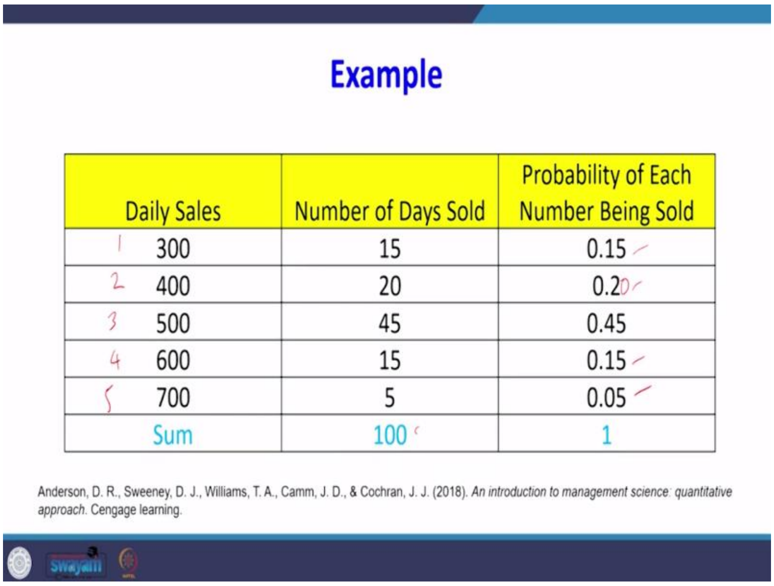
Another name for this model is the famous Newsboy problem. See the newspapers are another example of a product that is ordered one time and is either sold or not sold during a single period. So, if the newspaper is not sold that day, you cannot use it the next day. Although newspapers are ordered daily, they cannot be carried in inventory and sold in later periods.

Thus, newspaper orders may be treated as the sequence of single-period models. Each day is one period there that is each day or period is separate, and a single-period inventory decision must be made each period that is for each day because we order only once for the period the only inventory decision we must make is how much of the product to order, we need not bother about when to bother we are bothering about only we should know how much the product to order at the start of the period.



Obviously, if the demand were known for a single-period inventory situation, the solution would be easy. We would simply order the amount we knew would be demanded. However, in most single-period models, the exact demand is not known. Even though a newspaper boy may not know what is going to be demand for that day. In fact, forecasts may show the demand can have a wide variety of values.

If we are going to analyze this type of inventory problem in a quantitative manner, we need information about the probability associated with various demand values. So, we need to have if there is a forecast value for the demand then we need to have the probability also. Thus, the single-period model presented in this lecture is based on probabilistic demand.

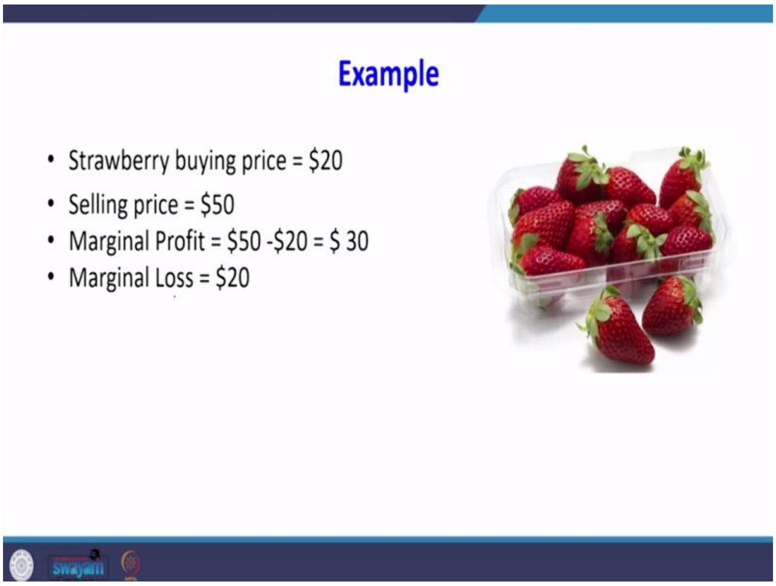


Daily Sales	Number of Days Sold	Probability of Each Number Being Sold
300	15	0.15
400	20	0.2
500	45	0.45
600	15	0.15
700	5	0.05
Sum	100	1

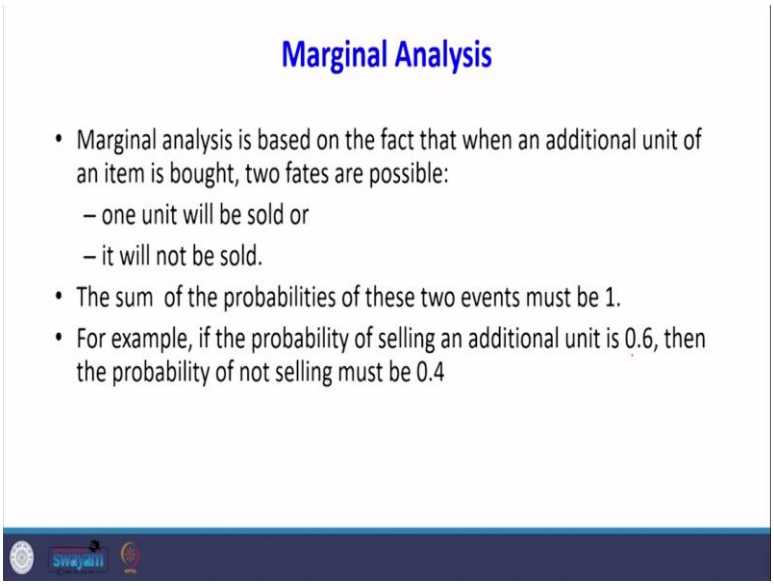
How to get these probabilities? The probabilities arrived from the past data. For example, suppose I have considered 1, 2, 3, 4, 5, say 5 days in a week except Saturday and Sunday. We have the past data; for example, on day 1, the number of daily sales was 300. So, that occurred for 15 days out of 100 days. In 15 days, the daily sales are 300, and in 20 days, the daily sales are 400.

For 45 days, the daily sales are 500, so this is a frequency number of days sold. What is the frequency? So, 300 newspapers are sold 15 times out of 100 days. So, 400 newspapers are 20 times out of 100 20 days the sales were 400. So, we have the frequency table from this. We can add this frequency, so then if you say this it is 100 days, so 15 divided by 100, so this is 15 percent, 20 percent, 45 percent, 15 percent, 5 percent.

So, what is the meaning of the probability of 300? The demand for 300 is 0.15. The probability for the demand of 400 is 0.20, so we now have the sales and corresponding probability. This is the way to arrive at the probability of the demand.

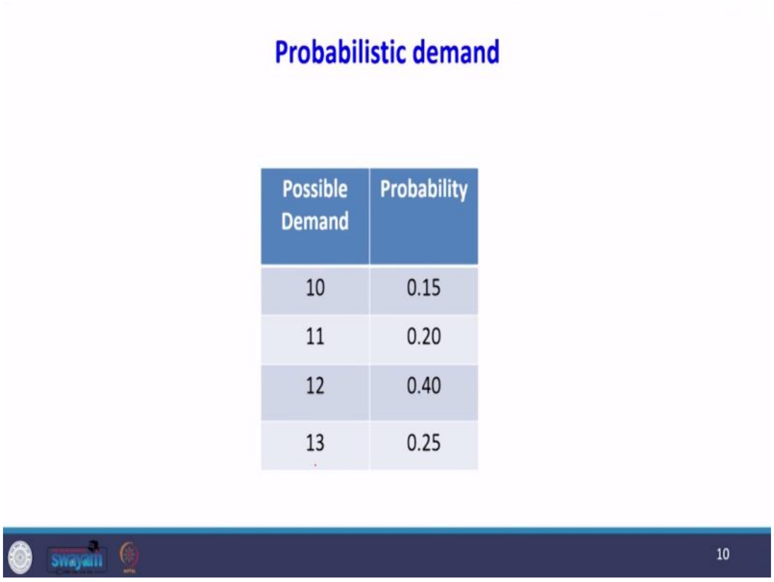


Now, we will take one example here: the strawberry buying price, say dollar 20, and the selling price is dollar 50. Now, I am going to discover the concept of marginal profit, which is one unit of profit. So, what will be the one-unit profit? It is a dollar 50 selling price - a dollar 20 buying price, so it is a dollar 30. What is the marginal loss? Because your buying price is 20, that cannot be carried for the next period, so if it is not sold, the loss is dollar 20.



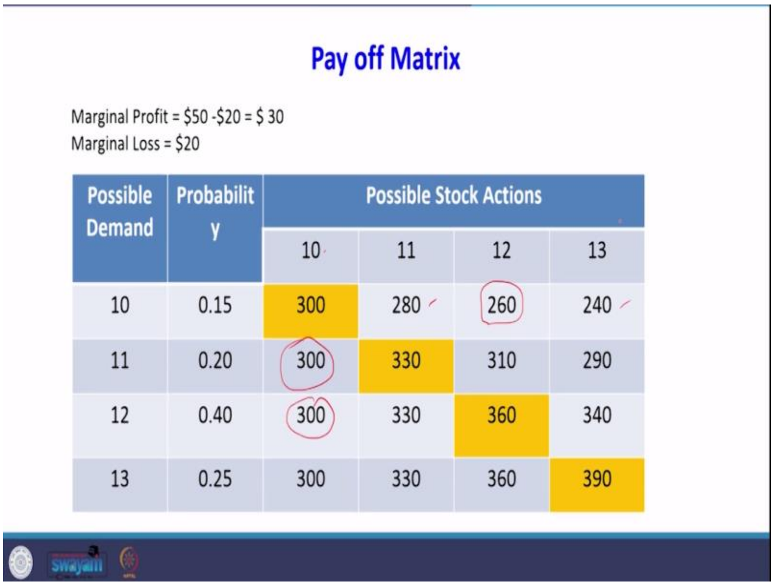
By using this concept of marginal analysis, we are going to find out how many quantities have to be ordered at the beginning of the period. First, you will see what marginal analysis is. So, the marginal analysis is based on the fact that when an additional unit of an item is bought. When an additional unit of item is bought two fates are possible. One is one unit will be sold, or it will not be sold.

Some of the probabilities of these two events must be 1. For example, if the probability of selling an additional unit is 0.6, then the probability of not selling must be 0.4 because the sum should be 1.



Possible Demand	Probability
10	0.15
11	0.20
12	0.40
13	0.25

Now we have an empirical distribution for the demand. So, what is that empirical distribution? The possible demand and corresponding probability. So, the demand is 15 percent. There is a 40 percent chance that the demand will be 12. Now what we need to know we should know what the ordering quantity for this kind of empirical distribution should be. Now the demand is an empirical distribution.

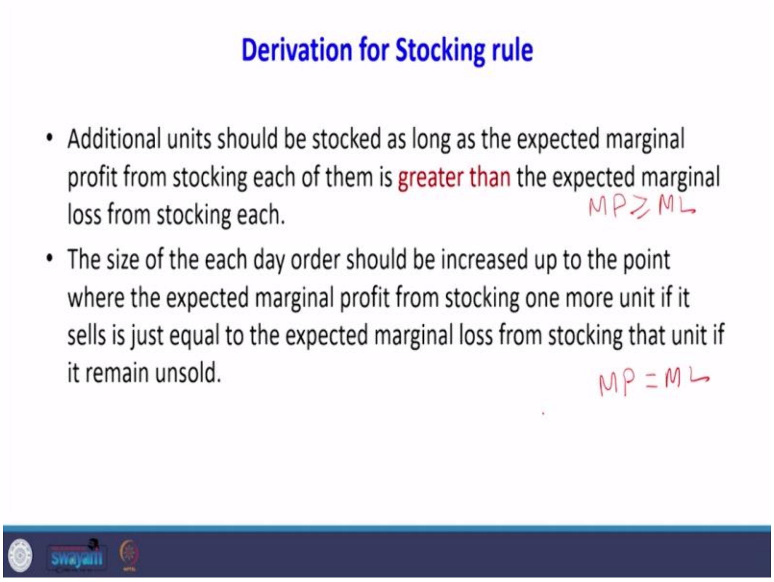


So, first, we will prepare a pay-off matrix. So, what is the payoff matrix? Suppose my demand is 10. If I stock 10 units here, I am going to write first. I am going to fill only the diagonal values. Here, I have written the probabilities, so my demand is 10. I have stocked 10 units, so what will be the profit for me 10 multiplied by 30, so marginal profit is dollar 30 for 1 unit, it is dollar 30 for 30 units, and it is dollar 300.

Then, see if I have 11. The demand is 11 units. I have stocked 11 units. So, what will happen here the payoff will be 330. The demand is 12 I have stocked 12 units 360. The demand is 13. I have stocked 13 units. It is 390. Now look at this category here. What has happened? My demand is only 10, but I stocked 11 units. So, in this case, I can sell 10 units for dollar 300. The one unit that is 11th unit I am not able to sell, so the loss of those 11th units is dollar 20.

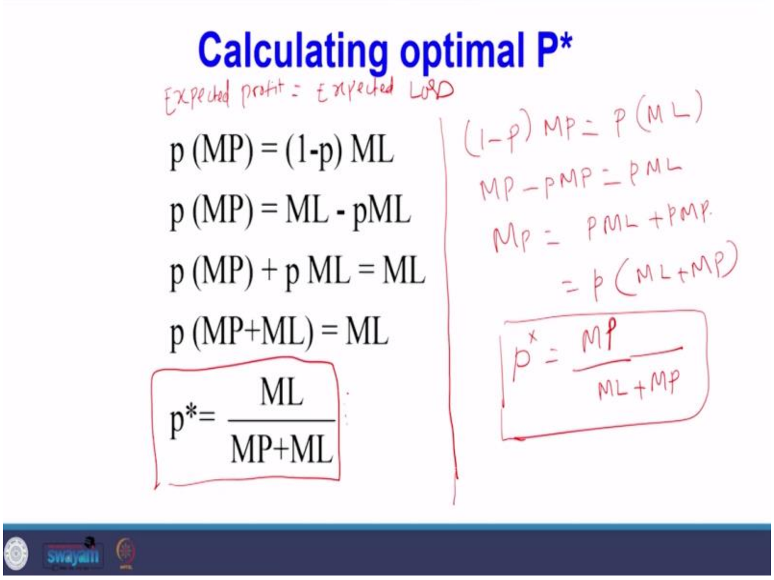
So, 300 – 20 it is 280. So, what is the meaning of this 12? Demand is 10, but I have stocked 12 units, so 10 units I can sell, so two units I am not able to sell, so 40. So, 300 - 40 will be my payoff here, as I have filled out for this 13 also. Now come to this category. What has happened here? My demand is 11, but I have stocked only 10 units, so I can sell only 10 units, so 10 into 330 it is 300.

Here is the meaning: demand is 12, but I have stocked only 10 units, so the payoff is 300 units. So, now we have the payoff matrix, so we need to know what the ordering quantity should be. Whether the stocking quantity should be 10, 11, 12, or 13, that is what we are going to do.



Now, to know what the stocking quantity should be, we will derive the stocking rule. What does the stocking rule say? Additional units should be stocked if the expected marginal profit from stocking each of them is greater than the expected marginal loss from stocking each. If the marginal profit is greater than or equal to the marginal loss, then we can go for stocking an additional unit.

So, the size of each day's order should be increased to the point where the expected marginal profit from stocking one more unit if it sells is equal to the expected marginal loss from stocking that unit if it remains unsold. So, the concept for stocking an additional unit is if the marginal profit is equal to the marginal loss up to that point, we can go to have some additional units.



Now, we have to find out the probabilities for which the marginal profit is equal to the marginal loss. So, what we have to do is know the probability and marginal profit, so this will be the expected profit; so, the expected profit is equal to the expected loss. So, what is the expected profit? Probability multiplied by marginal profit.

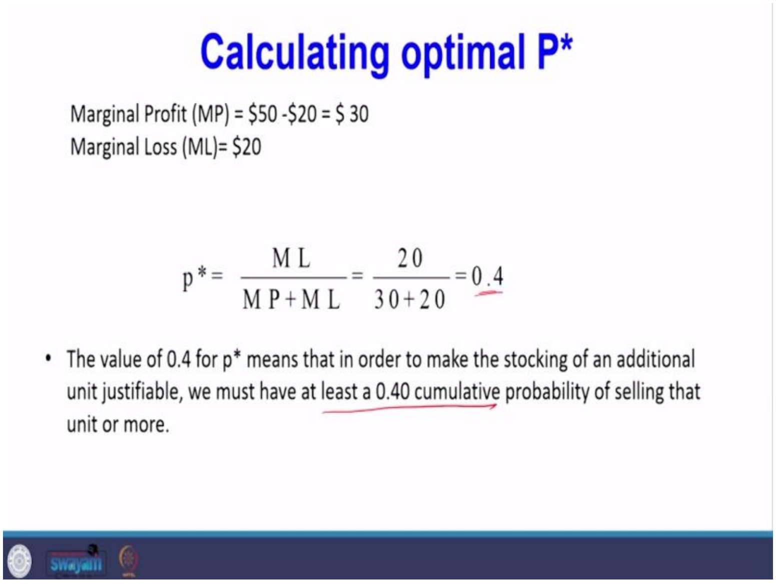
So, what we have to do for one value of this p will get a point at which the marginal expected profit is equal to the marginal expected loss. So, I have multiplied by p and marginal profit = 1 – p multiplied by marginal loss. So, when you simplify this, the left-hand side is okay, the right-hand side is marginal loss minus p marginal loss, and then you bring this marginal loss – p marginal loss to the left-hand side.

So, what will happen is that p marginal profit plus p marginal loss is equal to the marginal loss, so p you take it outside so it will become pl, so the p* is the probability at which the expected profit is equal to the expected loss. Then, we get this expression: marginal loss divided by marginal profit plus marginal loss. In some books, there is a possibility that instead of writing this way, suppose (1 – p). If we put (1 – p) marginal profit = p marginal loss, what will happen?

So, this will become marginal profit minus p marginal profit equal to p marginal loss. So, I will do again marginal profit equal to marginal p into marginal loss plus p into marginal profit. So, it will come up with marginal loss plus marginal profit so, if you want to know the p star so it will be marginal profit equal to marginal loss plus marginal profit. You should be very careful in some textbooks to follow this equation.

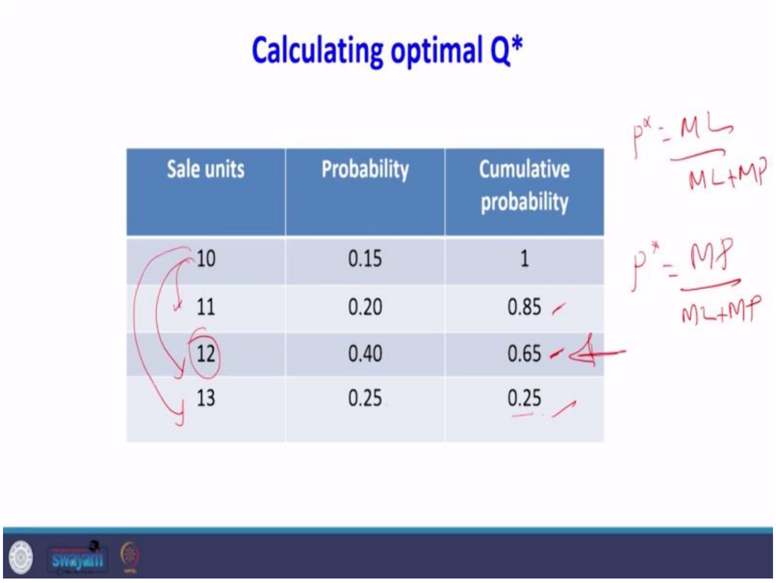
$$p\,(MP) = (1-p)\,ML$$
$$p\,(MP) = ML - pML$$
$$p\,(MP) + p\,ML = ML$$
$$p\,(MP+ML) = ML$$
$$p^* = \frac{ML}{MP+ML}$$

The numerator there is a marginal loss some books they follow this expression in numerator there is a marginal profit, but how this formula will affect our interpretation that I will explain with the help of a problem.



So, the marginal profit we know is dollars 30, and the marginal loss is dollars 20. So, we have just now derived the value for optimal probability, so that is marginal loss plus marginal profit plus marginal loss, and we are getting 0.4. Now the question is, what is the meaning of this 0.4? The value of 0.4 for p* means that in order to make the stocking of an additional unit justifiable, we must have at least a 0.40 cumulative probability of selling that unit or more. Here, there is at least a 0.4 cumulative probability of selling that unit or more.

$$p^* = \frac{ML}{MP+ML} = \frac{20}{30+20} = 0.4$$

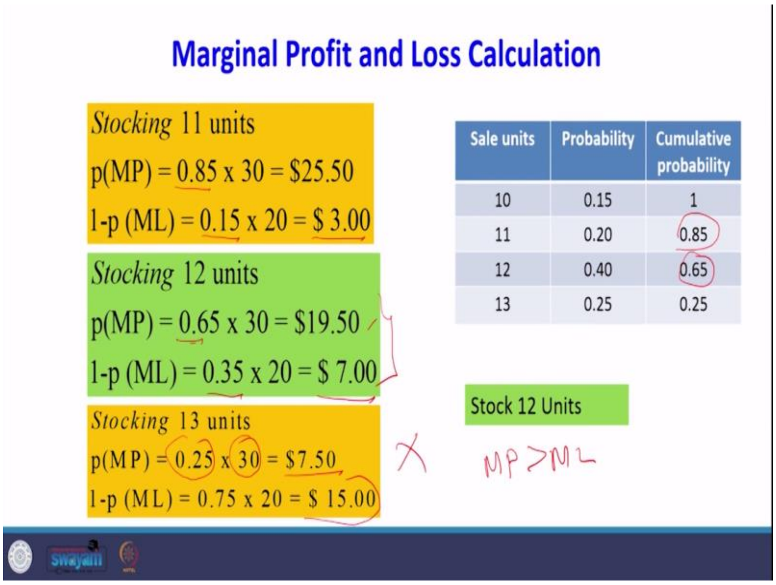


Now you see this I have found the cumulative probability from the bottom say here it is 0.25. So, 0.25 + 0.40 it is 0.65 so here 0.65 + 0.20 it is 0.85 so 0.85 + 15 is 1. The question is you may ask why do not find the cumulative probability from the bottom because see in our expression the expression which we are following the p* = marginal loss + marginal probability, but there is a reason for that.

Why I have done that I have 10 units in my hand. Suppose if I am going for 11th units there is a 85% chance that they will be sold in the market, but you see in my hand I have 10 units. Suppose if I go for 12 units (2 units more) there is only 65% chance that they will be sold. So, I have 10 units in my hand if I go for 13 units, so 3 units more, only 0.25 chance that will be sold. So, intuitively, we can say if you go for your small increment, there is a higher chance that additional units will be sold.

If your jump is very long, there is a lesser chance that it will be sold. That is why I wrote the cumulative probability from the bottom. This is applicable when we use this formula that is p* = marginal loss divided by marginal loss + marginal probability. In case we use these equations for finding the optimal p-value for finding the optimal Q value, so you have to find the cumulative probability from the top. That is the only difference, and the end result is the same.

Now, what do we have to do so what is our p* value? The p* is 0.4. So, from the bottom where is a 0.4 coming so this is a 0.25 so this is the point where we are getting the 0.4. So, this must be checked in our cumulative probability. So, as per this marginal analysis concept, we must stock the 12 units. So, the optimal stocking quantity is 12 units.

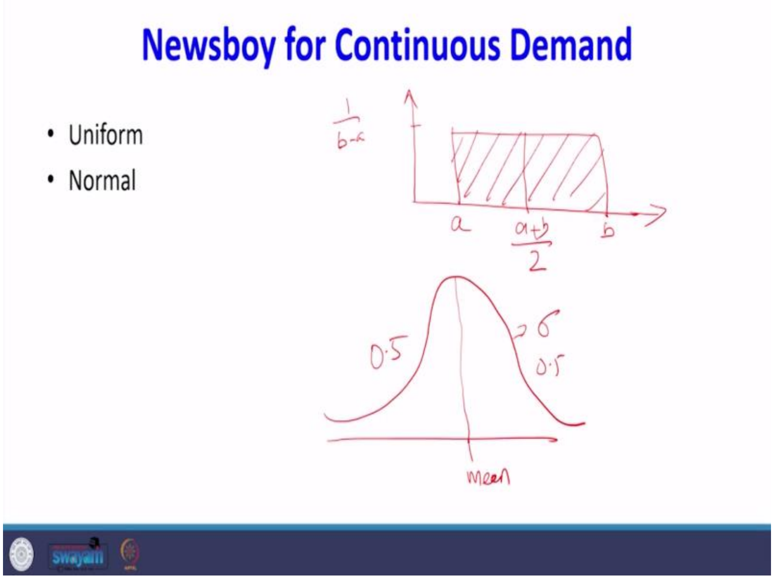


Now why it is 12 units, we will explain with the help of some numerical. So, marginal profit and marginal loss calculation. Suppose you see that I have 10 units. Suppose it is going for 11 units. What is the marginal profit? We know the marginal profit expected, so the probability is multiplied by the marginal profit. So, here, the probability is 0.85. This is 0.5; this is 0.85 multiplied by us knowing that the marginal profit is 30. So, it is dollar 25.5.

What is the expected marginal loss (1 – p) that is (1 – 0.85), so it will be 0.15. What is a marginal loss of 20? So the expected marginal loss for having one additional unit is from 10 units to 11 units. If we go for one additional unit, the expected loss is only 3 dollars. So, it is better to go for the 11th unit because the expected marginal profit is higher. Suppose if we go for two units from the 11th unit, we are going for the 12th unit.

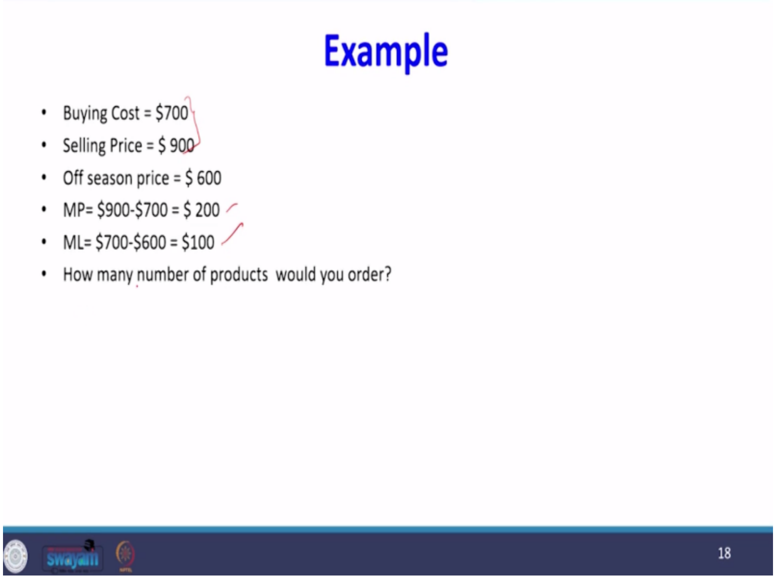
What will be the expected marginal profit 0.65 is from here 0.65 multiplied by 30, which is the marginal profit. We are getting dollar 19.5. So, what is the expected marginal loss (1 – p) (1 – 0.65)? It is 0.35, and the marginal loss is 20, so we are getting a dollar 7. Here, we have 12 units, and our expected marginal profit is larger than the expected marginal loss. Now you see the interesting things if we go for the 11th unit, so if you go for the 11th unit, the cumulative probability is 0.25, and the marginal profit is 30.

So, the expected marginal profit is only dollar 7.5, but the expected marginal loss is (1 – p) 0.75 multiplied by 20, which is dollar 15. So, here, what is happening is that the expected marginal loss is less than the expected marginal loss, so we should not go for that. So, we have to keep on increasing the additional units as long as our expected marginal profit is greater than the expected marginal loss. So, that happens if you go for only 12th units. So, that is why we got the optimal ordering quantity is 12.



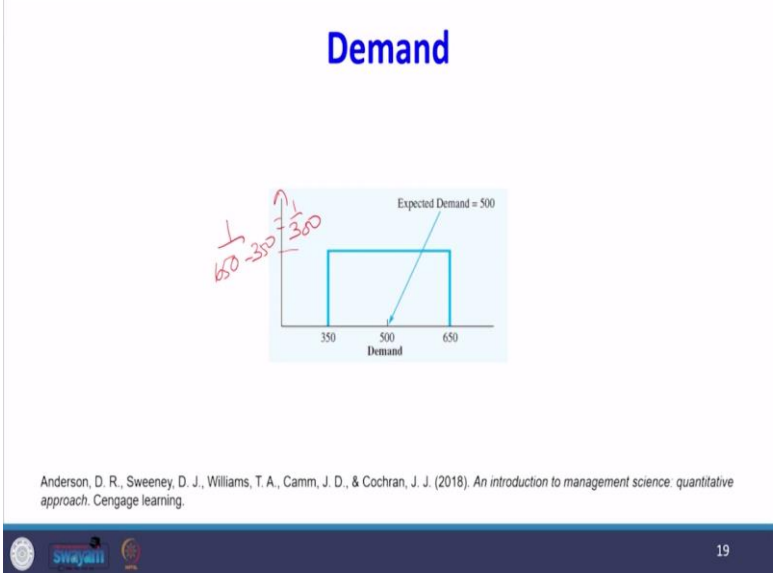
So far, we have seen an empirical distribution. Now we will use the same Newsboy problem for a continuous demand. So, we are going to consider two types of demand patterns: demand that follows a uniform distribution. What is the nature of uniform distribution? See that uniform distribution will be like this. Another name is rectangular distribution, so this is a, this is b, so this value is 1 divided by (b – a). This area is 1.

The mean of this distribution is (a + b) / 2. If the demand follows normal distribution, we know that here there will be a mean, and here will be the standard deviation. This is a shaped curve. It is symmetrical. So, the left side area is 0.5, and the right-side area is 0.5. Now, first, we will see if the demand follows uniform distribution and how to find out the optimal order quantity.



So, we have some products, see the buying cost is 700. Suppose the fashion good, say for example, a shoe or something else, the buying cost is 700, and the selling price is 900. If it is not sold in that season, the off-season price you can sell is dollar 600. So, now, if the product is not sold, the loss is not dollar 700. It is only 100 dollars. So, first, we will see what the marginal profit is; marginal profit is the selling price – buying cost, so 900 – 700. This is a dollar 200.

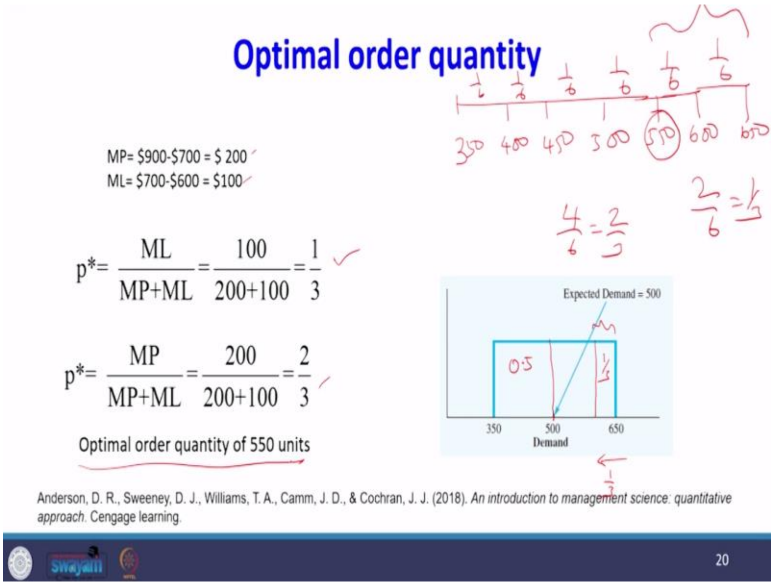
What is the marginal loss? You bought it for a dollar 700. If it is not sold in that season, it will be sold for dollar 600. Instead of 900, it will be sold at dollar 600. The loss is only dollar 100. Now the question is, how many products would you order?



Here the additional information is given the demand for that product is following uniform distribution. What is the uniform distribution saying the distribution starts from 350 to 550.

The mean is (a + b) into 500. There is an equal chance of why we are calling it a uniform distribution. So, the probability of selling 350 is 1 divided by (650 – 350) so that is equal to 1 / 300. So, the probability of selling the demand to be 350 is 1 / 300 even 351 also the same, and 352 is also the same because you see that the probability in the axis is the probability. The probability is 1/300 is fixed.

So, if the demand pattern follows this kind of uniform distribution, what should be the optimal ordering quantity?



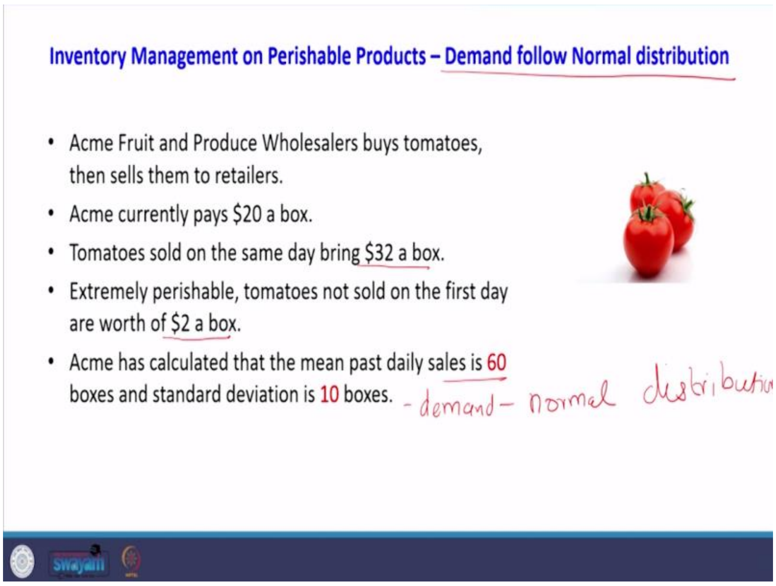
So, the marginal profit is dollar 200, marginal loss is dollar 100. First, you will find out the optimal probability. What is the meaning of this optimal probability? The probability at which your expected marginal profit is equal to the expected marginal loss. So, marginal loss is 100, marginal profit is 200 so we are getting 1/3. So, here 1/3 is your optimal probability. You remember this 1/3 we must because you see that in the empirical distribution, we have found cumulative probability from the bottom.

In the same way here, when the probability from right to left is 1/3, what should be the corresponding ordering quantity? In case we use this formula, we will be getting 2 upon 3, so here you can directly say that when the probability is 2 upon 3, what is the value of this demand that we can find out? at present, we are following this one. So, what we are doing is probability from right to left because, in the previous empirical problem, I explained that we have added I found the cumulative frequency from the bottom.

So, now I am going to find out if the probability from the right side is 1/3. For example, we know this is 1/2 because the probability is 0.5. We know the demand is 500. From the right-hand side, if it is, say, if it is 1/3, for example, here, 1/3 from right, you should be very careful. From the right, what should be the x value? So, what I am going to do is I am going to write one-third of this uniform distribution.

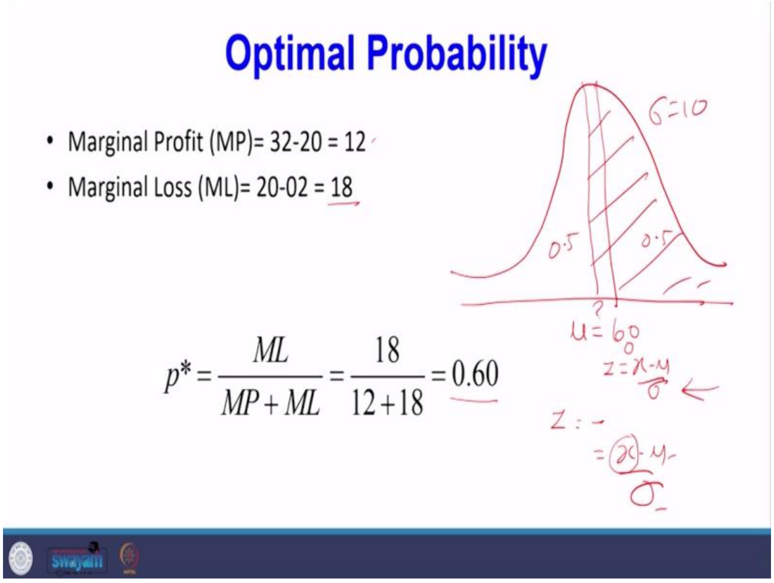
So, suppose I am writing this, say this is 350, 400, 450, 500, 550, 600, 650: 1, 2, 3, 4, 5, 6, so I have divided it into 6 equal parts. So, here this is 1/6; here, it is 1/6, here, it is 1/6, 1/6, here, 1/6, here, 1/6. So, I want to find out the probability where it is 1/3. So, if you come from this location, this is a 2/6, so 2 6 = 1/3. So, what is a 1/3? So, with the probability is 1/3, what is the quantity? See, this is 550.

So, the optimal ordering quantity is 550 units. Suppose I use this formula, what will happen here is 1/6, 2/6, 3/6, 4/6, so what is 4/6? 4/6 is 2/3. So, what is the corresponding value? It is 550, so the optimal ordering quantity is 550 units. One thing we must know from the probability we are finding random values. If the probability from right to left is 1/3 the corresponding random demand is 550 units. So, the optimal ordering quantity is 550 units.



Now, we will see another problem where the demand follows a normal distribution. So, the inventory management on perishable products, so here the demand follows normal distribution. See the Acme fruit and produce wholesalers buy tomatoes and then sell them to retailers. So, the buying price Acme currently pays dollar 20 for a box, and tomatoes sold on the same day bring dollar 32 for a box because the tomato is extremely perishable.

Tomatoes are not sold on the first day or the worth of dollar 2 of a box. They are buying for dollar 20, but if it is not sold, it is only for a dollar 2. So, this dollar 2 is called salvage value, so Acme has calculated that the mean past daily sale is 60 boxes, the standard deviation is 10 boxes, and the demand follows normal distribution. So, the demand follows normal distribution.



So, as usual, the marginal profit is we are buying for dollar 20, but we are selling for dollar 32, so 32 – 20 is a dollar 12. The marginal loss is if it is not sold on the same day, so we are buying for dollar 20. The loss of not selling on the same day is dollar 2, so the marginal loss is dollar 18. So, we can find out the optimal probability. So, it is a marginal loss divided by marginal profit + marginal loss. So, 18 divided by 12 + 18, so when you simplify, we are getting 0.6.

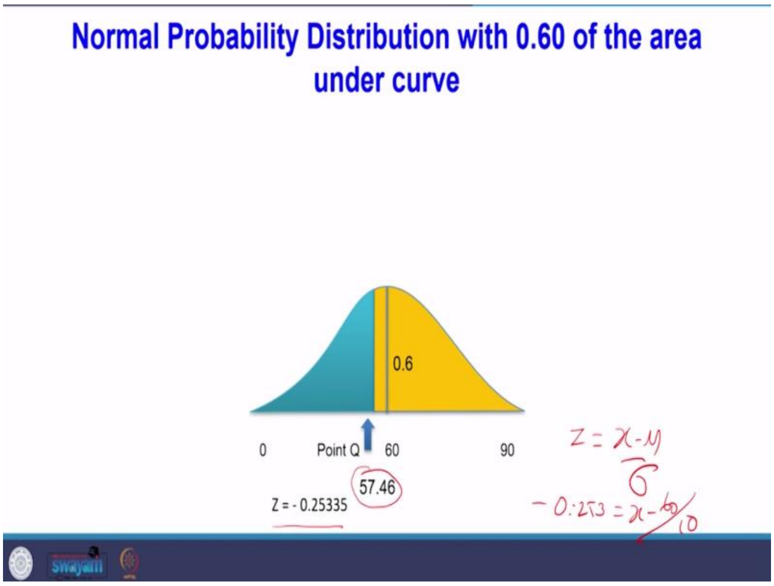
$$p^* = \frac{ML}{MP+ML} = \frac{18}{12+18} = 0.60$$

So, here what are we going to do? See the demand follow uniform distribution. Now, what is the mean of that demand? So, the mean is 60, and the standard deviation is 10. Now, if the optimal probability is 0.6 we should know what is our order and quantity. So, we know this side area is 0.5, and this side area is also 0.5. So, what we have to know from the right is that you should be very careful because you used the numerator, and we used the marginal loss.

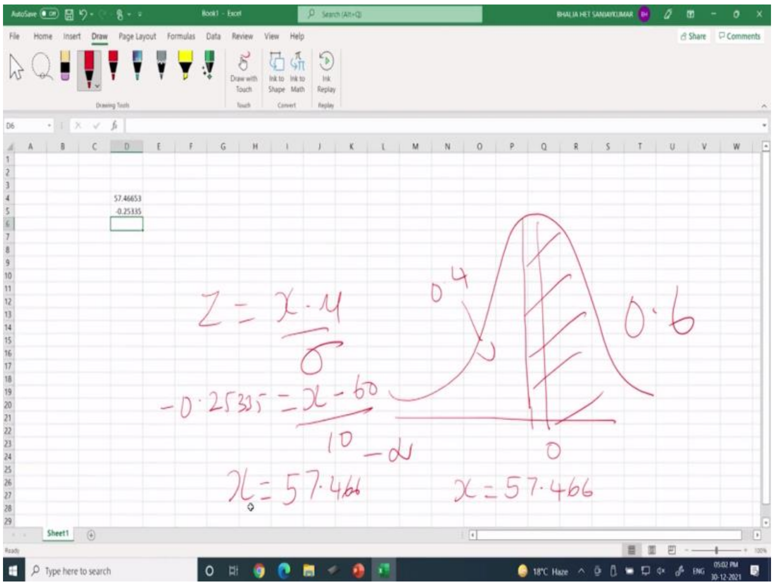
From the right, we must find out, for example, area 0.6 may be here. Suppose this is a 0.6 area. We should know what our order and quantity are. So, now the area of a normal distribution is given from right, it is 0.6, then we should know what this order and quantity is. There are two ways we can do that; one is we can use your statistical table if it is a statistical area, so if the area is 0.6, you should first get the z value.

And you see that if we are using the standard normal distribution, the value of z will be negative. you have to attach a minus sign because when you convert mu, we have to use this formula (x – u)/sigma because 60 -60, so this will be 0. So, the value of z will be negative, so it will be whatever. So first, we have to find out when the area is 0.6. We will get the z value. You have to attach a negative sign there.

So, we know that from that value we will get (x – u)/sigma is given sigma is given you have to find out the x value, but we are not going to use a statistical table we are going to use Excel to find out the area of a normal distribution not area of normal distribution if the area is 0.6 what is the corresponding x value.



So, the x value is 57.46. Now I am going to open my Excel, so with the help of Excel, I am going to tell you how to find out the area.

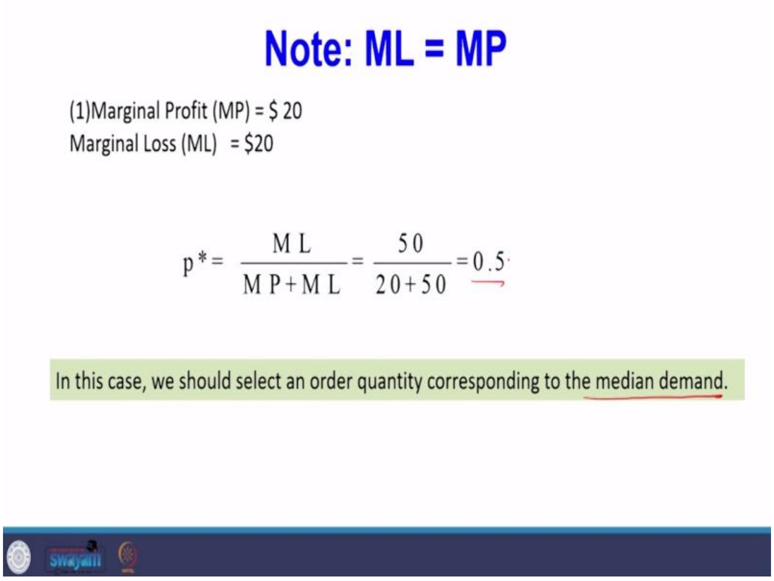


So, we know that the area that we are getting from Excel is the mean value, so we will be getting minus infinity to positive z value if it is a standardized normal distribution. So, now we must know if the area is 0.6. If this side area is 0.6, I should know what the corresponding x value is because Excel is given from the left side area. So, if the area is 0.4 and the left side area is 0.4, I should know what the corresponding x value is.

So, for that purpose I have to use a function equal to norm.inv, you see the probability is 0.4, the mean of the distribution is 60, and the standard deviation of the distribution is 10. You see that the quantity is 57, so we are getting a direct x value. The x value is 57.466. Instead of this, suppose we use a standardized normal distribution equal to the norm.s.inv,. Please see that; that is a standardized normal distribution.

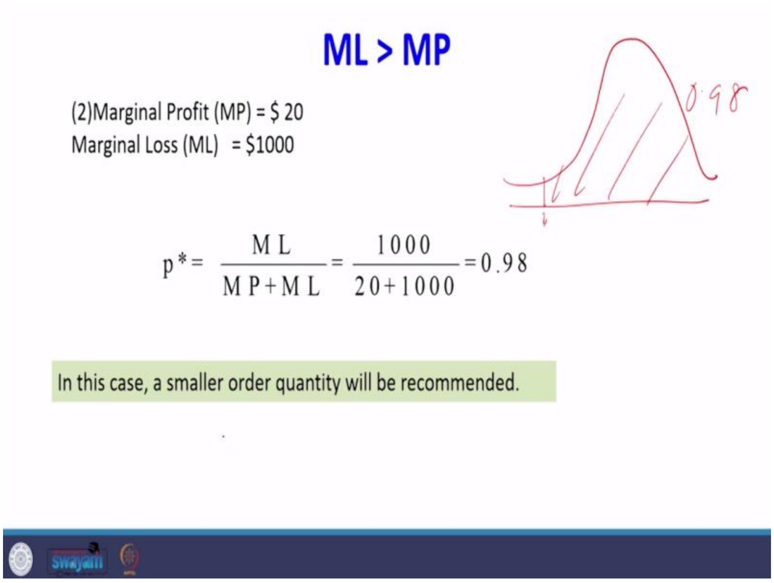
What is the probability of 0.4? My z-value is negative. We know that the z value is (x – u) / sigma, and my z value is – 0.25335. So, I know that the standard deviation is 10. From this, I can find out the x value. So, what is the x value – 2 points we are getting 57.4666, so this is the order and quantity. Now, I will go back to the presentation. if the area is 0.6 from the right-hand side, then what is the Q value?

So, the Q value is 57.46, so if we are using a standardized normal distribution, your z value is from this one. From this z value, you have to find out the z value is 0.253; (x – 60) upon 10. So, we will get the way we will get the same answer.



Now, some of the important notes you should remember are what will happen if the marginal loss is equal to the marginal profit. So, assume that now the marginal profit is 20, and the marginal loss is also 20. So, when you substitute here, you see that the optimal probability is 0.5, so what is the meaning of this 0.5? In this case, we should select and order quantity corresponding to the median demand because 0.5 divides the whole population into two parts.

So, when it is 0.5, the corresponding value is nothing but the median of the demand. This is the case 1.



Sometimes what happens is that marginal profit is greater than marginal loss. So, if I have that additional unit, suppose we will be getting more profit than the marginal loss. What will happen if I kept the marginal profit very high, say 1,000 units dollars and the marginal loss is only dollar 20? So, when I substitute here, what I am getting is that the p star is very low, 0.01. So, what is the meaning of 0.01? Assume this follows a normal distribution, for example.

So, from the right-side area, if it is 0.0196, I will get some Q value that Q value is a very high value. So, what will happen if the marginal profit is higher in this case, a larger order quantity will be recommended. So, what will happen if the reverse of this so the marginal loss is greater than the marginal profit? You see, the marginal profit is only dollar 20; I have reversed the marginal loss is 30. Now the optimal probability is 0.98. So, what is the meaning of 0.98?

So, from the right-hand side, it is 0.98, so this side area is 0.98. Obviously, we will be ordering a very small quantity. So, in this case a smaller ordering quantity will be recommended. Dear students, in this lecture, I have explained the famous problem called the Newsboy problem. The Newsboy problem is a single-period inventory model with probabilistic demand. We have seen three types of demand patterns. One is an empirical, uniform, and normal distribution.

By considering these three types of inventory patterns, we have suggested the optimal ordering quantity. In the next lecture, we will be discussing the multi-period inventory model with probabilistic demand. Thank you very much.