Design Making With Spreadsheet
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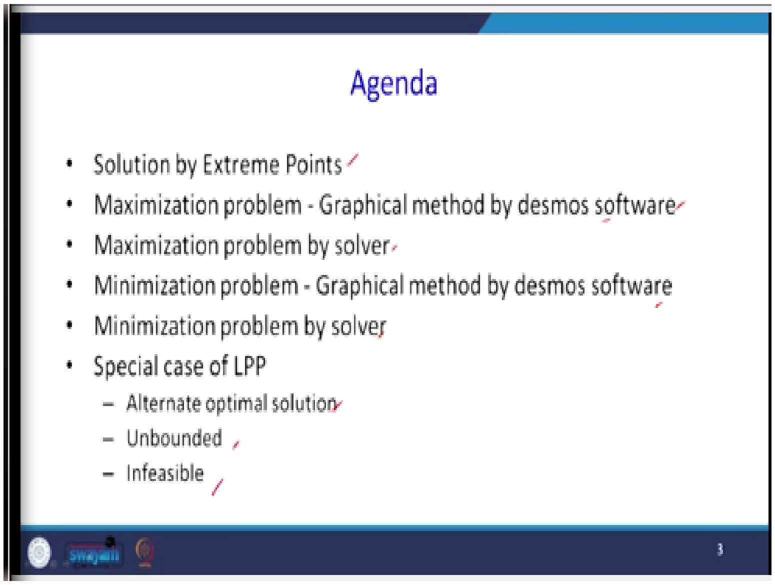
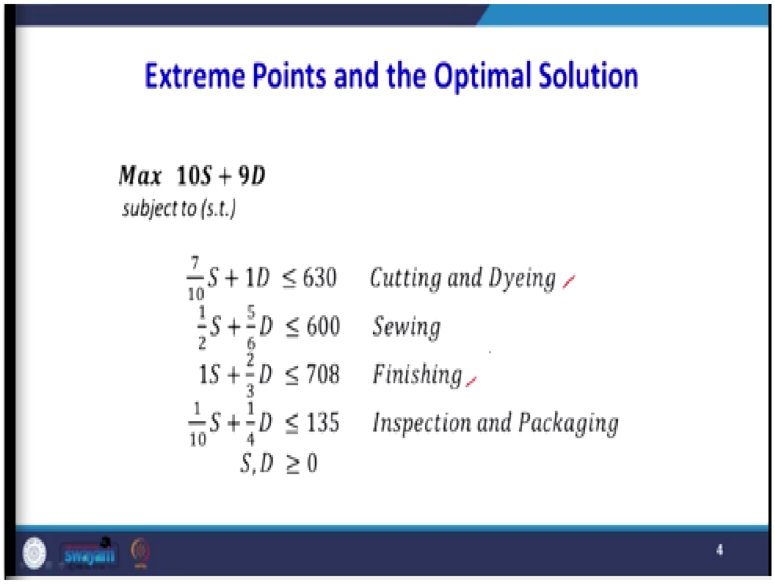
Lecture-05
Graphical Calculator and Excel Solver for Solving LPP

Dear students, in the previous classes, I have discussed the formulation of linear programming problems. Once the formulation is done, the second stage is the solution to the problem. One method is the graphical method, when there are only 2 decision variables which I have discussed in the previous lecture. Another important method is the simplex method. In this lecture, I will discuss how to use demos, a graphical calculator for solving linear programming problems graphically.

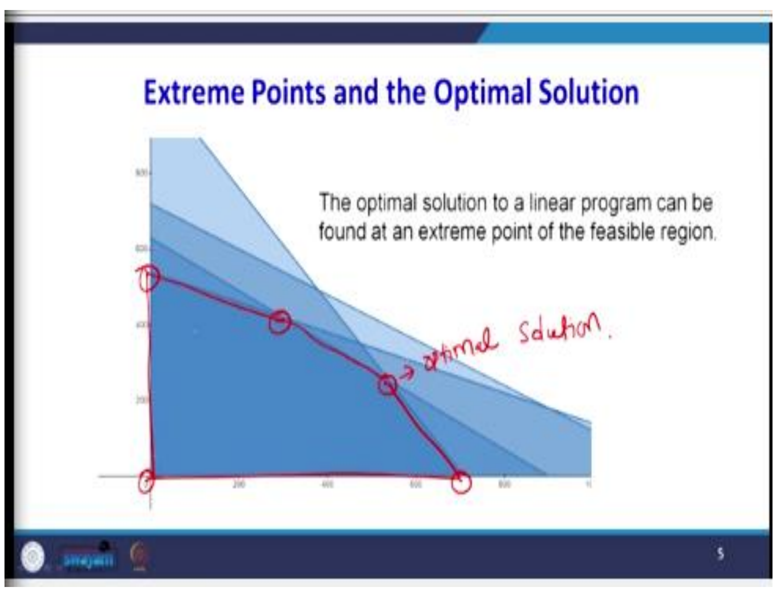
How do you use Excel solver to solve the LPP model? Also, I am going to discuss special cases of linear programming problems, and now we will go to the lecture.



The agenda for this lecture is the solution of linear programming problems by extreme points, then maximization problems using a software desmos software. Then the same problem I am going to solve with the help of an Excel solver. Then, for our understanding, I am going to take another problem, which is a minimization problem that I will also solve by desmos and solver. In the end, we can see the different special cases of our linear programming problem, what are the different cases, alternate optimal solutions, unbounded problems, and infeasible problems.



First, we will solve the problem by extreme points and the optimal solution. This was the problem that I formulated and solved using the graphical method in the previous lecture, which is 10S + 9D maximization subjected to; there are 4 constraints. We have seen in the graphical method previously out of these 4 constraints, the cutting dyeing, and finishing constraints, these are binding constraints.

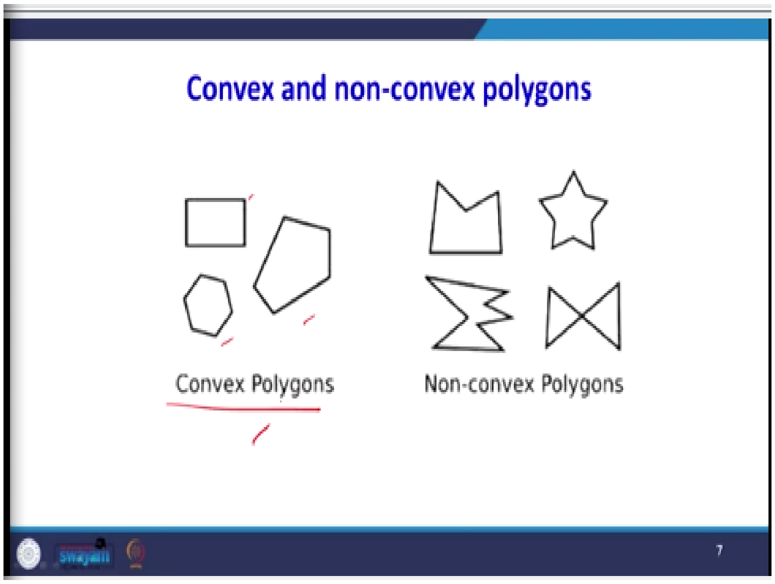


I have solved this graphically, and I will explain after a few minutes how I got this figure using this software. So, the optimal solution to a linear programming problem can be found at the extreme point of the feasible region. What are the extreme points? This is one extreme point, this is another extreme point, this is another extreme point, this is another extreme point, this is another extreme point. When we were solving this problem graphically, we found this point was our optimal solution; when you look at this space, this space is our feasible region. So, what are these feasible reasons? One of the solutions lies in these extreme points.

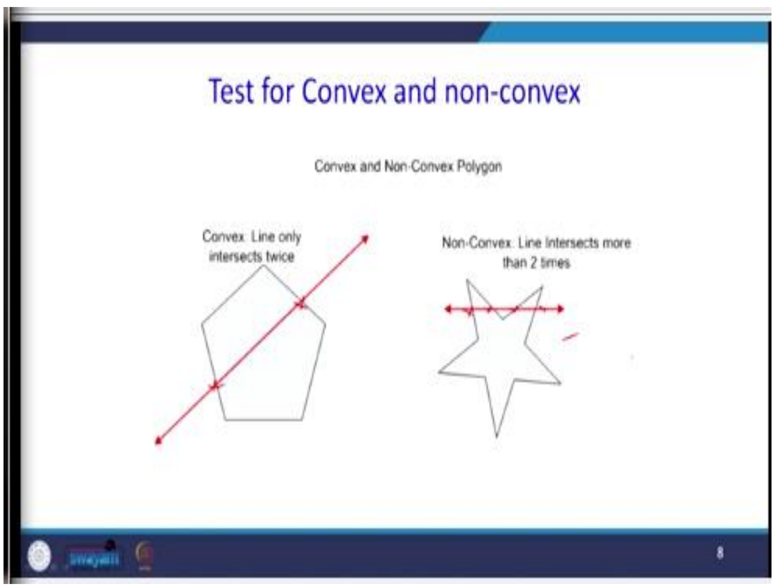
So, first, let us find out the extreme points. This point is 700, 0, this is 540, 252, 300, 420, this point is 0, 540. We know our objective function. What is the objective function? The objective function is 10S 10 standard bags and 9 deluxe bags; the 10 and 9 are the profit contribution by each bag. So, first, we can take this point (0, 550), so this is the first point we can substitute in our objective function.

When you substitute it, the value of Z = 4860 dollars. Next, we will take this point; how did I get this point? By solving these 2 equations, by solving this constraint and this constraint where it is intersecting when I solve that one, I will get this point. So, when I substitute this point in our objective function, I am getting 6780. So, the next point is (540, 252); how did I get these points? By solving these 2 equations that corresponding constraint, so 7668.

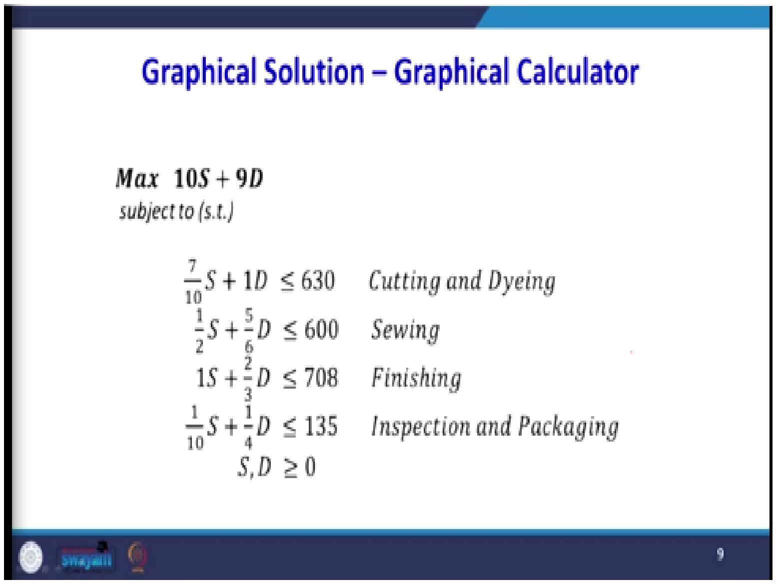
This is the last point that is (700, 0), that is 7000. So, out of these 4 extremes, this is obviously 0. The value of Z is 0. Out of these 4 extreme points, the point which provides maximizing our objective function is this point, where is that? That is 540 come out 252, so this is our optimal solution. So, this method is called solving an LP problem by extreme points.



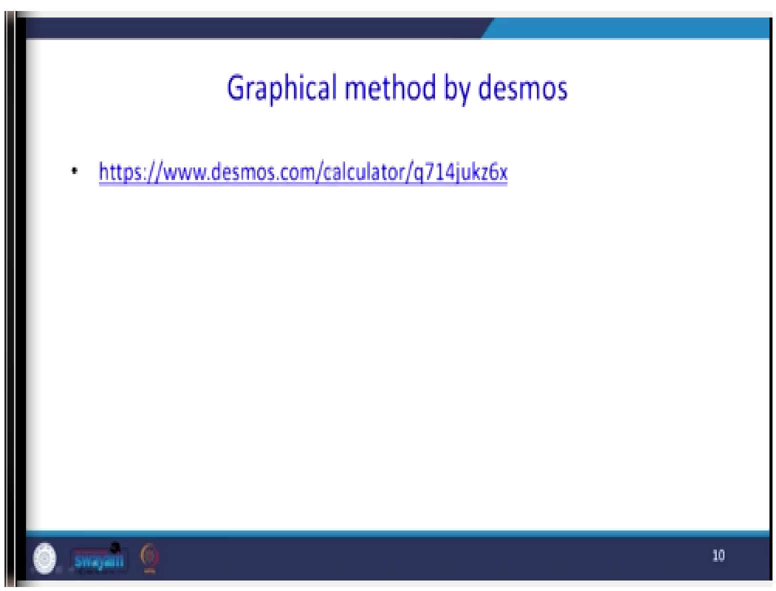
Here is one important point you should know: that a feasible region should be a convex polygon. What is the convex polygon? Here, when you go back here, you see this region is convex; some examples of a convex polygon may be rectangles like this shape. This is an example of nonconvex polygons. When we discuss nonlinear programming, there will be a lot of applications whether the problem is convex or nonconvex. So, what point am I trying to say here? The extreme points will be one of the solutions only if the feasible regions follow convex shapes. So, how to test?



Whether a region is convex or nonconvex. So, see convex, line only intersects twice, so when you draw a line, it intersects only twice. Then we can say this is a convex region or convex function if it is in equations. See the figure on the right-hand side; see here 1, 2, 3. There are more than 2 intersections, so this is a non-convex polygon. So, if our feasible region is non-convex polygons, finding extreme points and finding the corresponding value of Z will not ever be a solution. You should be very careful when you are finding the extreme points and corresponding value.



Now the same problem will be solved with the help of a graphical calculator, this was the problem which we have solved. The same problem in the previous lecture was solved graphically; in this lecture, I have solved it by using extreme points. Now we are going to use a graphical calculator.



There is a graphical calculator is available at, www.desmos.com. So, I have brought this link here. So, this software looks at this desmos.com, which is very useful to display any equations.

So, here you can type, say, when you type, plus here you can write an equation; I already entered it for our convenience.

So, the first constraint is (7/10)x + 1y = 630,

but in our constraint, there is less than or equal to; you can type here less than or equal to. So, this is the region; here, you see that I have restricted the value of x and y to only positive values. So, this is the region where the first constraint is represented.

So, when you remove this less than or equal to sign, you will get only a line; when you type this less than or equal to, we will get all the regions. So, this region's blue regions represent our first constraint, and the second constraint is the only line. But our constraint is less than or equal to I can type it here less than or equal to. So, this is the second region, so the third equation also shows that you can highlight it. If you want, you can have it; otherwise, we can remove it.

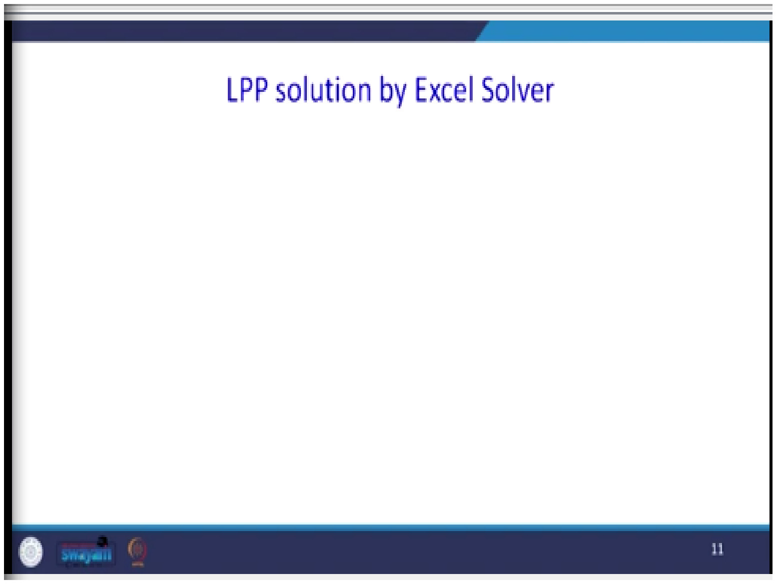
So, this is again less than or equal to this is my third constraint. This is our fourth constraint, so it is also less than or equal to this one. So, now we have the common region; what are the common regions? This point, this point, this point, and this point, the common regions, is the same thing that I have done. So, once we know the feasible region, the feasible region is this region where the dark blue side.

So, first I will take an arbitrary, objective function value of 1800.

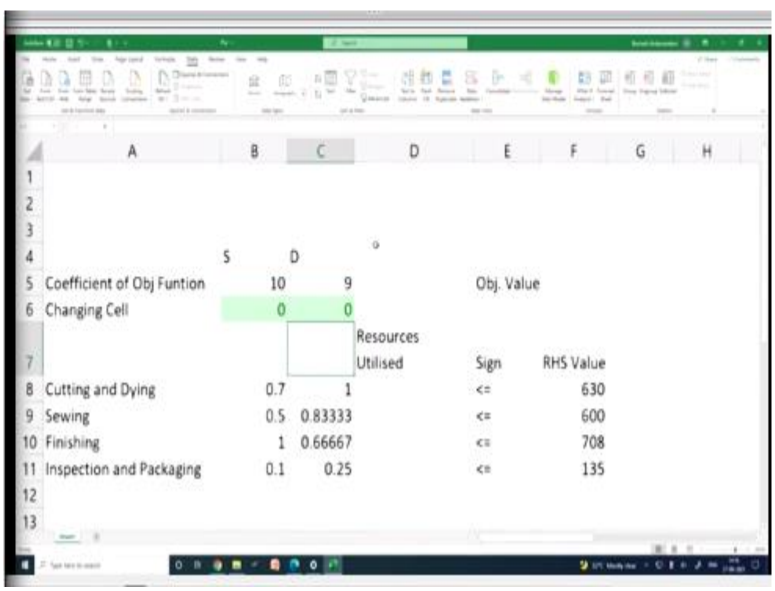
So, if I say 10x + 9 y = 1800,

this is the line I have drawn. So, here, I can get the value of x where it intersects with the axis. Similarly, I can get the value of y and substitute the z value. Then, if you increase the right-hand side equal to 3600, this is another line. So, when you keep on moving this objective function line away from the origin parallel to each other, that is what we discussed in the previous class.

At one point, you see that this is an extreme point beyond which, if you move on the right-hand side, you will land on the infeasible region. So, this is an extreme point where you are landing in the feasible region. So, this is the way to use your desmos software for solving a linear programming problem graphically. This is a very important point when we go for the graphical method only when there are 2 decision variables. If there are more than 2 decision variables that cannot be displayed in a 2-dimensional figure, the graphical method is not suitable.



Students have learned how to use desmos to solve the problem graphically. Now let us take the same problem and solve it by using a solver, that is, Excel solver. This Excel solver is an inbuilt function in Microsoft Excel.



Students now will use Excel solver to solve the same problem, which is our maximization problem, which we have solved graphically. So, I have MS Office 2016; I opened an Excel in which I entered all the data. So, you see the first one, A5, coefficient of objective function, A6 changing cells. So, here in the B5, I have written 10 because this is the coefficient or objective function 10S + 9D.

Below that, I have entered (0, 0), and I have highlighted it in green; there is a reason for that; that is called a changing cell. This is the place where we will get the answer, nothing but the value of our decision variables. Then I entered all 4 constraints. The first constraint is cutting dyeing, that is, (7/10)S + 1D. The second constraint is sewing constraint (1/2)S + (5/6)D; the third constraint is finishing 1S + (2/3)D, and the fourth constraint is (1/10)S + (1/4)D. After that, I utilized written resources.

So, what is the resource utilized? I will use a formula equal to the sum-product; I am going to select this changing cell, I am going to freeze it by selecting the function key F4, press F4, then select this cutting dyeing, that is B8 to C8, close the bracket, and enter. So, what is the meaning of these resources utilized? So, I am going to enter values 1 and 1 below the changing cell, so what is the meaning?

If I produce 1 standard bag and 1 deluxe bag, how many resources will I utilize from the cutting and dyeing department? 1.7 hours, but on the right-hand side, the maximum hours available is 630; this is the meaning of resources utilized. Why have we used the dollar symbol by pressing F4? Because I am going to drag it, when I drag it, this value should be constant. This is called an absolute reference.

So, I am going to drag it; here in E5, I have written objective function value; in F5, I am going to use one formula that is equal to the sum-product, select the coefficient of the objective function, change cells. So, how does this sum product function work? So,

10*1+19 = 19.

What is 19? Suppose if I put 2, for example, it is a different number, so I will put 2, 3, so how would the value we got be?

10 * 2 = 20,

9 * 3 =27,

so 20 + 27 = 47,

this is objective function value. Now I have entered the given problem into excel; now I am going to use Solver. So, when you go to this data tab, see no solver option on the extreme right. So, what do you have to do? You have to go to file, so there is an option, click on options, then there is an option called add-in; here, you have to select the solver, solver, and go select all.

Because the euro that is the analysis tool pack may be used for any statistical analysis, press ok, yes. Now you see that when I go for the data tab, there are 2 options available. One is for data analysis and solver; click on solver. For the solver, first, we have to select an objective, so select the cell; the objective value is F F5, and the next one is the maximization problem, changing cells; this is where I have given a color, green color, which is the changing cell.

Now I have to add the constraint with the problem minimization. You have to select minimization; if you want to get a particular value, then you have to choose a value of. So, now I want to add the constraint and click add. So, I am going to select all these cells, this is, that is D8 to D11. Then, you see that there are 3 different options less than or equal to, equal to, greater than or equal to, integer, binary, and so on.

So, our problem is less than or equal to the select constraint, so now you select this right side value; what is that? That is F8 to F11; again, click there and press ok. Now unconstrained variables are nonnegative, which is a default value. Then you see that there are 3 options: GRG, nonlinear, simplex excel, and evolutionary. So, we are going to follow the simplex algorithm and then solve it.

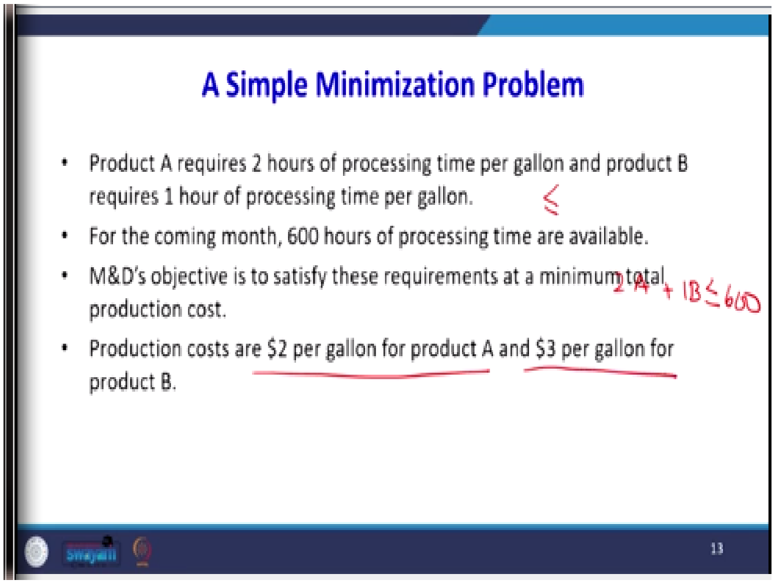
You see that on the right-hand side, there is a report answer, sensitivity limits; you select all this, then you press ok, and now we have the answer; what is the answer? The value of see B6 is 540, and C6 is 252; when you go to this answer report, click this, see here, and you will get the answer. So, look at this value: that is, the final value B6 is 540, which is the value of our standard bag, and then C6 is 252, which is the exact value that we got from our graphical method problem.

I solved the same problem with the extreme point method, the same problem now I use Excel; I solve it. There are many other values; there is a slack, which I will interpret in the coming lecture. There is one more coming in a few minutes; then, there is another sensitivity report that we will discuss in detail in the next lecture. I will explain the limit report in the next lecture.

So far, we have taken one maximization problem, and we solved that problem graphically. Then, the same problem we solved using a graphical calculator, then the same problem we solved by extreme points, and then the same problem is used by the solver. Now, we will take another problem, which is the minimization problem.

We are going to solve this problem also with the help of a graphical calculator and with the help of a solver. First, we will formulate the problem, and then we will go for solving by graphical method. Now, we will go to an example of a minimization problem. This problem is taken from the book by Anderson, Sweeney, and Williams et al.; The problem is like this. M and D Chemicals produces 2 products that are sold as raw materials to companies manufacturing bath soaps and laundry detergents.

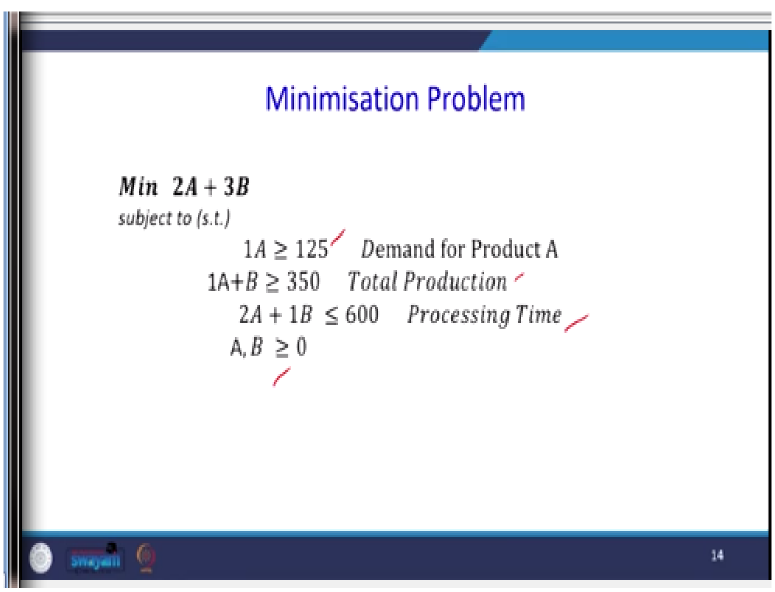
Based on an analysis of current inventory levels and potential demand for the coming month, M and D management specified that the combined production of products A and B, there are 2 products A and B must total at least 350 gallons. So, A and B at least whenever it is at least it is the meaning is greater than or equal to. Separately, a major customer order of 125 gallons of product A must also be satisfied. So, a product also should be greater than or equal to 125 A.



Product A requires 2 hours of processing time per gallon, and product B requires 1 hour of processing time per gallon. So, these are the resources, this is capacity, it is limited, so it should be less than or equal to. For the coming month, 600 hours of processing time are available. So, how can you write this constraint?

So, 2A + 1B <= 600

M and D's objective is to satisfy these requirements at minimum production cost. Look at the problem here: we have to minimize cost, so this is an example of a minimization problem. The production costs are 2 dollars per gallon for product A and 3 dollars per gallon for product B.



So, we can write in the form of a mathematical model.

So, the minimization of cost 2A + 3B

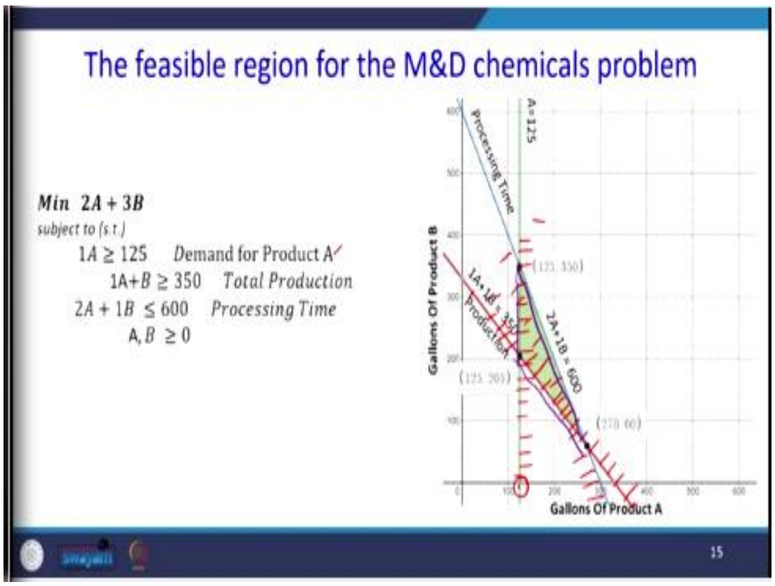
1A>= 125,

and the total production constraints. So, 1A + B >= 350.

then there is a processing time, and there is a limitation. So, 2A + 1B <= 600,

A, B >= 0.

So, this is an example of a minimization problem.



We know that the first step is we have to plot all the constraints. So, first, I am going to take this constraint number 1, that is, A >=125,

so this is the point of 125.

So, the value of 1A >= 25.

Look at this: If it is a greater than type, this side should be shaded.

And the second constraint is 1A + B >=350, so this is the red color.

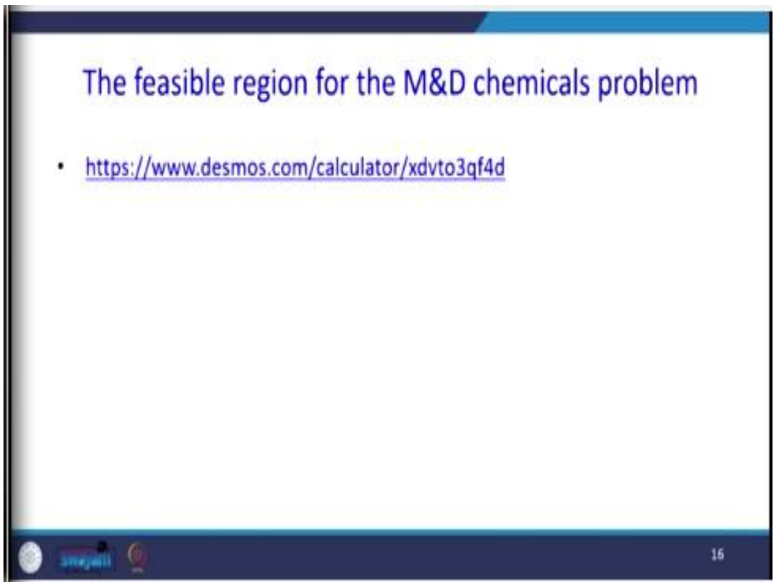
It is 1A + 1B >= 350.

So, it is greater than, so you have to shade away from the origin side.

The third constraint 2A + 1B <= 600,

so if it is less than or equal to, you have to. This is the blue line. So, that should be shaded towards the origin, so this**.** Now, we must look at the feasible region. So, what is the feasible region? This is our feasible region, where I have used the purple color, this is our feasible region. What is the meaning of feasible regions?

All the points in this region will satisfy our constraints. What is the next step? We have to select 1 point from here, so we can do it in trial and error methods, a point where the objective function is minimized; that is our solution. But now we will draw an arbitrary value for our objective function.

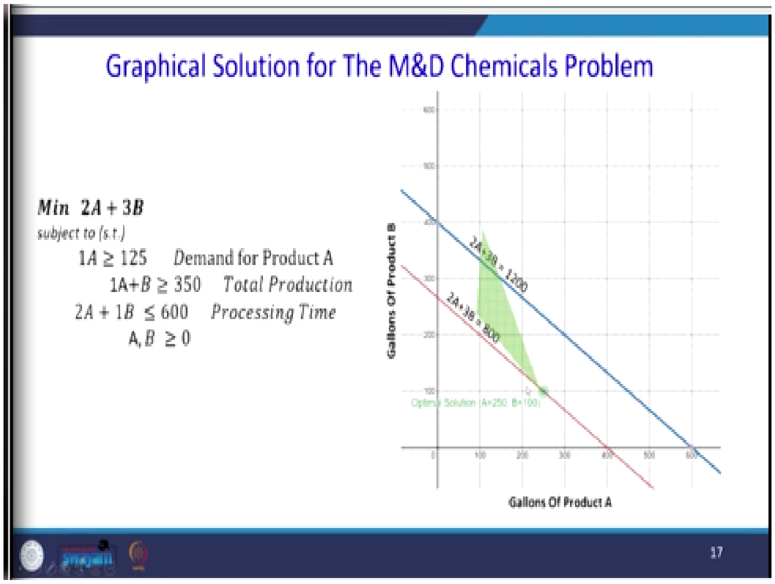


I will explain this with the help of Desmos.

So, I have entered the value x >=125 and x + y >= 350, 2x + y <= 600.

So, in this region, we got one point, (250, 0), and the other point is (125, 350); there is one more point here that is (125, 225)

. So, what do we have to do? These values have to be substituted in our objective function; the values that provide the minimum z value and the points that provide the minimum z value are our decision values.



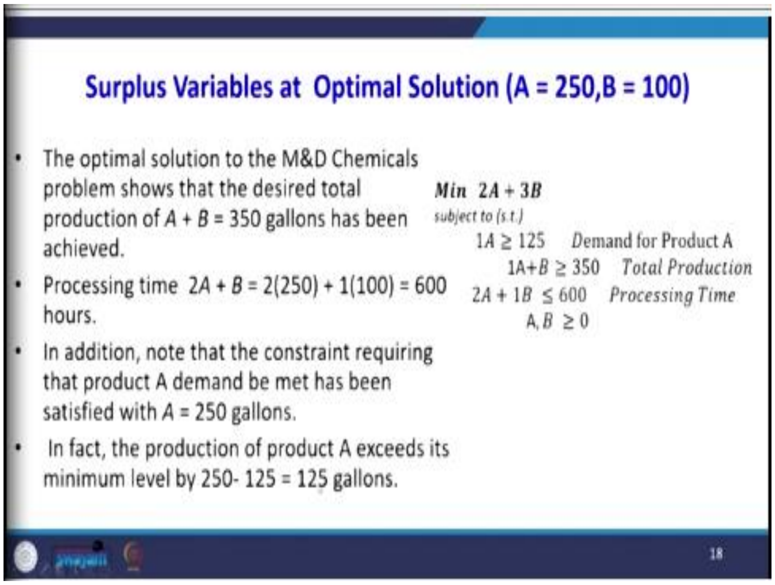
The graphical solution of M and D chemicals problem. First, I have drawn all the constraints, and then I have taken an arbitrary value for our objective function; what is the arbitrary value? 1200,

2A + 3B = 1200.

Now, I have to move these objective functions towards the origin because it is a minimization problem.

Then I have taken another arbitrary value 2A + 3B = 800.

So, at this point that extreme points where A = 250 and B = 100 that is the extreme points where we will be getting our optimal solutions. If I go below this point towards the origin will be moving away from our feasible region. So, the extreme point is A = 250, B = 100 is our optimal solution that will minimize our objective function.



Next, we will discuss another important term called surplus variable at optimal solutions

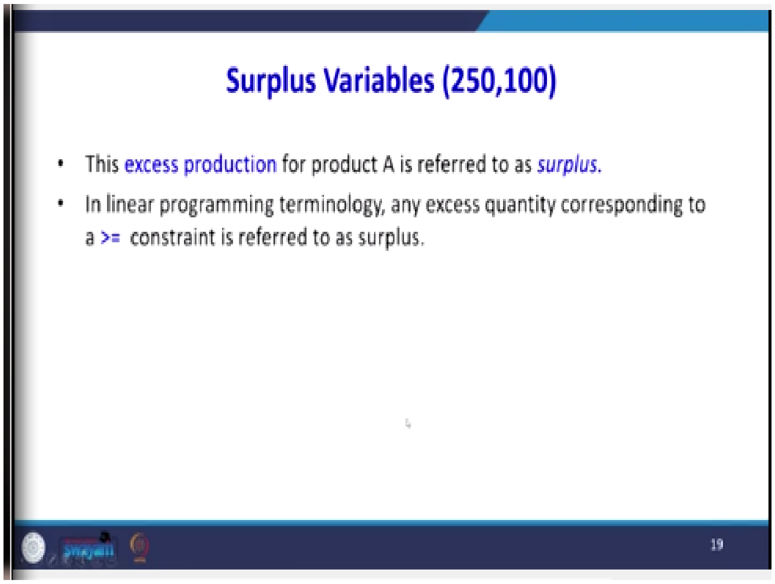
A = 250, B = 100. So, this is our given problem:

2A + 3B, 1A >=125, A + B >=350, and

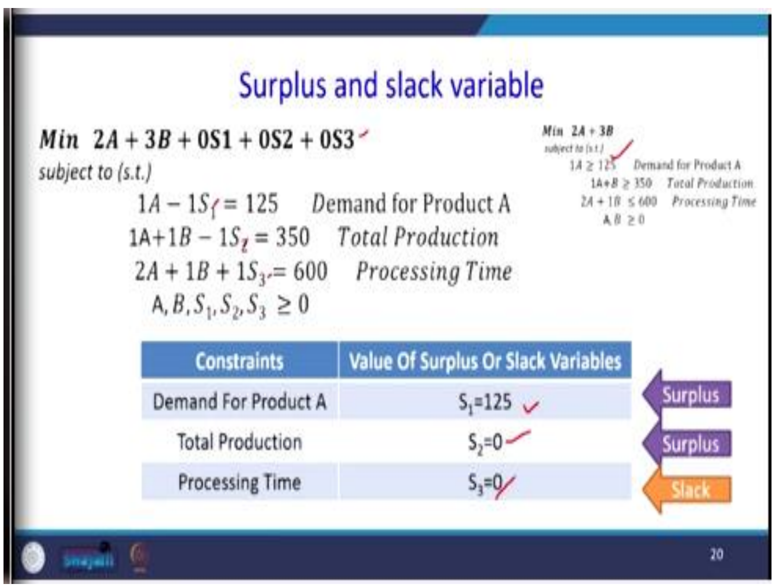
2A + 1B <= 600.

In the previous lecture, we saw slack variable; what is the meaning of slack variable unutilized resources? Whenever your constraint is less than or equal to type, the variable that you are adding on the left-hand side is called the slack variable. Now there are 2 constraints out of 3 that are greater than or equal to, so you will be adding another variable here. That variable is called the surplus variable. Let us see what the meaning of this surplus variable is. The optimal solution to the M and D chemicals problem shows that the desired total production of A + B = 350 gallons has been achieved, which is this second constraint.

And look at the third constraint 2A + B when you substitute it is equal to 600; this is also fully utilized. In addition, note that the constraint requiring that product A demand be met has been satisfied. A = 250 gallons, but the minimum is 125, but we are producing 250 gallons. In fact, the production of product A exceeds it is minimum level by how much? 250 - 125 = 125 gallons, these values is called our surplus variable. We need only 125, but we are producing 250, so the extra unit is called the surplus value, or the variable that we are going to add is called a surplus variable.



So, this excess production of product A is referred to as surplus. In linear programming terminology, any excess quantity corresponding to a greater than or equal to constraint is referred as surplus.



Look at this problem. So, now we have added a surplus variable - S_1. Why have I added - S_1? Because our constraint, look at this our constraint is greater than or equal to type. Look at the second constraint that is also greater than or equal to type, so I have added $-S_2 = 350$. So, S_1 and S_2 are called our surplus variables. The third constraint, where the constraint is less than or equal to type I, has added another variable, S_3, which is called our slack variable.

So, we know that the contribution of the surplus and slack variable for the objective function is 0. So, since we have introduced 2 surplus variables and 1 slack variable in our objective function, they also need to be introduced by having a 0 coefficient.

Now the standard form of our LP problem is $2A + 3B + 0S_1 + 0S_2 + 0S_3$

subjected to

$1A - 1S_1 = 125$

$1A + 1B - S_2 = 350$

$2A + 1B + S_3 = 600.$

A, B, S_1, S_2, S_3 all are greater than or equal to 0. So, the surplus value for the constraint number 1 is 125. So, here is 125; the feasible meaning of this 125, is an excess quantity that we have produced. $S_2 = 0$, which is also a surplus variable. Because we are fully achieved, we produce exactly only 350 for the second constraint, so that is no extra quantity. The third one is also, the constant is fully satisfied; here also there is no slack variable.

Now, we have formulated a minimization problem, and we have solved it graphically. Now, the same problem is with the help of a solver. I have entered the given values; for example, here, this is 2A + 3B; this is the place where you are going to get the answer. So, I am writing a decision variable, the decision variable is this I am going to give different colors.

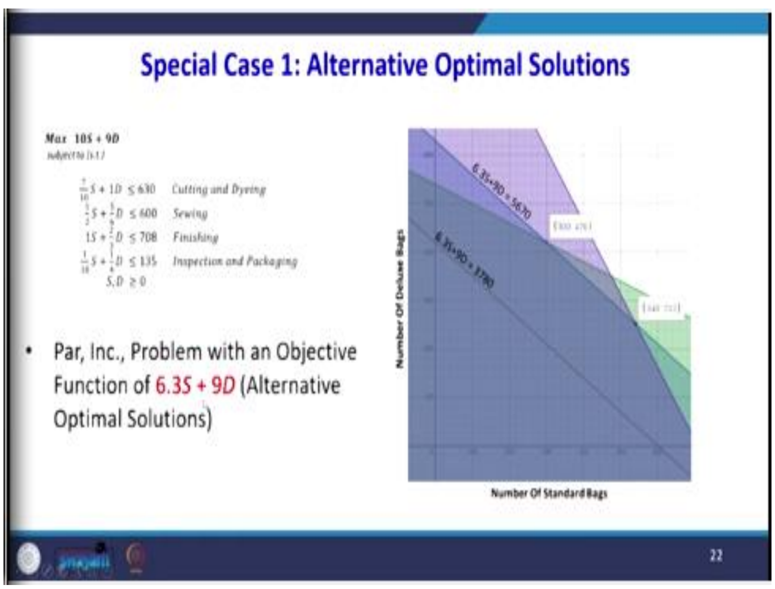
So, I have written there are 3 constraints: C_1, C_2, and C_3. I have written the corresponding coefficients. For example, C_1, only constraint A is there, so 1A, for constraint 2, 1A + 1B, and for constraint 3, it is a 2A + 1B. So, you can enter 0 for our convenience. Here, I am going to use the sum-product, so I am going to select this where the green color is and press F4 for freezing. Now select these values, close the bracket, enter, and drag it, these are the values RU resources utilized. This is our sign; this is our right-hand side value.

So, now I have entered the values, now I am going to solve this problem using a solver. So, go to data, then go to the solver; here, first, I have to select the objective function values; I do not enter the objective function value here. So, the objective function values are equal to the sum-product, select the coefficient of the objective function, decision variables, or the changing cells, and enter. So, if you want to check the logic, just press 1, 1, this is 2 + 3 = 5, now it is correct.

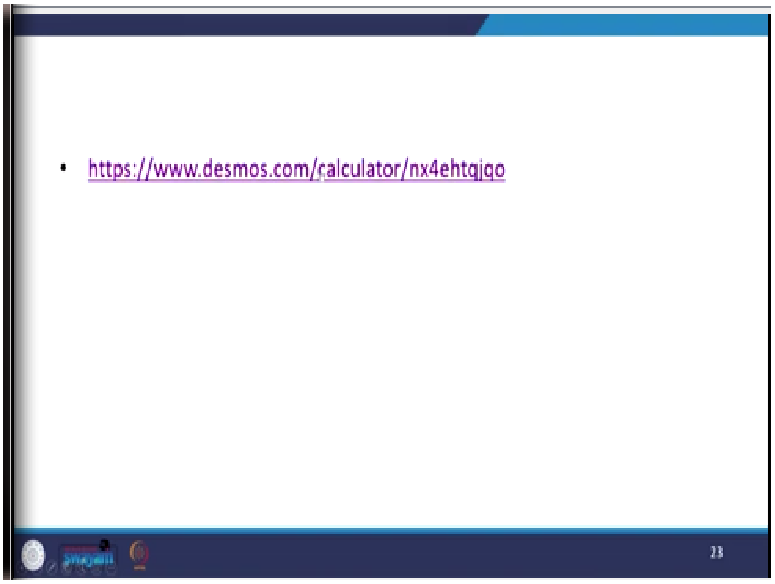
Now you can go for (0, 0). That is ok, now solver, so the objective function value is this location. Then there should be careful; this is a minimization problem, so select the minimization problem. Changing cells, this is changing cell, then I am going to select the constraint; you see that here I cannot select all the constraints at a time.

So, first, I am going to select 2 constraints that are greater than or equal to type, so select this. I have selected these 2 values. Both are greater than or equal to the I have selected on the right-hand side value also, add it. The third constraint is less than or equal to, so I have selected this value, so it is less than or equal to 600, so press ok. Now I am going to solve it using the simplex method, press solve. So, I want to answer sensitivity limits; press ok, now the answer has come.

So, the value of A is 250, and the value of B is 100, so when you go for the answer report, we can see this, the value of A is 250, the value of B is 100. So, we will interpret what is the sensitivity report and limit report in the next lecture.

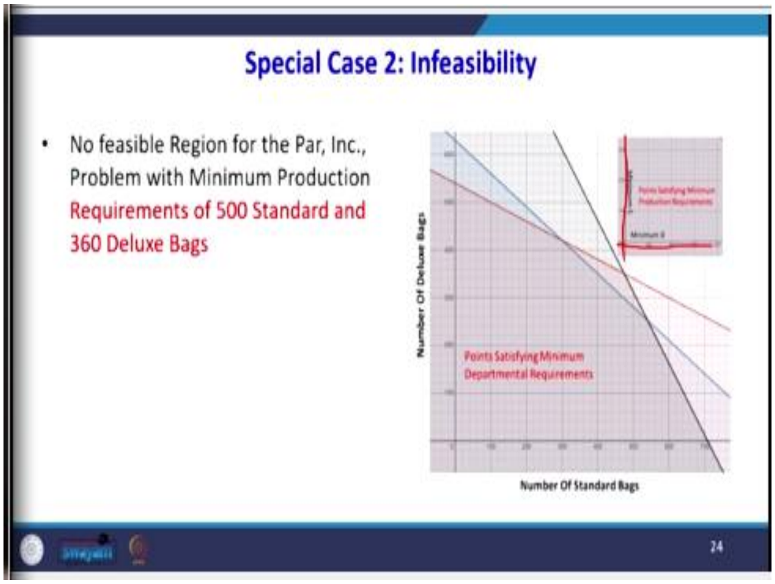


Now we will go to the special case. Special case 1 is alternate optimal solutions, so we will go back to our original problem, which is at 10S + 9D, there are 4 constraints. Now assume that the coefficient of the objective function is for S. It is 6.3 in step 10, and now it is 6.3S + 9D. So, what will happen when the question objective function changes?

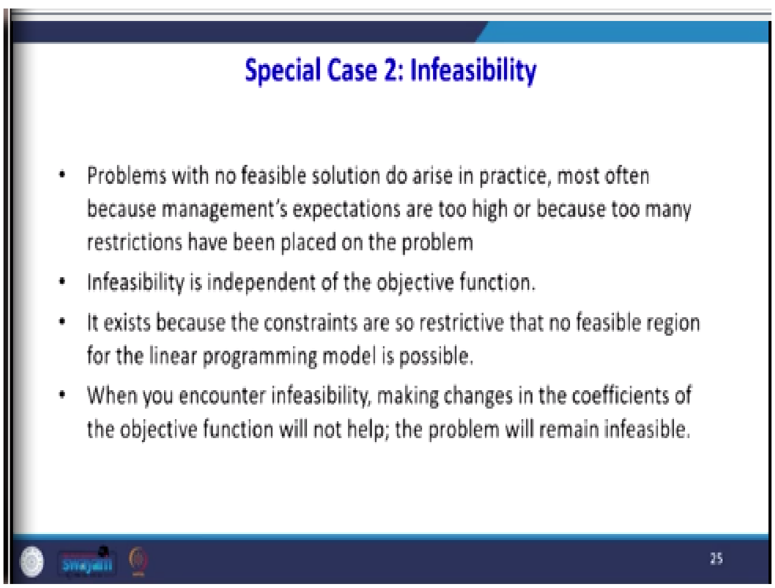


So, I have solved in desmos when you go here. You see that previously, this 548, 252 was the solution, but now our objective function is parallel to these 2 locations. This location and this location between these 2 points that is 300, 420 and 540, 252, so what will happen? All these points are now solution points, so we can minimize this by first switching off these lines. For example, this is the first constraint I have drawn, this is the second constraint, and this is our third constraint.

So, now, if I plot the first objective function value, the 3780 I have taken arbitrarily. So, when I select another point, how is it happening now? Now this black line, which is in black colour, it is between these 2 points. Now the solution points lie, all the points you see coincide with this extreme line. So, now all the points in this region will satisfy our objective function, now what is happening here? We are getting our multiple optimal solutions, that is called alternate optimal solutions; this is case number 1.

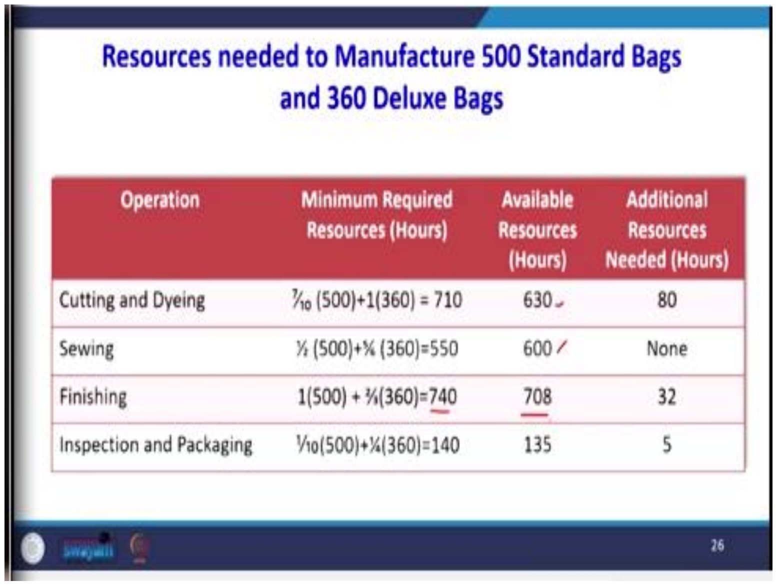


Case number 2 is infeasibility; when the infeasibility situation comes, for example, the company is creating another 2 new constraints; what are the 2 new constraints? So, they want to have more than, the requirement of 500 standard bags and 360 deluxe bags minimum is required. So, what will happen when you draw this minimum requirement? This is your 1 constraint, and this is our other constraint. So, why do we say the infeasible? You see that there are no common regions that satisfy all over constraints, so thus, situations are called infeasible solutions.



The problem with no feasible solution does arise in practice, most often because the management's expectations are too high or because too many restrictions have been placed on the problem. Another important point should remember, this infeasibility is independent of the objective functions. If the problem is infeasible, it has nothing to do with the objective functions; it is only with the constraint.

It exists because the constraints are so restrictive that no feasible region for the linear programming model is possible. When you encounter infeasibility, making changes in the coefficient of the objective function will not help; that makes the problem will remain infeasible.


Resources needed to Manufacture 500 Standard Bags and 360 Deluxe Bags

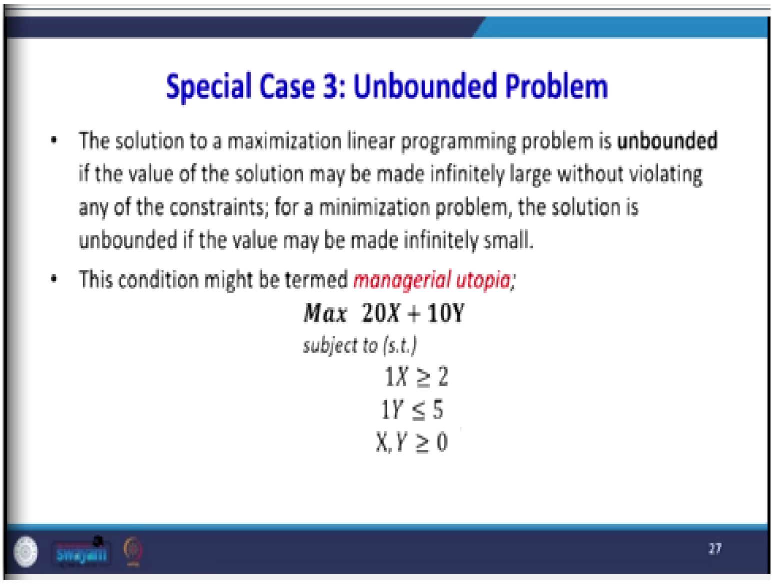
Operation	Minimum Required Resources (Hours)	Available Resources (Hours)	Additional Resources Needed (Hours)
Cutting and Dyeing	$\frac{7}{10}(500)+1(360) = 710$	630	80
Sewing	$\frac{1}{2}(500)+\frac{5}{6}(360)=550$	600	None
Finishing	$1(500) + \frac{2}{3}(360)=740$	708	32
Inspection and Packaging	$\frac{1}{10}(500)+\frac{1}{4}(360)=140$	135	5

For example, let us see how this infeasibility occurs. We have seen that the company is bringing a new constraint, which is the resources needed to manufacture 500 standard bags and 360 deluxe bags. This 500 and 360 were imposed by the company; suppose when you substitute these values in our constraint

(7/10) *500 + 360 = 710.

So, the resource required is 710, but the resources available are only 630. So, additional resources of 800 hours are required.

For the second sewing constraint, there is no problem because the minimum resource is only 550, but the available resource is 600, so there is no problem. But look at the finishing constraint and definition constraint, the minimum required resource is 740 hours, but available resource is only 708, so 32 hours are required. Look at the last constraint inspection packaging. The hours required are 140, and the hours available are 135, so 5 additional resources are required. So, when you bring these constraints that are not achievable, then you will end up with infeasible solutions.



Then the third case is an unbounded problem; the solution to a maximization linear programming problem is unbounded if the value of the solution may be made infinitely large without violating any of the constraints. Now, the feasible region is infinitely large, but it does not violate any constraint. For a minimization problem, the solution is unbounded, the volume may be made infinitely small.

For the maximization problem, it is all the constraints not bounded, but for the minimization problem, whenever the value is infinitely small, then you will encounter the unbounded problem. For example, we see this constraint, so this condition might be termed as a managerial utopia that is not possible.

So, maximize 20X + 10Y,

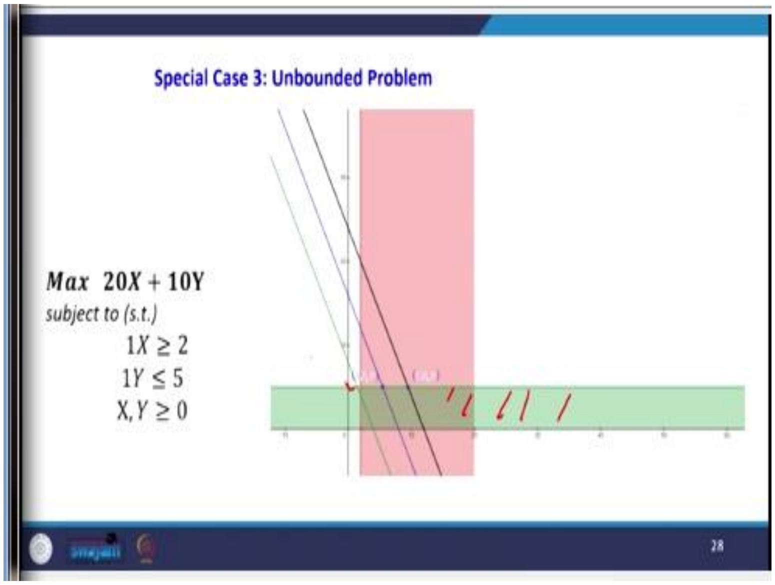
so the value of X is utopia, which means that it is unimaginable, something that can be present in the imagination.

So, X >= 200,

Y <= 5,

X > 2,

Y < 5.



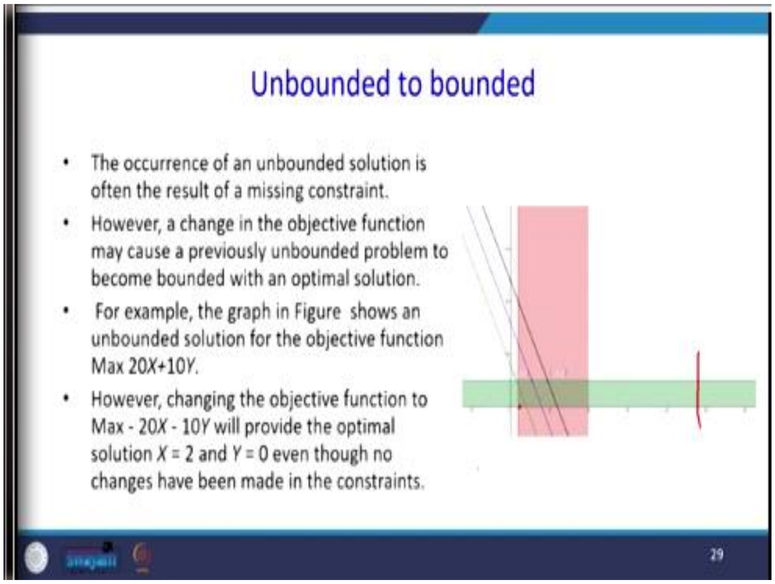
When you plot graphically, this is the point where

Y <= 5,

X > 2,

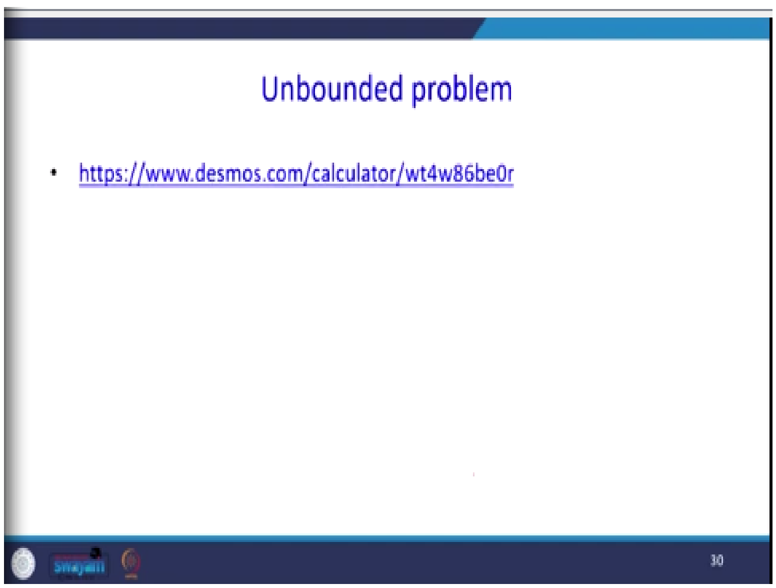
Y < 5

So, now some concerns are missing, it is not bounded. Because this region satisfy our constraints, but there is no boundary for that, this situation is called an unbounded problem.



How to make unbounded to bounded? The occurrence of an unbounded solution is often the result of a missing constraint because there is no constraint here to make the boundary. However, a change in the objective function may cause a previously unbounded problem to become bonded with optimal solution. You see, this problem, which I have taken is maximization type; if you make some adjustments to our objective function, our unbounded problem may become a bounded problem.

For example, a graph in figure shows an unbounded solution for the objective function
20X - 10Y.
However, changing the objective function to -20X, -10Y will provide the optimal solution when X = 0 and Y = 0.
So, what will happen when you multiply by minus your maximization problem will become a minimization problem?
Now this X = 2, this X = 2 will be your solution even though there are no changes that have been made in the constraint.



This we can see graphically.
First, I have drawn my first constraint, X >= 2, and
the second constraint, X <= 2.
So, in this region, there is no because I did not restrict the value of X and Y, which is positive; that is why it is going. But these regions, this rectangular region, is our feasible region; there is no boundary. Suppose I make an objective function like this right-hand side equal to 80, so (160, 240) like that. There will be a lot of solutions for this because there is no boundary.

So, how we can make this one into a bounded problem? When you multiply the coefficient of the objective function by minus, this will become a minimization problem. When it becomes a minimization problem, the value X = 2 will be one of your solutions. Summary of this lecture; we have taken a maximization problem that we have solved using Desmos software, the same problem we have solved using Solver.

After that, we took a minimization problem, and we formulated a minimization problem that we graphically solved using Desmos software, and then the same problem was solved by using a solver. Then we have seen special cases of linear programming problems, what are the special cases, alternate optimal solutions, unbounded problems, and infeasible problems. In the next class, we will start with a new topic that is sensitivity analysis; thank you very much.