Decision Making with Spreadsheet
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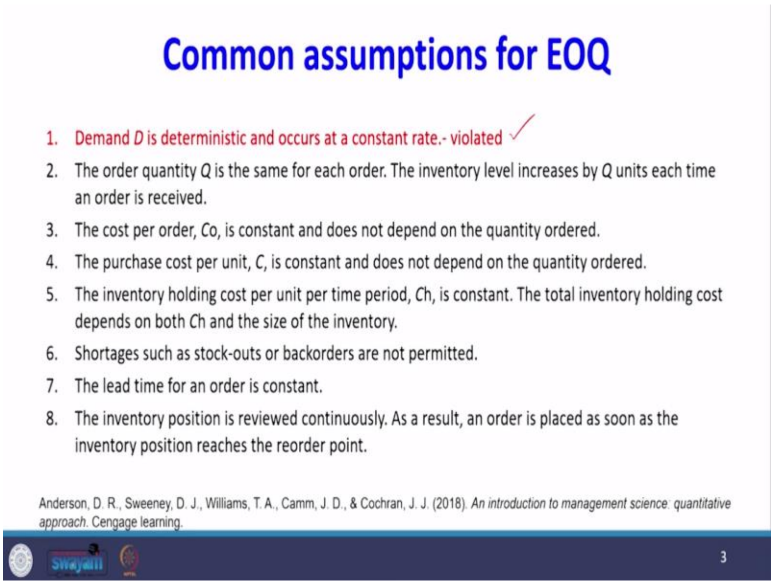
Lecture – 39
Multi-Period Order-Quality Reorder Point Model with Probabilistic Demand

Dear students. In the previous lecture, I discussed the Newsboy problem. For the Newsboy problem, we have considered only one period of the cycle inventory cycle. In this lecture, we are going to consider multi-period order quantity and reorder point model with probabilistic demand. So, the difference is in the previous lecture, we considered only one period. Now, we are going to consider multiple periods, and here, the demand is going to be probabilistic.

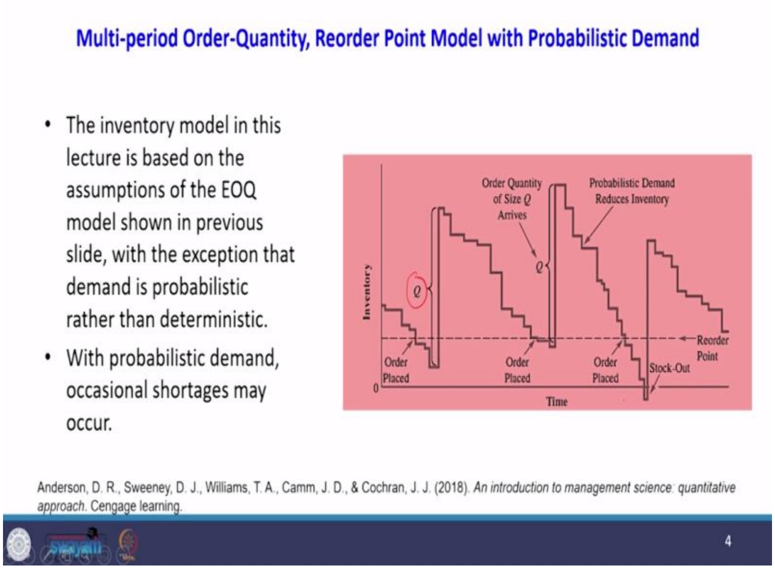
So, the agenda for this lecture is multi-period order quantity and a reorder point model with probabilistic demand.



You remember, at the beginning of this topic on inventory management, I have shown you what the assumptions we have followed are. So, this is for the traditional EOQ model; what are the assumptions? Demand D is deterministic and occurs at a constant rate. So, this assumption is violated by this model, which I am going to discuss now here. The demand is not deterministic, and it is a multi-period like that. There are 7 more assumptions. All other assumptions are valid. For example, if the order quantity Q is the same for each order, that is okay.

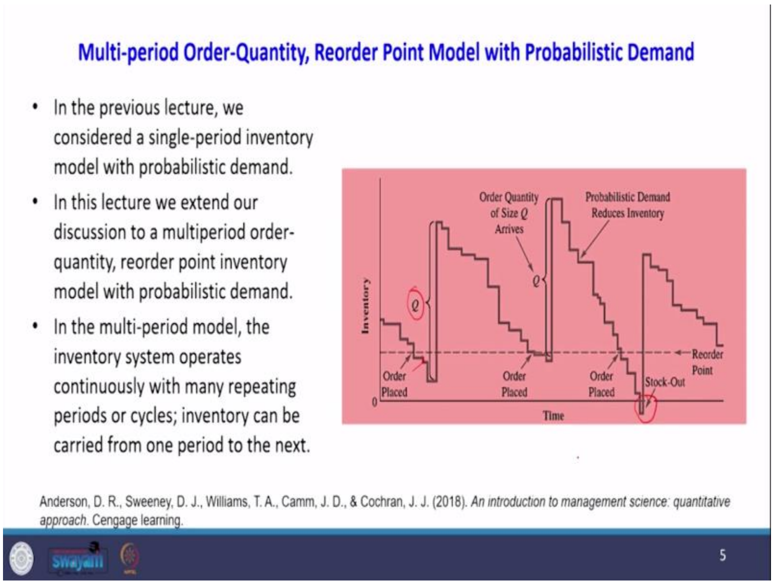
The inventory level increases by Q unit each time an order is received; the cost per order is constant and does not depend on the quantity ordered. The purchase cost per unit C is constant and does not depend on the quantity ordered. The inventory holding cost per unit time period Ch is constant. The total inventory holding cost depends on both the holding cost and the size of the inventory.

Shortages such as stock out and backorders are not permitted. The lead time for an order is constant; the inventory position is reviewed continuously. As a result, an order is placed as soon as the inventory position reaches the reorder points. These are the eight traditional EOQ models. These are the eight assumptions. The model in which I am going to discuss the first assumption is that demand is deterministic and occurs at a constant rate when that assumption is violated.



The inventory model in this lecture is based on the assumptions of the EOQ model shown in the previous slide with the exception that the demand is probabilistic rather than deterministic. Look at this x-axis. There is a time. In the y-axis, it is inventory, and there is a reorder point where you see the demand is not constant. It is probabilistic. As soon as the Q inventory is over again, we are going to order, but when the order comes, the order replenishes instantaneously. The Q is the order and quantity.

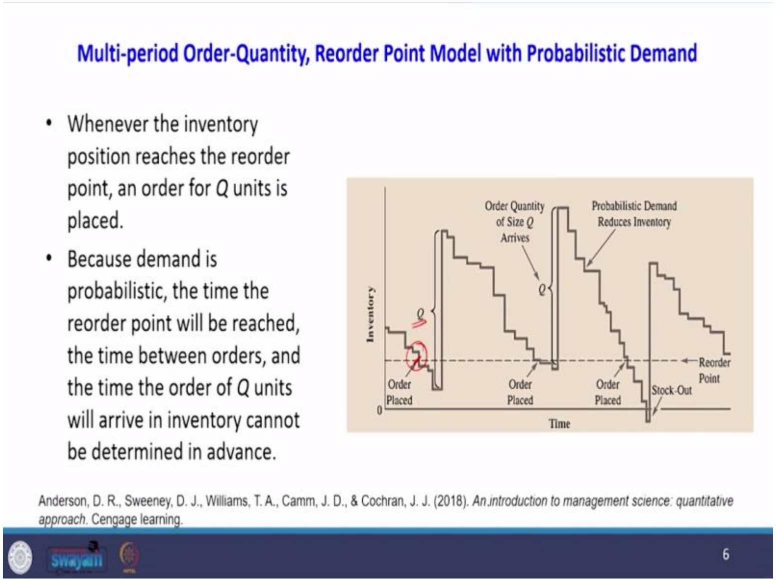
This is our reorder point. Sometimes, there is a chance that since the demand is probabilistic, there is a chance there will be a stock out. So, we are going to consider this problem in this lecture.



In the previous lecture, we considered a single-period inventory model with the probabilistic demand, which is what we have done in the Newsboy problem. Still, in this lecture, we extend our discussion to a multi-period order quantity reorder point inventory model with the probabilistic demand. In the previous lecture, we did not discuss the reorder point; in the previous lecture, we considered only one period. Still, we are going to consider multiple periods and the reorder point.

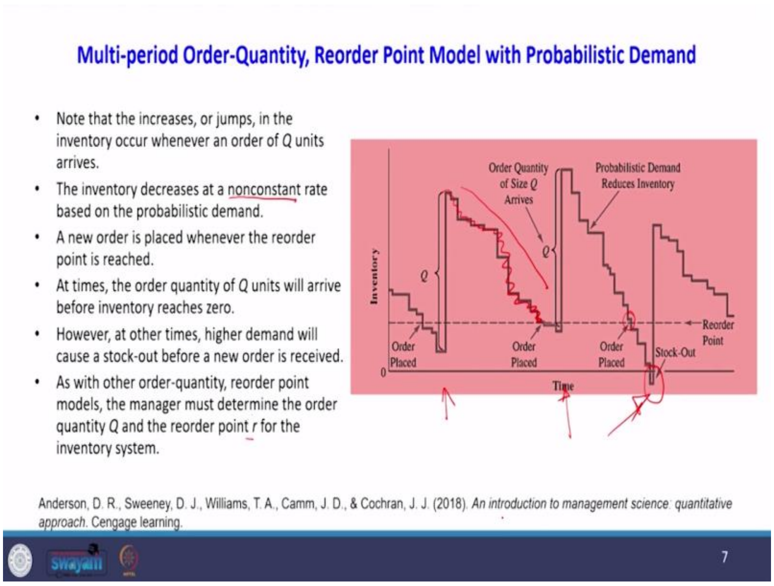
Sometimes, there may also be a stock out, and the demand is probabilistic. So, in the multi-period model, the inventory system operates continuously with many repeated periods or cycles. So, inventory can be carried from one period to the next one. The Newsboy problem was not the assumption that inventory could not be carried out. So, here, the assumption is that what should be the Q when you have to make the order without minimizing the stock out?

And one important point I wanted to make is that since the demand is probabilistic, there is a chance for stockouts. Here is what we are going to do: We are going to order quantity Q so that the number of stocks out is minimized. That is the basic idea behind this model.



So, whenever the inventory position reaches the reorder point, for example, here, whenever the inventory position reaches the reorder point, an order of Q units is placed. So, we are making this Q unit because the demand is probabilistic. The time the reorder point will be reached, the time between orders, and the time order of Q units will arrive in inventory cannot be determined in advance.

So, since the demand is probabilistic, it is very difficult to say the time the reorder point will be reached, the time between the orders, and the time the order Q units will arrive in inventory, so this cannot be clearly predicted in advance because the demand is probabilistic, uncertain.

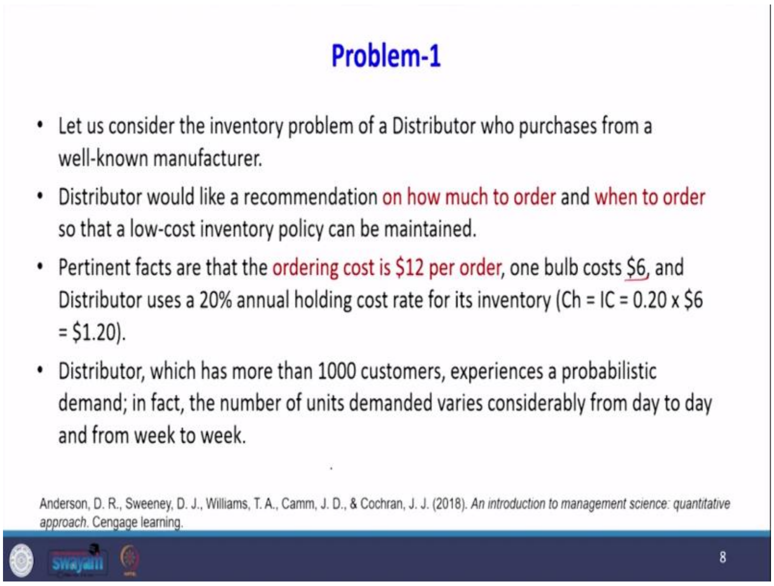


Note that the increase and jumps in the inventory occur whenever an order of Q units arrives. You see that. Suppose we arrive at this: this is not the increase. There is a jump. Previously, it was a straight line like this, but now there is a jump. So, the inventory decreases at a nonconstant rate based on the probabilistic demand. You remember previously that the assumption for demand decreases at a constant rate.

Now the demand decreases, and inventory decreases at a nonconstant rate. A new order is placed whenever the reorder point is reached; for example, whenever the inventory reaches this point, we make another order, so that point is a reorder point. At times the order quantity of Q units will arrive before inventory reaches 0. So, what will happen is this Q unit may arrive before it reaches the zero you see here. Before it reaches, we have started to order reaches the 0.

However, at other times, a higher demand will cause a stock out before a new order is received. See this situation where there is a chance for stock out. You see here, for example, this category, so at this point, you see that even the inventory before it reaches, we are making the order. Here, also see this point: before the inventory reaches 0, we are making the order, but you see the last one, there is a point that we may end up with the stock out also.

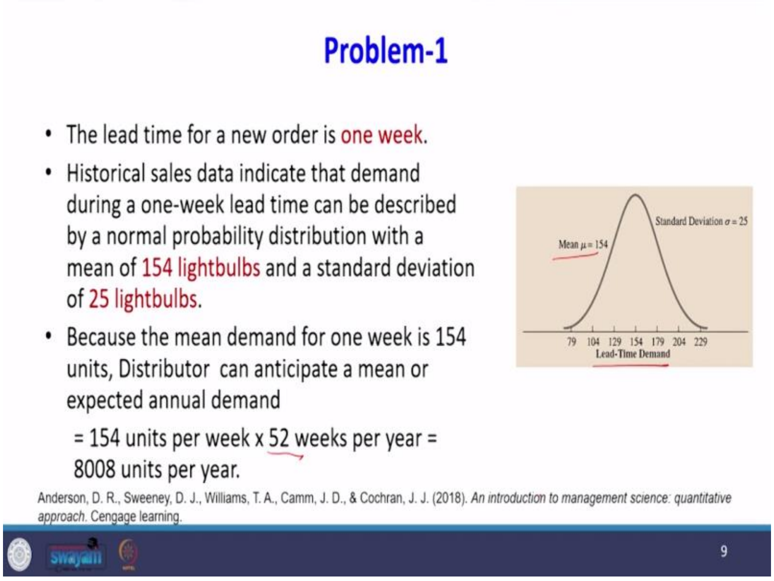
As with other order quantity reorder point models, the manager must determine the order quantity Q, which is the order point for the inventory system. So, here, we also have to specify the reorder point. Remember, in the Newsboy problem, we do not discuss the reorder point because this model is multi-period, so here we need to specify the reorder point for the probabilistic demand.



So, I will explain the concept with the problem. The reference for this problem is the Anderson et al. book. Let us consider the inventory problem of a distributor who purchases from a well-known manufacture there is a distributor he buys from the manufacturer. The distributor would like a recommendation on how much to order and when to order so that a low-cost inventory policy can be maintained.

So, the distributor is placing orders for the manufacturer, so we have to suggest how much to order and when to order. The pertinent fact is that the ordering cost is dollar 12 per order, and he is purchasing the light bulb. This is z distributor, so one bulb costs dollar 6, and the distributor uses 20 percent of the annual holding cost rate for its inventory. So, what will be the annual holding cost, the percentage interest percentage, say 20, multiply the unit cost of 6, so this will be a dollar. 1.2.

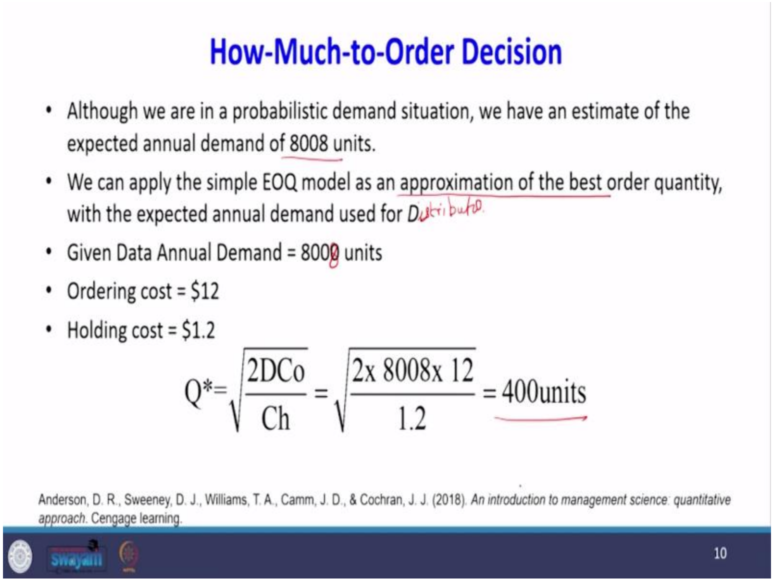
So, the distributor, which has more than 1,000 customers, experiences a probabilistic demand. In fact, the number of units demanded varies considerably from day to day and from week to week.



The lead time for a new order is one week. So, historical sales data indicate that the demand during one week lead time can be described by a normal probability distribution with a mean of, say, 155 here. The mean is 144 bulbs, and the standard deviation is 25. So, this was the lead time demand because what is the lead time demand? The lead time demand follows a normal distribution whose mean is 154 and standard deviation is 25 because the mean demand for one week is 154, which is lead and demand of 154 units. The distributor can anticipate a mean or expected annual demand.

How much? So, there are 52 weeks per annum, and one week's demand is 154, so the annual demand is 8,008. In some other books for multi-period probabilistic models, there are very detailed mathematical models presented, but in this lecture, we are going to do a kind of approximate method. So, this approximation method will provide you with a better result because we have already studied that if we do the sensitivity analysis, the order quantity will not greatly affect the total cost.

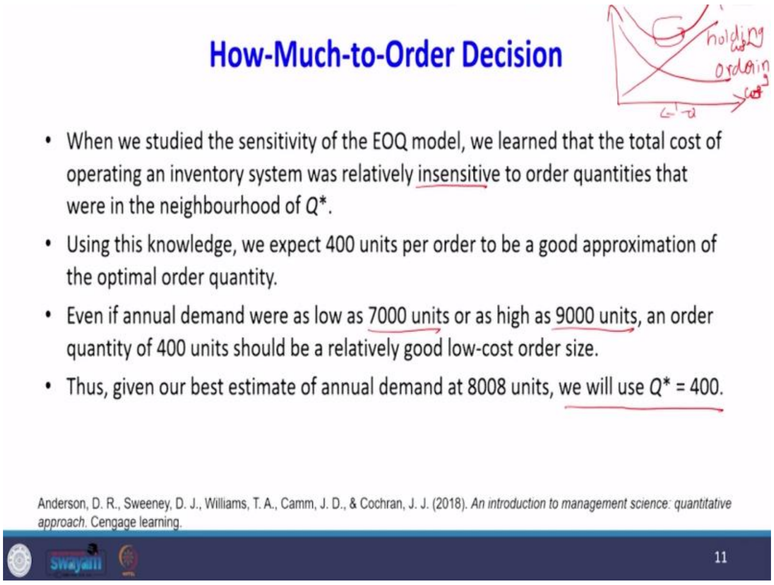
So, now, based on what we got from the lead time demand, we have found that the annual demand is 8,008 units per year.



Now we have to decide how much to order, so although we are in a probabilistic demand situation, we have an estimate of the expected annual demand of 8,008 units. How did we get these 8,008 units? We know the weekly demand during the lead time that we have multiplied by 52, so we now have annual demand. Now, we can apply the simple EOQ model as an approximation of the best order quantity.

So, the model that we are discussing is an approximation of the best order quantity with the expected annual demand used for the D used for this distributor. So, what is the data given for the annual demand? So, the annual demand is 8,008, which is 8,008. The ordering cost is 12, and the holding cost is 1.2. So, we know our traditional formula 2DCo/Ch, Co is the ordering cost, so 12Ch is the holding cost. We know the demand, so it is 400 units. So, for a multi-period probabilistic model, the optimal order quantity is 400 units.

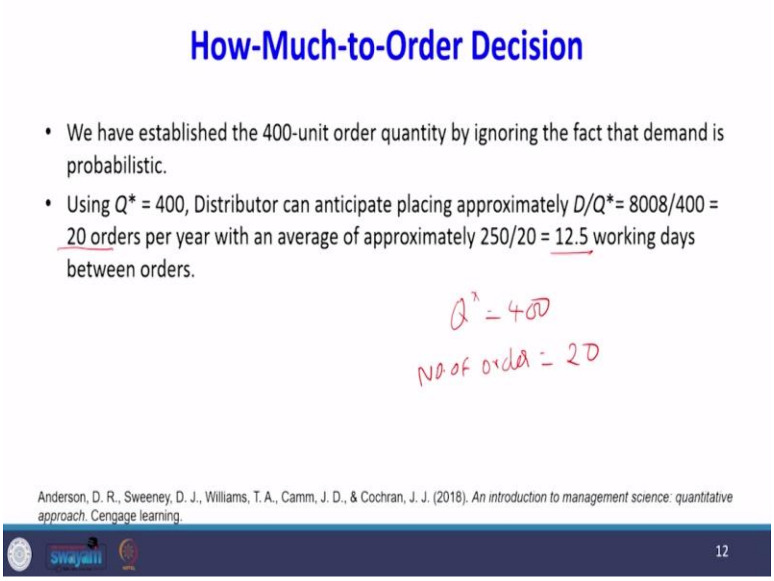
$$Q^{*}=\sqrt{\frac{2DCo}{Ch}}=\sqrt{\frac{2x\ 8008x\ 12}{1.2}}=400\text{units}$$



You remember when we studied the sensitivity analysis of the EOQ model, we learned that the total cost of operating an inventory system was relatively insensitive to the order quantities in the Q star neighborhood. Remember, when we are doing sensitivity analysis, this will be your holding cost, your ordering cost, and your total cost.

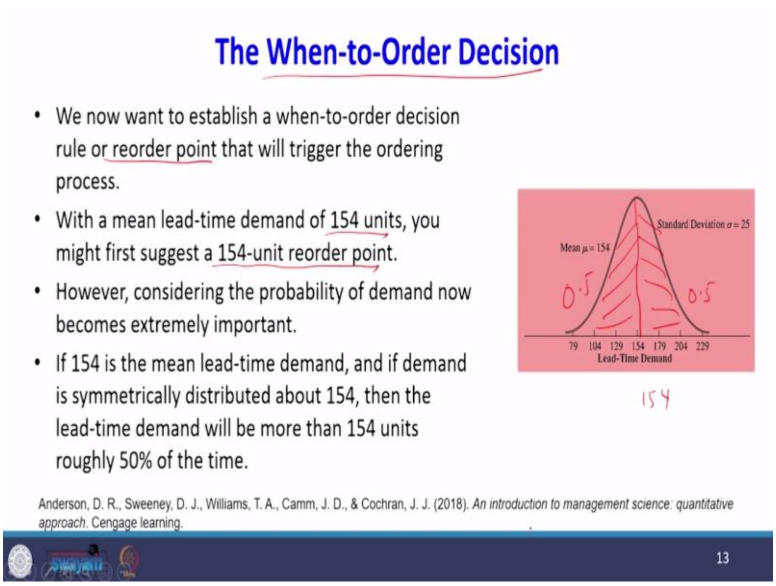
So, the total cost at a particular point is flat. So, this flat indicates that even though your Q is increased by or decreased by some quantity, the net effect on the total cost is the total cost, which is your holding cost; this is your ordering cost. So, what will happen even though there is a slight change in the ordering quantity so the total annual cost will not be changed?

So, using this concept of sensitivity analysis, we expect 400 units per order to be a good approximation of the optimal order quantity. Even if the annual demand was as low as 7,000 units or as high as 9,000 units, an order quantity of 400 units should be a relatively good low-cost order size. Thus, given our best estimate of annual demand at 8,008 units, we will use the optimal order quantity of 400 units. Now we have the answer for how much to order a decision.



So, we have established that 400 units order quantity by ignoring the fact that the demand is probabilistic. Using Q* optimal order quantity = 400 distributors can anticipate placing approximately how many orders? D/Q. So, D is 8,008, and Q* is 400, so he is making 20 orders per year, so we got the number of orders. So, we got Q* 400, we got a number of orders, which was 20, and assumed that the industry's distributor operates 250 days per year.

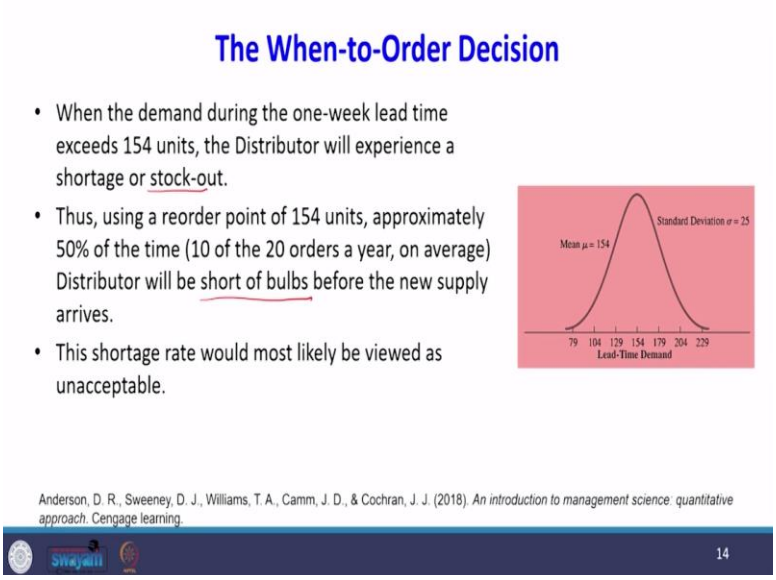
So, when you divide these 250 days by 20, that is the number of orders, so the 12.5 working days between the orders. So, the order cycle is 12,5 working days. What is the meaning of that? Every 12.5 working days, he has to make an order. How much to order? He has to order 400 units.



Now we have the answer for how much to order. The next point is when to order a decision. So, when to make a decision, we need to know the reorder point. We know we now want to establish when to order the decision rule or reorder point that will trigger the ordering processes. With a mean lead time demand of 154 units, you might first suggest a 154 units reorder point because the lead time demand is 154, which is the average lead time demand.

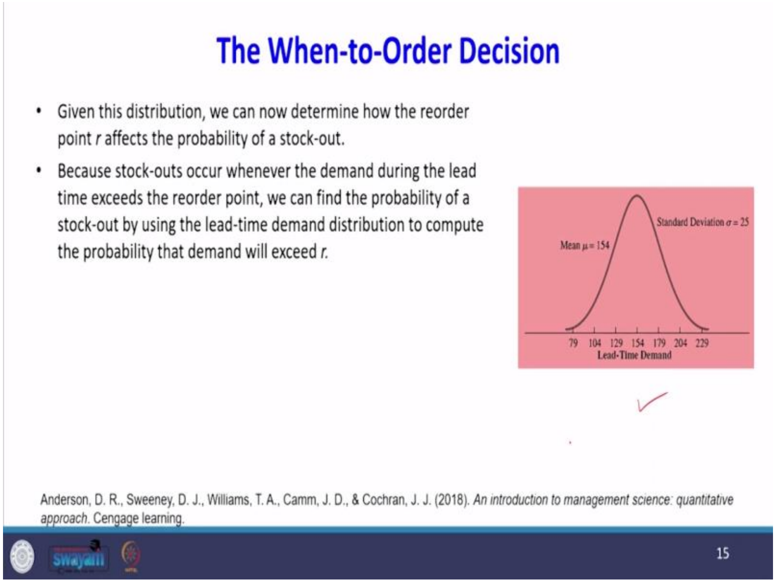
You may think that you can order 154 units; however, considering the probability of demand now becomes extremely important because the probability of demand changing, there is a chance that the demand may increase by more than 154 units during the lead time. So, if 154 is the mean lead time demand and if the demand is symmetrically distributed about 154, then the lead time demand will be more than 154 minutes, roughly 50 percent of the time.

So, we know that this is a mean of 154, so this side probability is 0.5, which means that for weekly demand, 50% of the time, the demand will be less than the mean. So, the remaining 50 percent of the time, the demand can be more than 154. Suppose you order 154 units, so 50 percent of the time, the demand may increase. So, what will happen is you will end up with the stock out.



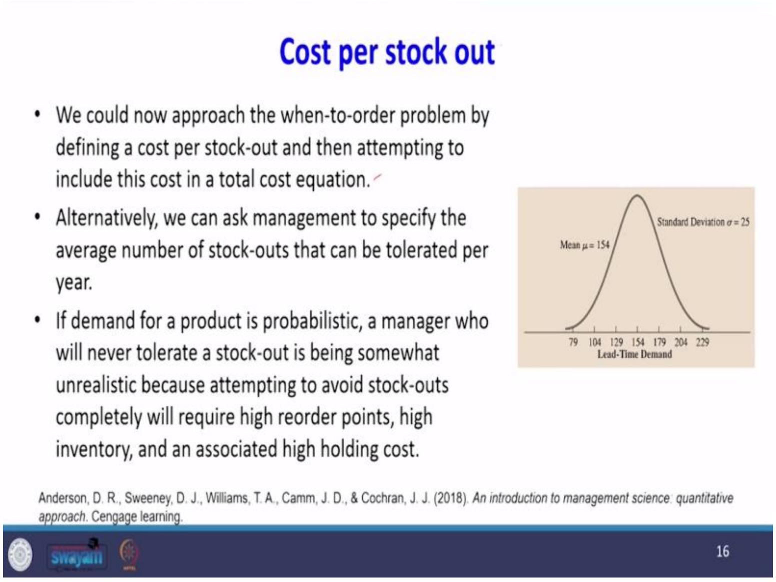
So, when the demand during the one-week lead time exceeds 154 units, the distributor will experience a shortage or stock out. Thus, using a reorder point of 154 units, approximately 50 percent of the time, that is 10 of the 20 orders a year, on average, the distributor will be short of bulbs before a new supply arrives. So, we should not go for 154 and see 154; 50 percent of the time out of 20 orders, so 10 orders 10 times you will come out with a shortage.

So, this shortage rate would most likely be viewed as unacceptable. So, we should not order 154.



So, given this distribution, we can now determine how the reorder point r affects the probability of stock out because the stock out occurs whenever the demand during lead time exceeds the reorder points, we can find the probability of stock out using the lead time demand distribution to compute the probability that the demand will exceed the reorder point. So, we have that reorder point demand, which is 154, and the standard deviation is 25.

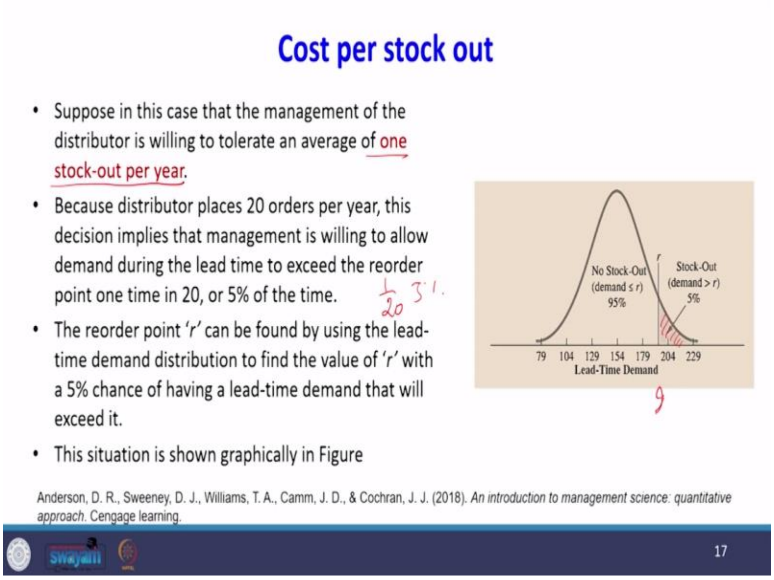
Now, by knowing this information, we can find out the probability of stock; otherwise, the probability that the demand will exceed the r, r is the reorder point.



Now we are going to find out if there is a stock out and what is the cost per stock out. We could now approach the when-to-order problem by defining the cost per stock out and then attempting to include this cost in a total cost equation. So, we know that suppose if the order is 150, the demand goes beyond the reorder point, there will be a stock out. We can assign some cost for each stock out like this we can find out the annual stock out cost.

So, alternatively, we can ask the management to specify the average number of stockouts that can be tolerated per year. So, we can ask how many stocks the company can tolerate. If a demand for a product is probabilistic, a manager who will never tolerate a stock out is somewhat unrealistic because attempting to avoid stock out completely will require a high reorder point, high inventory, and associated high holding cost.

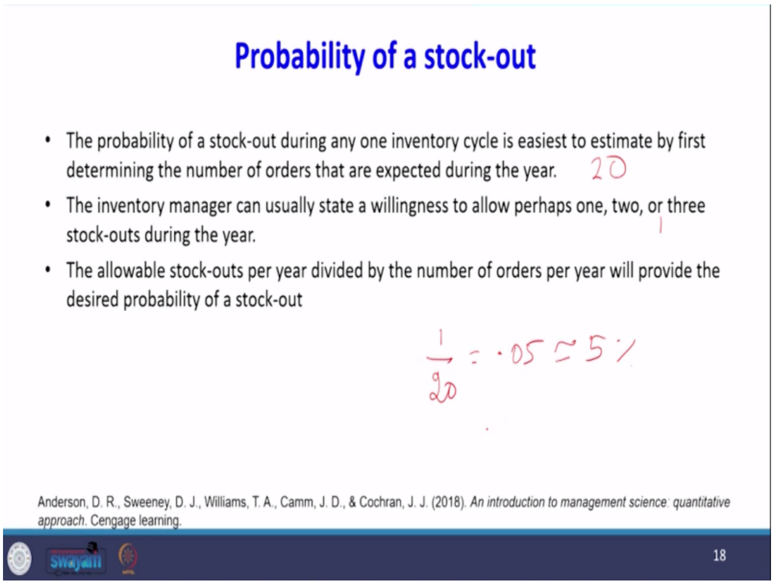
Since the nature of the demand is probabilistic, the distributor can tolerate some stock outs. So, he cannot say that I do not want any stock-out situation because going for not having any stock-out situation is a highly costly decision because we need to have more inventory. There will be more holding costs, so that is not good. So, what I am trying to say is that since the demand is probabilistic in nature, the buyer and the manager have to tolerate sudden stock out.



Suppose, in this case, the management of the distributor is willing to tolerate, on average, one stock out per year. You remember that we are making 20 orders because the distributor places 20 orders per year this decision implies that the management is willing to allow demand during the lead time to exceed the reorder point one time in 20. So, 1 upon 20 times, they are allowing for stock out, which means the demand the lead time demand exceeds the reorder point you take.

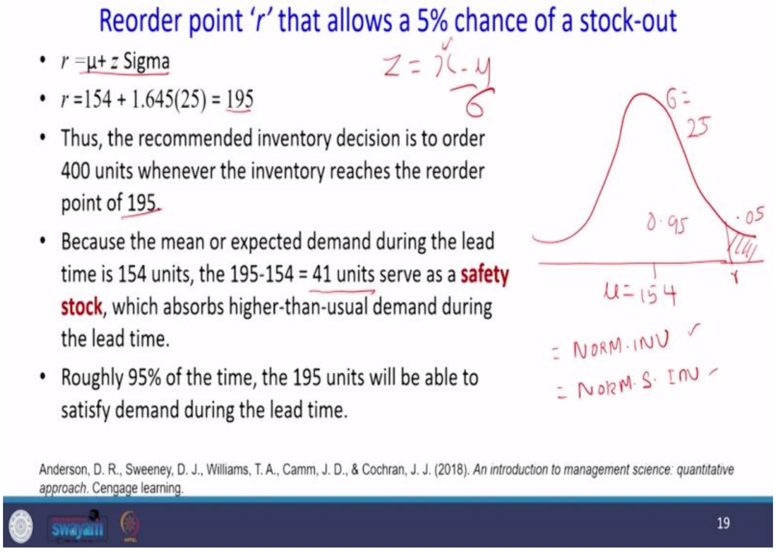
The reorder point r can be found by using the lead time demand distribution to find the value of the reorder point with a 5 percent. When I say 1/20 it is 5 percent. So, 5 percent of the time, they are willing to tolerate the stock out. So, that means the lead time demand distribution of the reorder r can be found by using the lead time demand distribution to find the value of the reorder point with a 5-percent chance of having lead time demand that will exceed.

Do you see that? Look at this site, this picture, so 5 percent of the time, the demand can exceed the reorder point. Whenever the demand exceeds the reorder point, there is a stock out. So, the company is willing to wait 5 percent of the time they are willing to accept the stockouts. So, look at this situation, which is shown graphically. Now, we should know this r value.



How to calculate the probability of stock out. So, the probability of stock out during any one inventory cycle is easiest to estimate by first determining the number of orders that are expected during the year. For example, here we have found 20 orders the inventory manager can usually stay willing to allow perhaps 1, 2, or 3 stock outs during the year. In our problem if the manager is willing to tolerate one stock out.

So, the allowable stock out per year that is allowable stock out per year divided by the number of orders, say 20, equal to will provide the desired probability of stock out. So, 1 / 20 is 0.05, which is 5 percent.

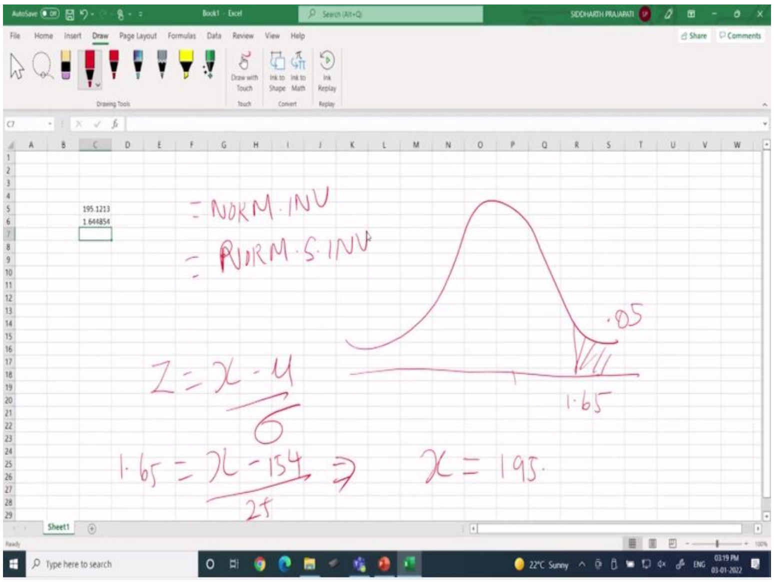


Now we will find out the reorder point. So, what we know is that this side area is 0.05, and 5% is stock-outs. So, what is this side? The remaining is 0.05. So, now we must find out this reorder point and what the mean of this distribution mean is 154. What is the standard deviation of this distribution? 25. . Now we have to find out what is the r point.

$r = \mu + z$ Sigma

$r = 154 + 1.645(25) = 195$

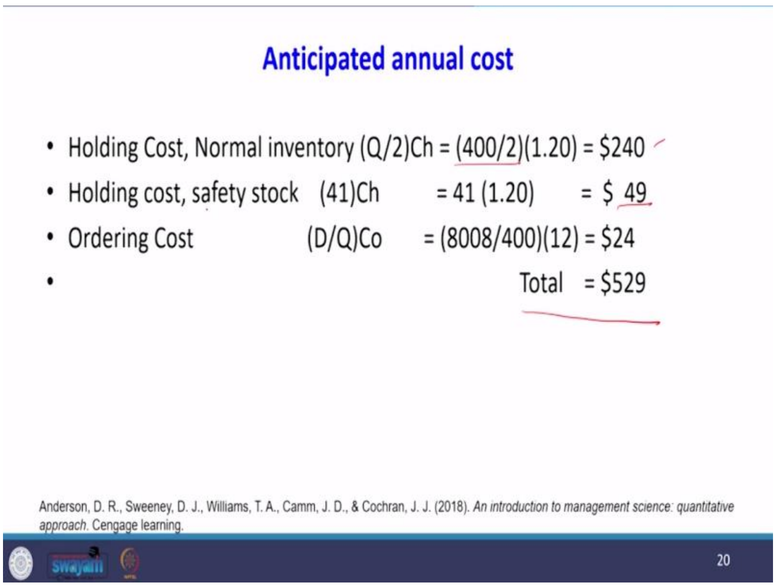
This r we are going to do with the help of excel. We know that in Excel, there is a function, so there are two functions equal to NORM.INV is there otherwise NORM.S.INV. So, S means standardized normal distribution. So, INV means for any distribution so we can use any formula to find out what is the reorder point the value for the reorder point. So, now I will open I will excel I will explain how to find out the reorder point.



So, equal to NORM.INV, so you see, because the right side area is 5 percent, the left side area is 0.95, the mean is 154, and the standard deviation is 25. You see that the reorder point is 195. There is another formula instead of using this formula, NORM.S.INV probability is 0.95. So, the value of Z is 1.65. You see that, so we got the Z value of 1.65.

Since the value of Z is positive on the right-hand side, so 1.65x, we do not know if the mu is 154 and the sigma is 25. So, from this expression, we can find out the value of x, so the value of x will be 195. So, both ways, we can find out the reorder point. What are both way there are two formulas in Excel? One is NORM.INV otherwise NORM.S.INV. Now I will go back to the presentation. So, the recommended inventory decision is to order 400 units whenever the inventory reaches the reorder point of 195.

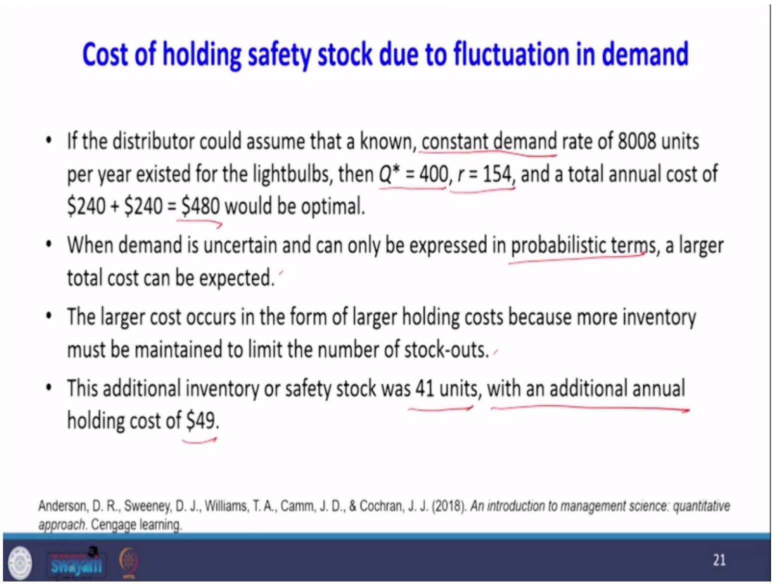
So, 195 – 154 these 41 units serve as a safety stock that absorbs higher than usual demand during the lead time roughly 95 percent of the time the 195 units will be able to satisfy the demand during the lead time that is the meaning of this side area 0.95 only 5 percentage of time we may not able to satisfy the demand, but that is also accepted by the companies or managers.



So, now we can find out the anticipated annual cost. The first cost is holding cost for normal inventory, which is the traditional formula of average inventory multiplied by holding cost. The average inventory is 400/2 multiplied by holding cost, there is a dollar 240. Now, the safety stock, you remember that apart from 154, we have 41 units additional units. So, that stock also has to pay the holding cost.

So, 41 multiplied by 1.2 is dollar49. What is the ordering cost number of orders multiplied by the cost of ordering? So, a number of orders is 8,800 / 400 multiplied by 20, which is the ordering cost. So, the total annual cost is 529 only additional cost we are incurring due to probabilistic demand is this safety stock. So, the safety stock is how much dollar 49.

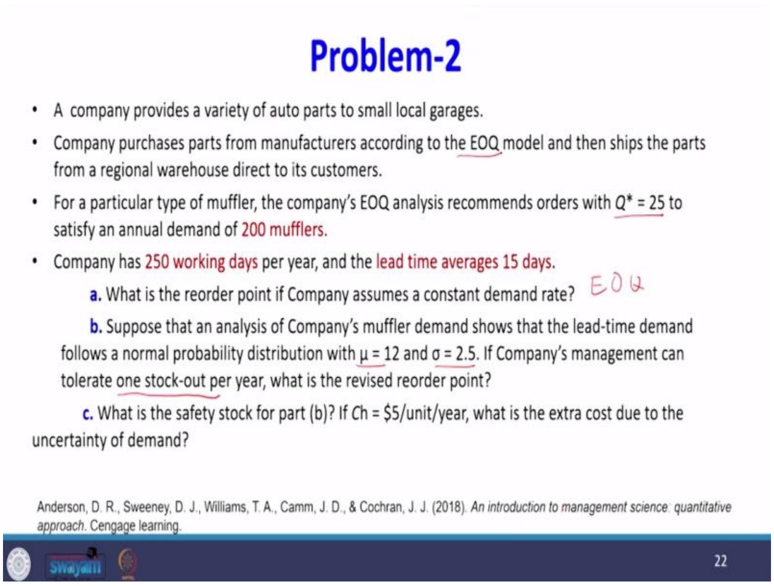
Holding Cost, Normal inventory	(Q/2)Ch	= (400/2)(1.20)	= $240
Holding cost, safety stock	(41)Ch	= 41 (1.20)	= $ 49
Ordering Cost	(D/Q)Co	= (8008/400)(12)	= $24
		Total	= $529



If the distributor could assume that a known constant demand rate of 8,000 units per year existed for the light bulb product. The optimal order quantity will be 400, the reorder point will be 154, and the total annual cost will be 240 + 240, only 480. This will happen when the demand is constant demand, but that is not the case here. Here the case is the demand is probabilistic in nature.

When the demand is uncertain and can only be expressed in probabilistic terms a large total cost can be expected. The higher cost occurs in the form of higher holding costs because more inventory must be maintained to limit the number of stockouts. This additional inventory or safety inventory stock was 41 units for our problem. So, because of these 41 units, we are paying an additional annual holding cost of 49.

Remember that the assumed demand during lead time is a normal distribution. There may be some other distribution the demand can follow during the lead time if that is the case the whole problem will be completely different, but since the normal distribution is a common approximation of any distribution even though the demand may be any distribution you can assume a normal distribution at the end there would not be much variation in the total cost.

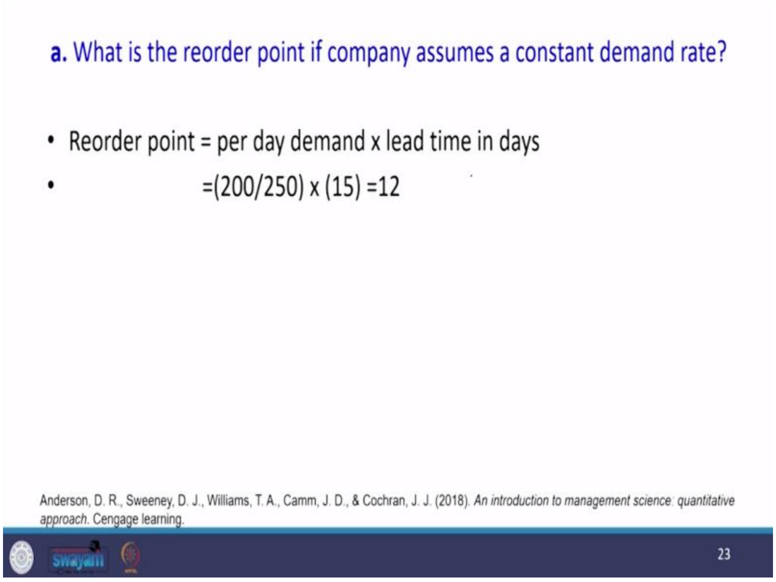


Now, we will take another sample problem I will explain this concept with the help of one more problem. A company provides a variety of auto parts to small local garages. The company purchases parts from manufacturers according to the EOQ model and then ships the parts from a regional warehouse directly to its customers. Traditionally, we follow the EOQ model where the constant demand rate.

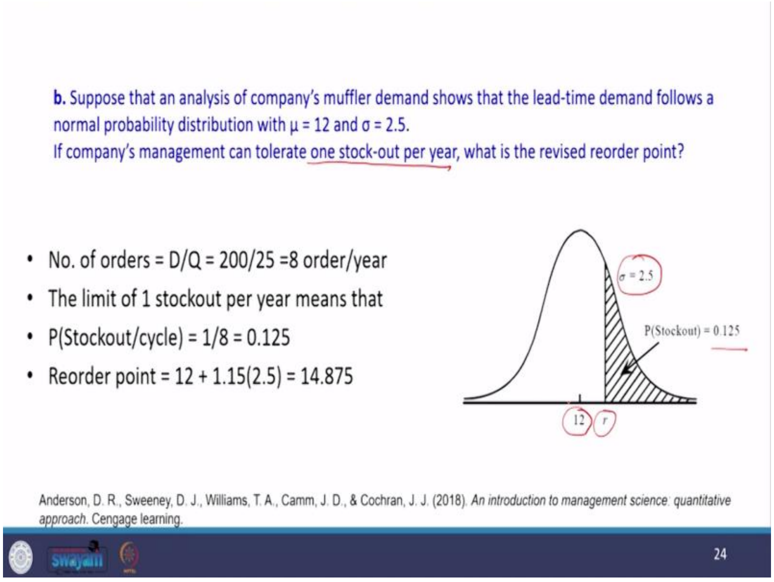
For a particular type of muffler that is an automobile component, the company EOQ analysis recommends an order that Q* = 25. The optimal order quantity is 25 to satisfy an annual demand of 200 mufflers. The company has 250 working days per year, and the lead time average is 15 days. What is the reorder point if the company assumes a constant demand rate in our traditional EOQ?

Suppose that analysis of the company's muffler demand shows that the lead time demand follows normal probability distribution with a mean of 12 and a standard deviation of 12.5. If the company's management can tolerate one stock out per year, what is the revised reorder point? In the second case, what is the safety stock if the holding cost is dollar 5 per unit per year? What is the extra cost due to the uncertainty of demand?

So, this problem is a multi-period problem. The demand is probabilistic, so there is a lead time. So, we are going to find out when to order and how much to order.



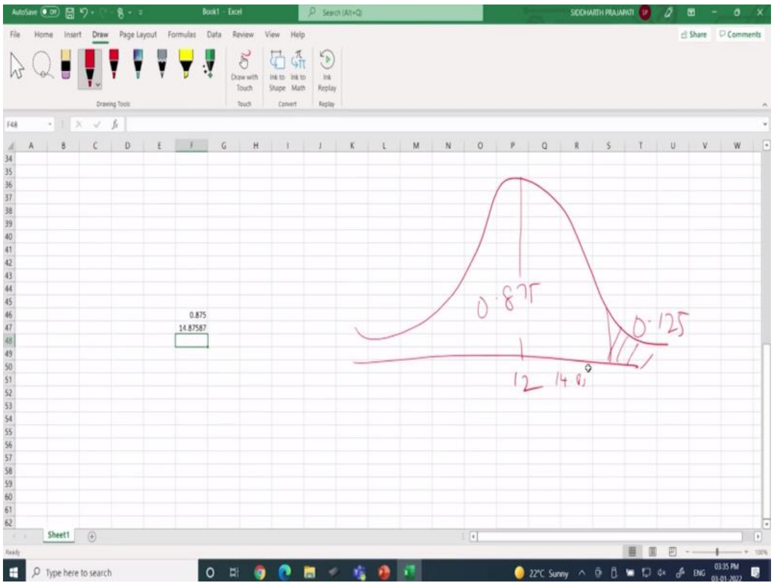
The first question is, what is the reorder point if the company assumes a constant demand rate? We know that a reorder point is in demand during the lead time. So, what is the per-day demand? We know the annual demand is 200, assuming that the company operates for 250 days? So, 200 divided by a number of days, you will get a per day demand and lead time in 15 days, so when you simplify, the reorder point is 12 units.



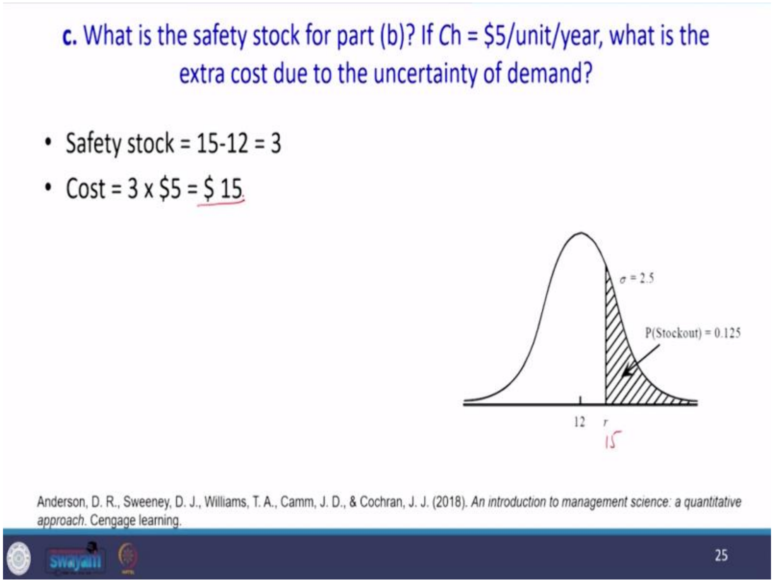
Suppose that analysis of the company's muffler demand shows that the lead time demand follows normal probability distribution with the mean = 12 and standard deviation is 2.5. So, this is the meantime; this is our standard deviation. The company's management can tolerate one stock out per year. So, they are willing to tolerate one stock out per year what is the revised reorder point?

First we have to find out the reorder of the stock out probability that they can tolerate. For that, we should know what the number of orders is. So, the number of orders = D upon Q star. So, the Q star we got already is given in problem 25. So, 200 divided into 25 is 8 orders per year. The limit of one stock out per year means that what is the probability of stock out 1 upon 8. So, 1 upon 8 is 0.125 so this area is 0.125.

Now, with the help of Excel, we are going to find out what the reorder point is. So, first, we should know the right side area and what is the right side area 1 – 0.125.



So, we should know this left side area that is = 1 – 0.125. So, this area is 0.875. Now open equal to NORM.INV probability is 0.875 12, 2.5, so it is 14, so this means 12, so it is coming to 14.87, approximately 15 units. I will go back to the presentation now. The reorder point is 14.875. Approximately, the reorder point is 15.



So, what is the safety stock for part b? What is the safety stock? In case the demand is not fluctuating, we will order only 12 units, but now the demand is fluctuating; instead of 12, we are ordering 15 units. So, the safety stock is 15 – 12 = 3. The holding cost for each unit is 5, so the total cost of safety stock is 15 dollars. Dear students in this class, I have discussed multi-period order quantity, and reorder point model with the probabilistic demand.

So, I have two problems. With the help of these two problems, I have discussed how much to order and when to order. In the next lecture, we will be discussing the periodic inventory system. Thank you very much.