Decision Making with Spreadsheet
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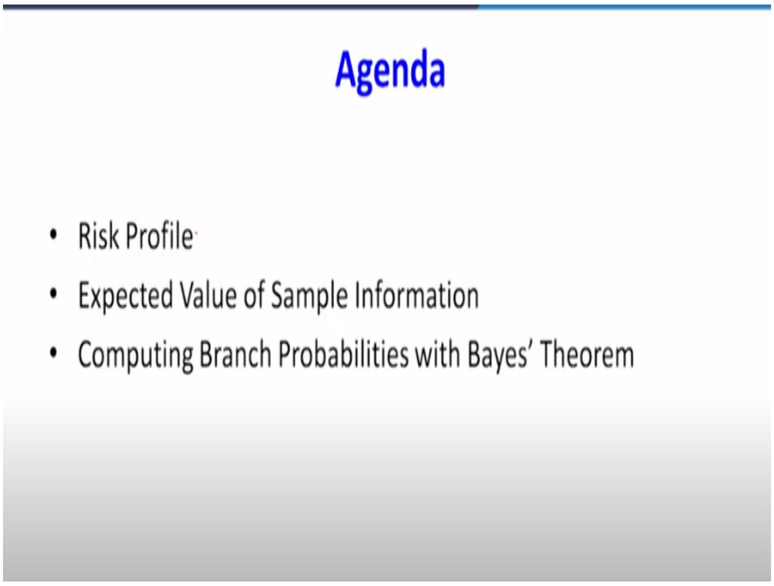
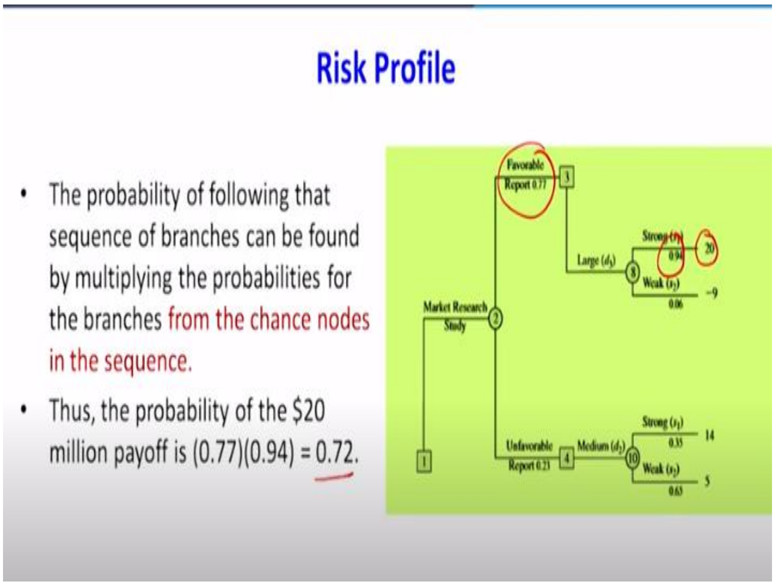
Lecture - 50
Decision Analysis - V

Dear students, in this lecture, I am going to discuss the risk profile and expected value of sample information. Finally, branch probabilities are computed with the help of Bayes' Theorem.

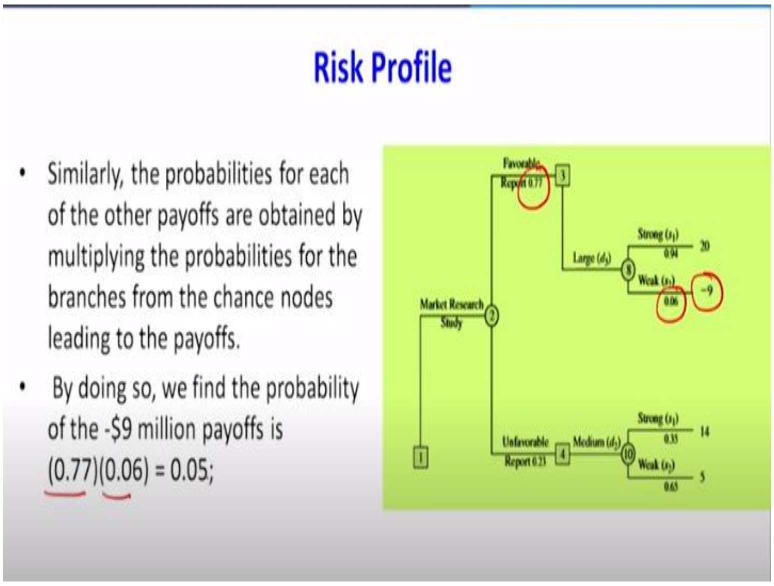


So, the agenda for this lecture is the risk profile, the expected value of sample information, and computing the branch probabilities using Bayes' Theorem.



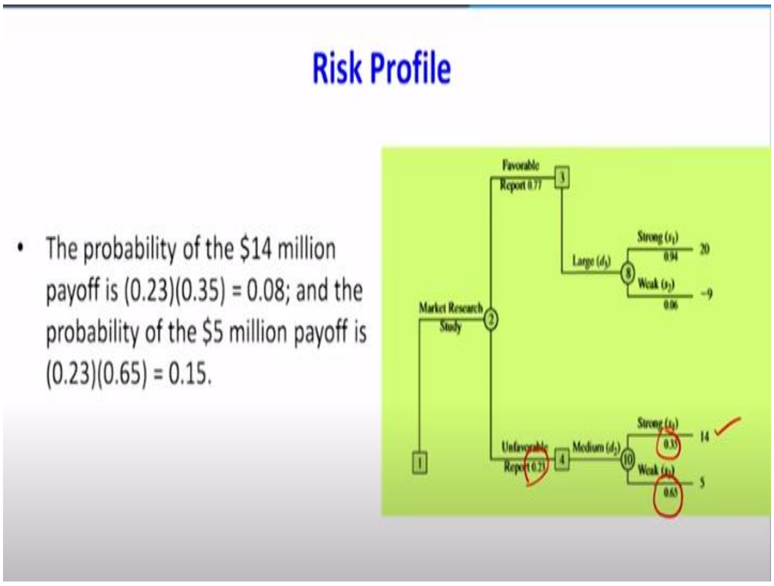
First, we will discuss what a risk profile is. The right-hand side was our final solution for our decision tree problem by using a backward strategy approach. So, for this solution, we have to provide the risk profile. So, for the risk profile, we know that to draw the risk profile, we need to know the probability and the payoff. For example, here, there are the 20s. One payoff is there.

Now we have to find out what the probability of that is. So, the probability of following the sequence of branches can be found by multiplying the probabilities for the branches from the chance node in the sequence. So, for example, the payoff is 20. So, what is the sequence? Here, there is one sequence, and here, there is another sequence. So, the probability of the 20 million payoff is the multiplication of 0.77 and 0.94. We are getting 0.72.

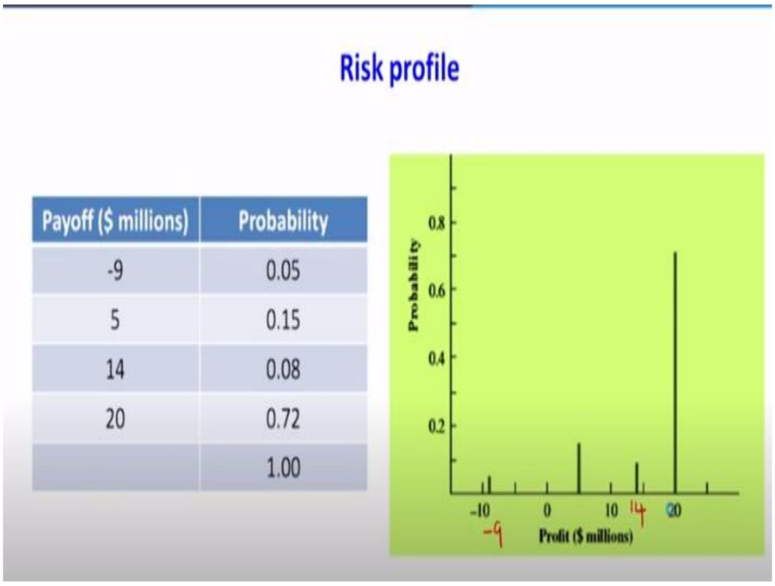


Similarly, the probabilities for each of the other payoffs are obtained by multiplying the probabilities for the branches from the chance nodes leading to the payoff. By doing so, we find the probability of -$9 million payoff. For example, is this one, How do we get the probability for this payoff? So we have to multiply this by 0.77, then 0.06 by this one. We got 0.05.

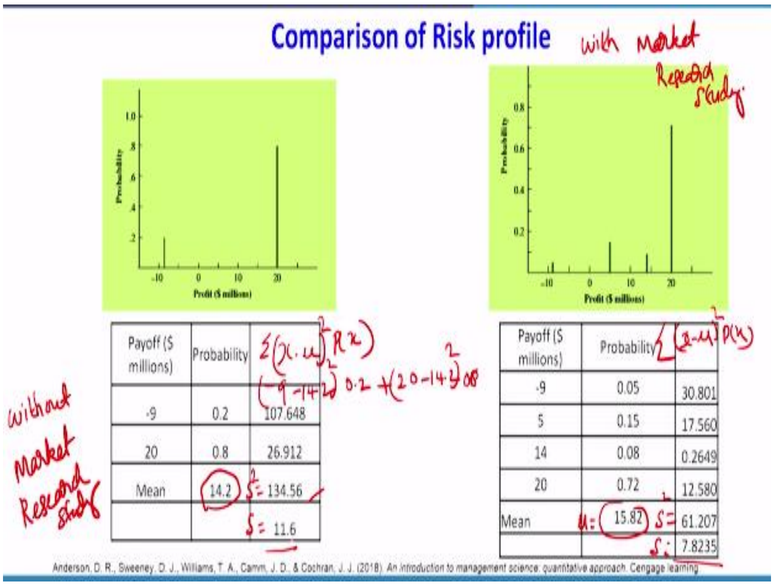
By doing so, we find the probability of the -$9 million payoffs is (0.77) (0.06) = 0.05.



The probability of a 14 million payoff. Here, we have to multiply this by 0.23 and 0.35. So, we are getting 0.08. And the probability of a $5 million payoff is again 0.23 multiplied by this 0.65. So, the probability is 0.15.



This says the payoff on corresponding probability. When you plot this, it is called a risk profile. What is that? There is a -9 and 0.05. Similarly, 5, 0.15. 14 here 14, 0.08. 20, 0.72.



Now we can compare the risk profiles. What is this comparison? The left side picture shows the risk of making the decision without a sample. That is without market research study. The left-hand side is without a market research study, okay. So, what is that payoff on the corresponding probability? -9, 0.2, 20, 0.8. So here is what I am going to find out: I am going to find out the variance.

After that, I am going to find out the standard deviation. The standard deviation says that the risk. We know what is the formula for the variance. What is the formula for variance? Sigma of x minus mu whole square multiplied by P(x).

The given expression can be written mathematically as:

$$\sum (x-\mu)^2 P(x)$$

So here mu is this 14.2. How did we get this 14.2? In Excel, we have to use some product formula. So, we have to select -9 20, 0.2, and 0.8.

We will be getting 14.2. That is nothing but -9 multiplied by 0.2 plus 20 multiplied by 0.8, so we will be getting 14.2. So first, we will find the variance. Calculations of the above can be shown as below;

Given values:

- $x_1 = -9, x_2 = 20$
- $\mu = 14.2$
- $P(x_1) = 0.2, P(x_2) = 0.8$

Now, the variance calculation:

$$\sigma^2 = (-9 - 14.2)^2 \cdot 0.2 + (20 - 14.2)^2 \cdot 0.8$$

$$\sigma^2 = (-23.2)^2 \cdot 0.2 + (5.8)^2 \cdot 0.8$$

$$\sigma^2 = 538.24 \cdot 0.2 + 33.64 \cdot 0.8$$

$$\sigma^2 = 107.648 + 26.912$$

$$\sigma^2 = 134.56$$

That is the variance. This we can say variance. When you take the square root of that we are getting the standard deviation. The risk of deciding without a market research study is 11.6. But come to the right-hand side. This is with market research study. So, we have the probability -9, 0.05, 5, 0.15, 14, 0.08, 20, 0.72.

Here also, first, we have to find out the mean. Next, we must find out the variance. The same formula, what is that? x minus mu whole square P(x). So here x is -9 minus 15.82 whole square multiplied by 0.05 plus 5 minus 15.82 whole square multiplied by 0.15 plus 14 minus 15.82 whole square multiplied by 0.08 plus 20 minus 15.82 whole square multiplied by 0.72.

The calculations of the above variance in details are given below;

The given expression represents the variance calculation:

$$\sigma^2 = \sum (x - \mu)^2 P(x)$$

Given values:

- $x_1 = -9, x_2 = 5, x_3 = 14, x_4 = 20$
- $\mu = 15.82$
- $P(x_1) = 0.05, P(x_2) = 0.15, P(x_3) = 0.08, P(x_4) = 0.72$

Now, the variance calculation:

$$\sigma^2 = (-9 - 15.82)^2 \cdot 0.05 + (5 - 15.82)^2 \cdot 0.15 + (14 - 15.82)^2 \cdot 0.08 + (20 - 15.82)^2 \cdot 0.72$$

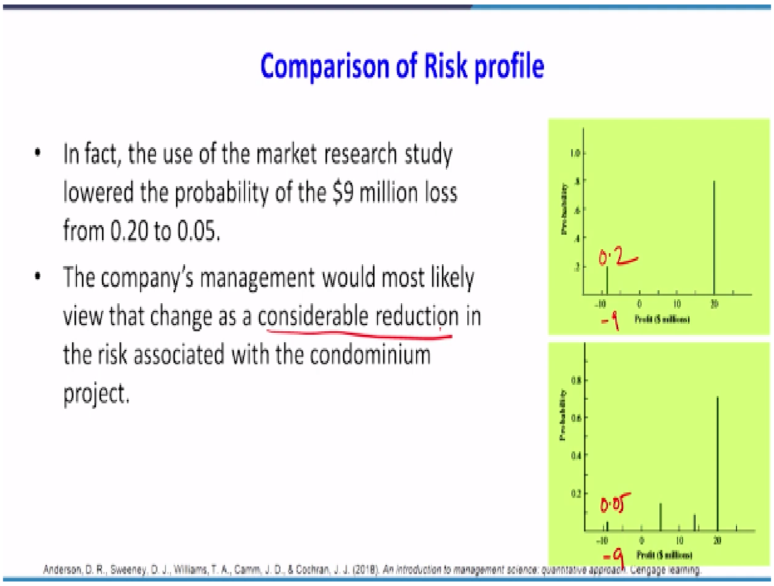
$$\sigma^2 = (-24.82)^2 \cdot 0.05 + (-10.82)^2 \cdot 0.15 + (-1.82)^2 \cdot 0.08 + (4.18)^2 \cdot 0.72$$

$$\sigma^2 = 616.32 \cdot 0.05 + 117.01 \cdot 0.15 + 3.31 \cdot 0.08 + 17.47 \cdot 0.72$$

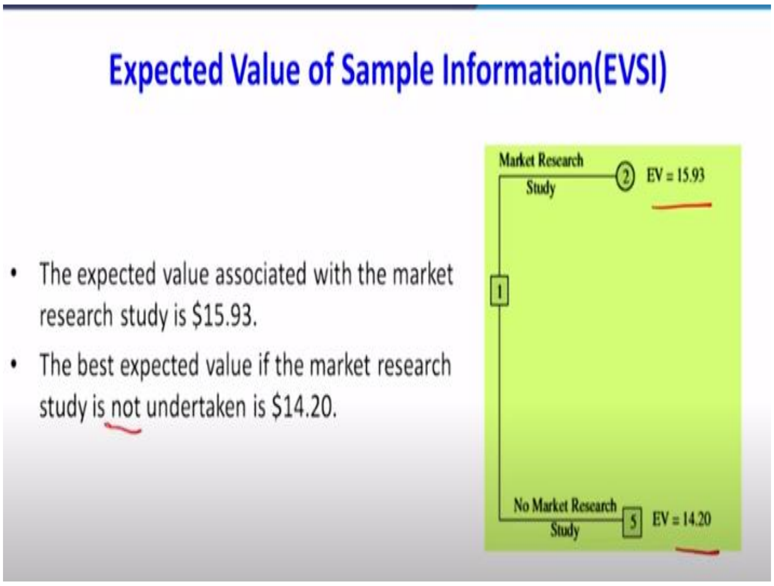
$$\sigma^2 = 30.816 + 17.5515 + 0.2648 + 12.5784$$

$$\sigma^2 = 61.21$$

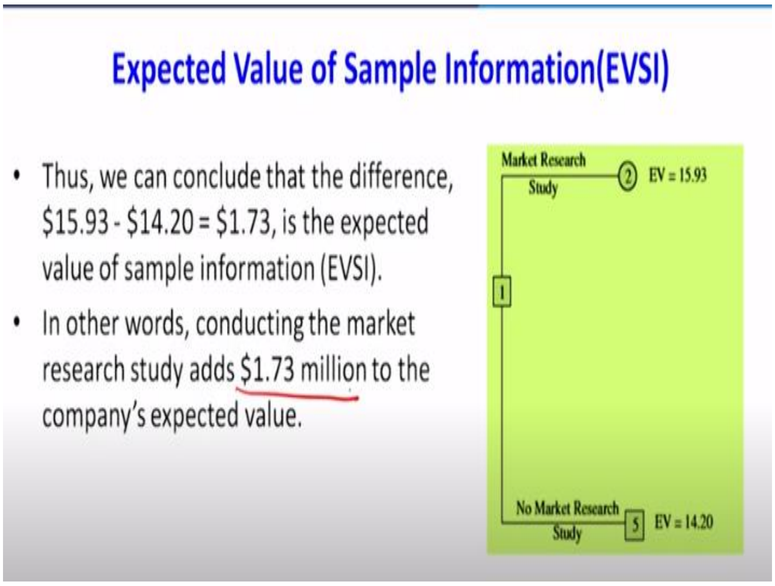
When you sum it up, you will get this one, variance 61.207. So, when you take the square root of that, we are getting 7.8. Now you see the comparison. Here, the risk is only 7.82. But on the left-hand side, the risk is 11.6. So, what we are learning from this risk profile picture is that when you make the decision with a market research study, your risk is minimized.



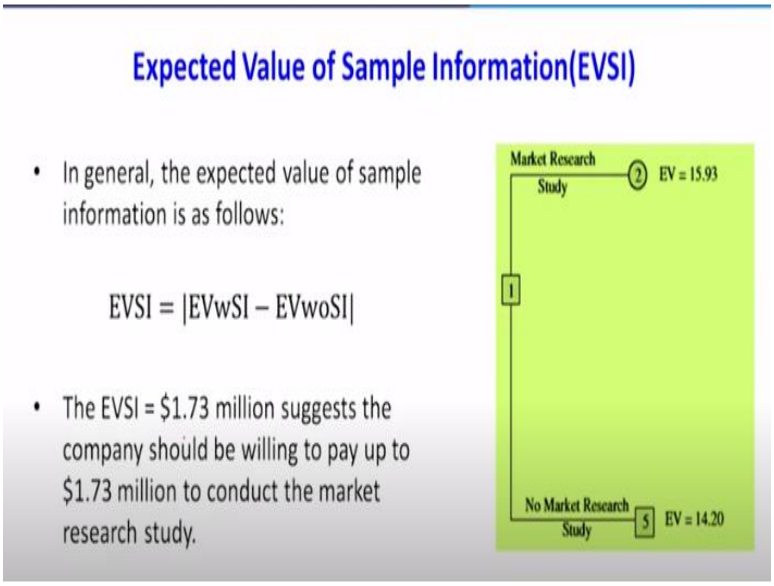
In fact, the use of a market research study lowered the probability of a $9 million loss from 0.2 to 0.05. You see that previously -9, 0.2. But now, it is -9 only 0.05. So, the company's management would most likely view the change as a considerable reduction because of this market research study, there is a considerable reduction in the risk associated with the condominium project. So, it is better to go for market research study.



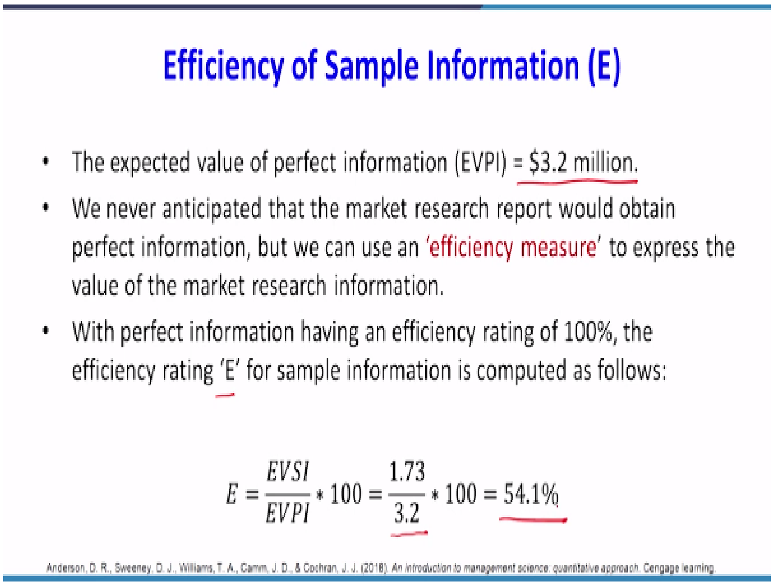
The following agenda is finding the expected value of sample information. The right-side picture shows our final result. What does this result say? The expected value associated with the market research study is 15.93. The best-expected value if the market research study is not undertaken is 14.20.



Thus, we can conclude that the difference of $15.93 minus 14.20 equal to 1.73 is the expected value of sample information, EVSI. In other words, another interpretation is conducting the market research study adds $1.73 million to the company's expected value. So it is better to go for a market research study.



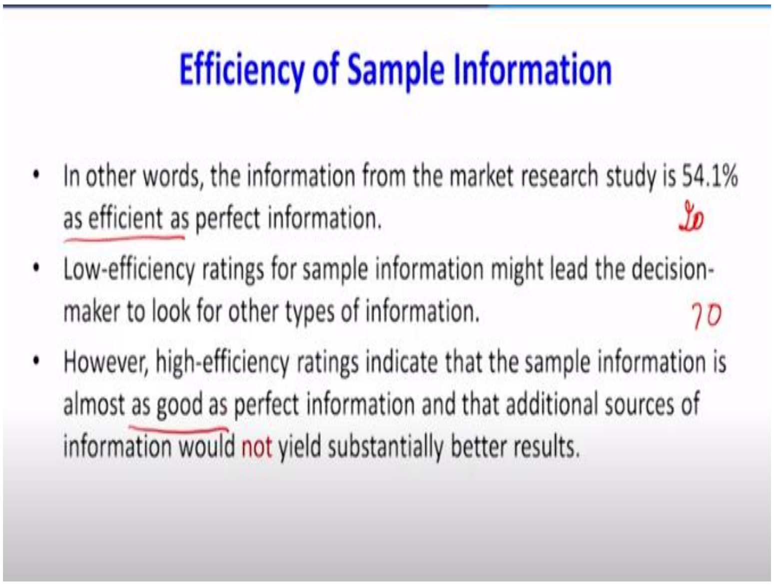
In general, the expected value of sample information is the difference between the expected value with sample information and the expected value without sample information. So, the EVSI $1.73 million suggests the company should be willing to pay up to seven points up to $1.73 million to conduct the market research study. So, this is another interpretation of your expected value of sample information.



Now another point that you are going to learn today is the efficiency of sample information. The meaning is whether there is any worth of going for this sample information, okay. So that worthiness, we will find out with the help of efficiency. So the expected value of perfect information EVPI, in the previous lecture, we have got this one $3.2 million. That is the expected value of perfect information.

But at present, we are not thinking of perfect information because we are inferring from the sample information. So, we never anticipated that the market research report would obtain perfect information; that is not possible. However, we can use efficient measures to express the value of market research information.

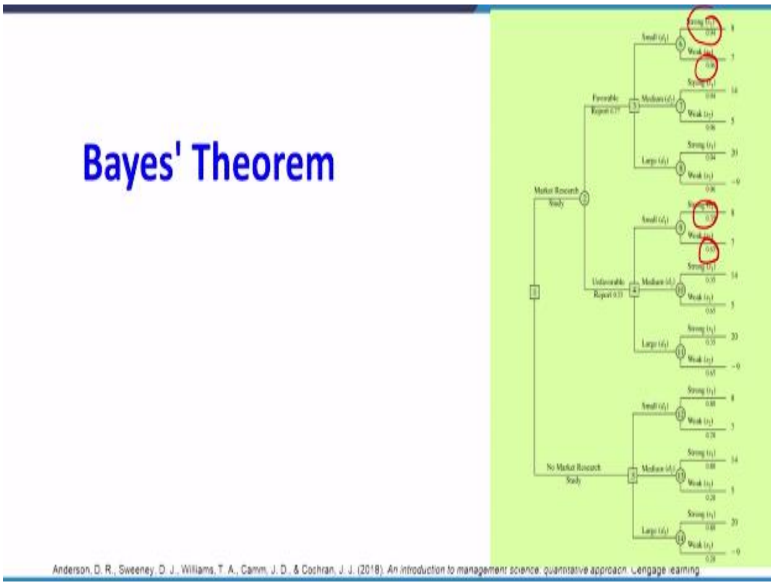
So, with perfect information having an efficiency rating of 100%, the efficiency rating E for sample information is computed as follows. How? The expected value of sample information above the expected value of perfect information multiplied by 100. So 1.73, just now we got it. In the previous lecture, we got the expected value of perfect information 3.2. So, the ratio is 54.1%.



So what is an interpretation of this 54.1%? In other words, the information from the market research study is 54.1% as efficient as perfect information, as efficient as perfect information. A low efficiency rating for sample information might lead the decision-maker to look for another type of information. Suppose it is, say, 20. That means what is the meaning of this 20?

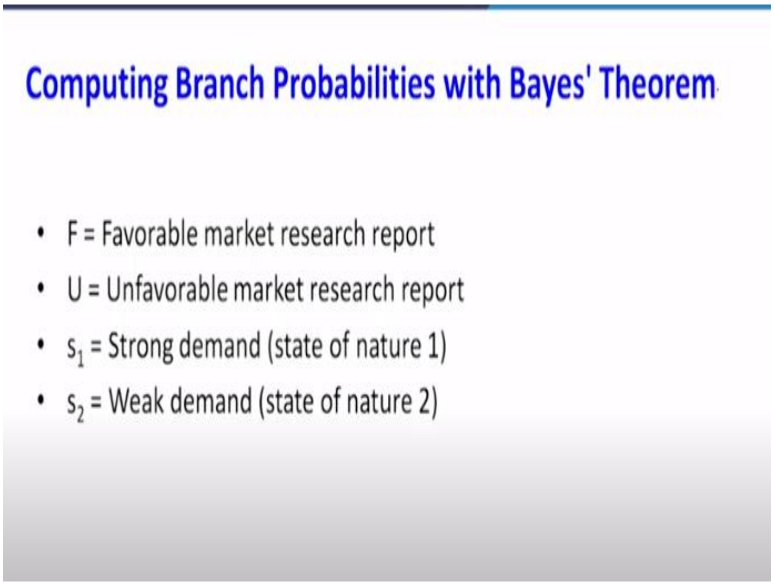
So, the sample information is only 20% as efficient as perfect information. So, there is no benefit to going for the sample information. However, a high-efficiency rating, say it is 70, indicates that the sample information is almost as good as perfect information, and the additional sources of information would not yield substantially better results.

So many times, with the help of sample information, you can get the perfect information. When is it possible? Whenever the efficiency rating is high.

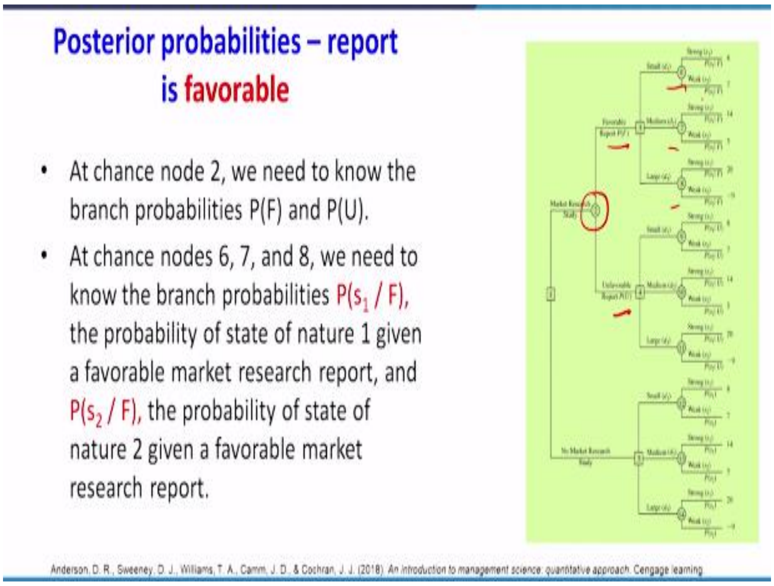


The last agenda is the Bayes' Theorem. It is a very interesting concept. We are going to use this idea in our decision tree problem to find out the branch probability. Remember, in our previous lecture, the values of 0.94 and 0.06 and 0.35 and 0.65 were given to you in the problem. That means all the information about the decision tree is given to you.

Then we used this backward strategy, and we found the result. In this lecture, I will explain how we got this value of 0.94 or 0.06.

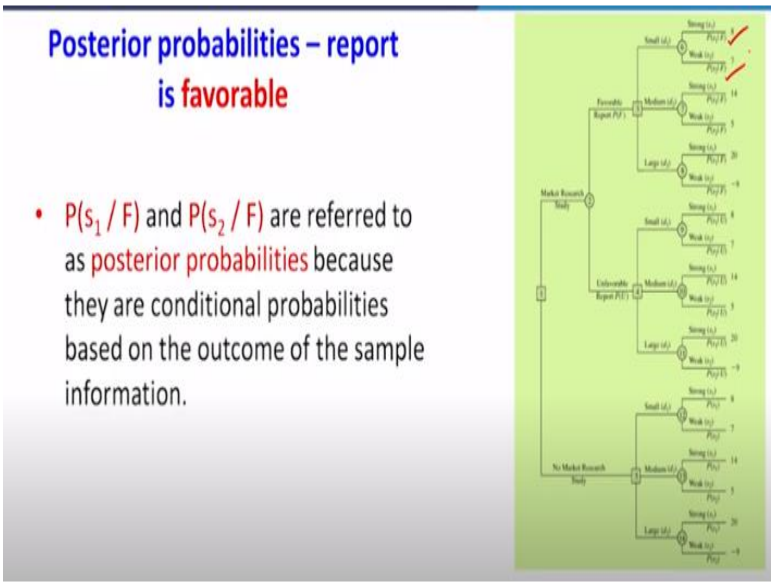


So, branch probabilities are computed using Bayes' Theorem. Now the company is going for a market research study; we know that. So, the F is a favorable market research report that the market research company may provide. F means favorable market research report. Or they may provide U with an unfavorable market research report. s_1 represents a strong demand state of nature 1. s_2 weak demand, state of nature 2.

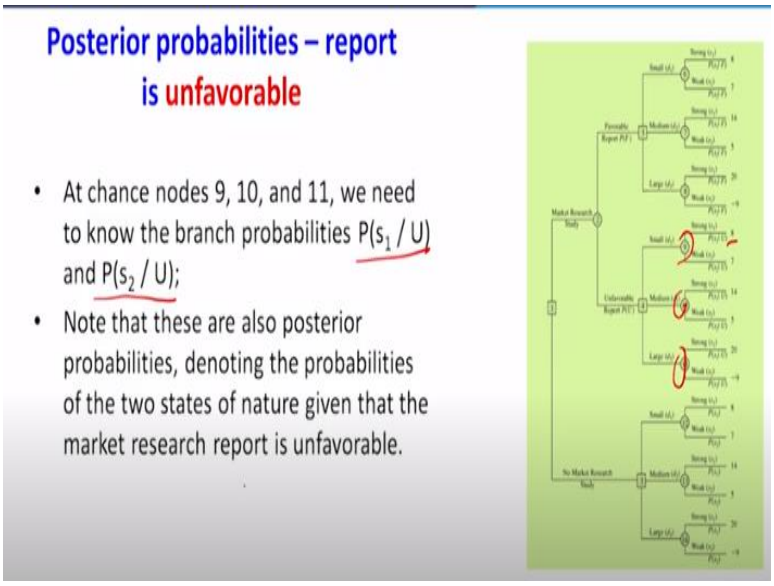


Posterior probabilities when the report is favorable. At chance node 2, please see this: at chance node 2, we need to know the branch probabilities P(F) and P(U), this one P(F) and P(U). At chance, we need to know the branch probabilities for nodes 6, 7, 8, and 6, 7, and 8. What are the branch probabilities? P(s_1) given F. Here it is a conditional probability. What is the interpretation of P(s_1) given F?

If the report is favorable, what is the probability that the demand will be strong? Similarly, what is the meaning of P(s_2) F? What is the probability that the demand is favorable? What is the probability of getting weaker demand?

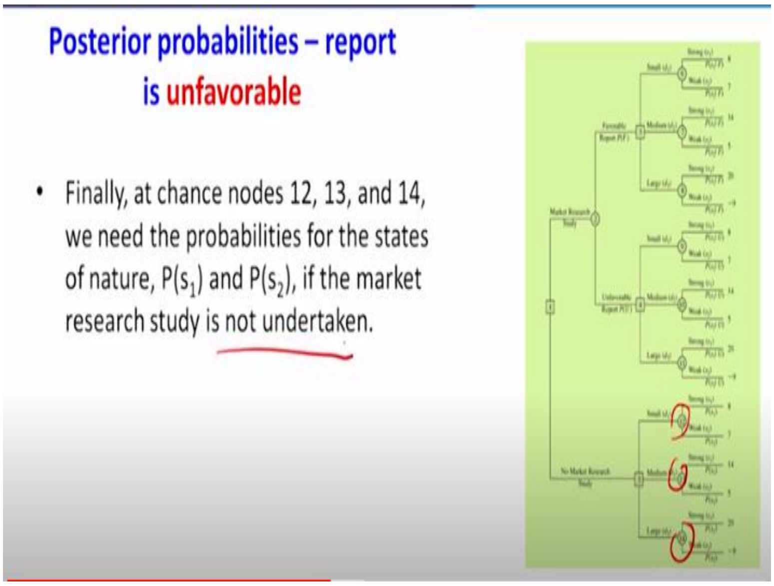


So the $P(s_1)$ is given F; $P(s_1/ F)$, and the $P(s_2)$ is given F at this point. $P(s_1)$ given F, $P(s_2)$ given F; $P(s_2/F)$ are referred to as posterior probability because they are conditional probabilities based on the outcome of the sample information. So, we are going to find out about this.

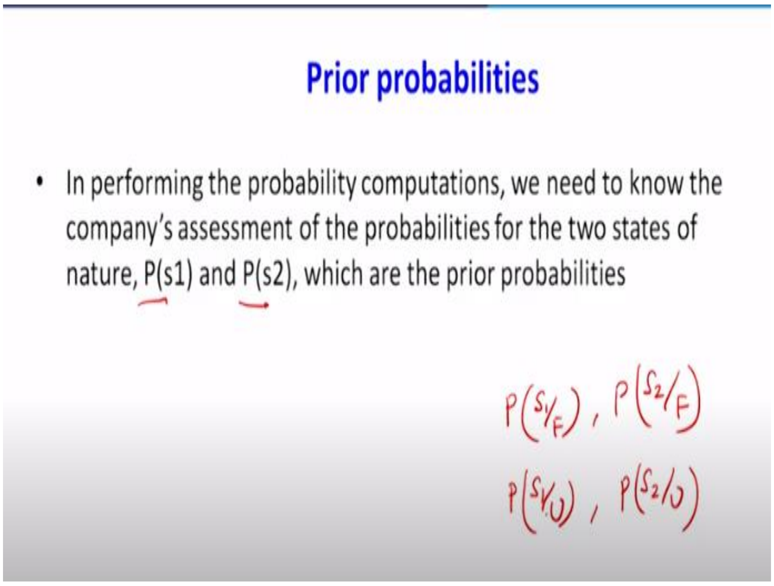


At chance nodes 9, 10, and 11, we need to know the branch probabilities $P(s_1 / U)$ and $P(s_2 / U)$. Note that these are also posterior probabilities, denoting the probabilities of the two states of nature given that the market research report is unfavorable. If the report is unfavorable, what is the probability that demand will be strong demand? That is the $P(s_1 / U)$. Similarly, if the report is unfavorable, what is the probability that the demand will be weak?

Note that these are also posterior probability denoting the probabilities of the two states of nature, given that the market research report is unfavorable.

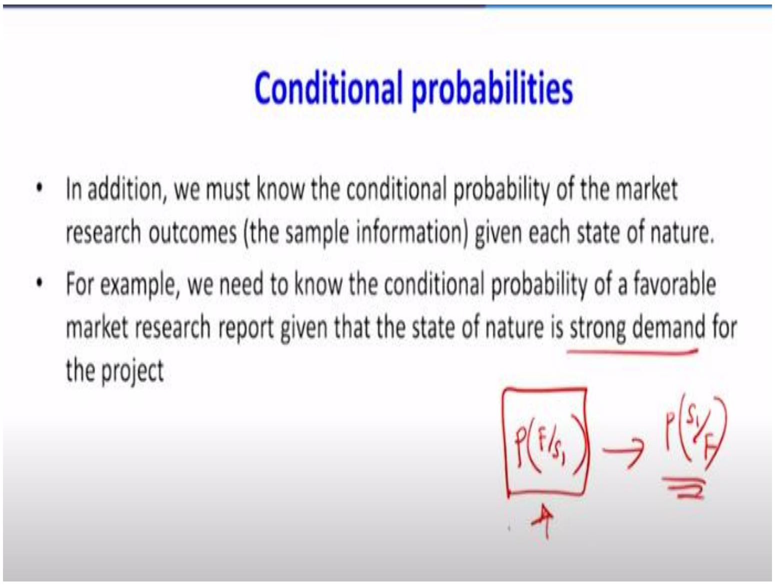


Finally, at chance nodes 12 here, 12, 13, and 14, we need the probabilities for the states of nature $P(s_1)$ $P(s_2)$ if the market research study is not undertaken.



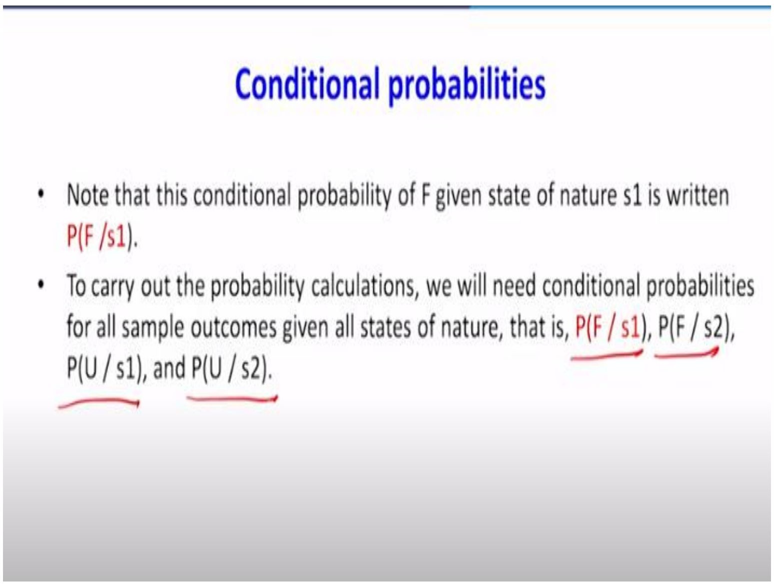
So far, I have been discussing posterior probability. Now, we should know what the prior probability is. In performing the probability computations, we need to know the company's assessment of the probabilities for the two states of nature, $P(s_1)$ and $P(s_2)$. So this $P(s_1)$ and $P(s_2)$ is called prior probability. So $P(s_1/F)$, this is our posterior probability. Or P(s2/F), which is also a posterior probability when the report is favorable.

$P(s_1 / U)$ or $P(s_2 / U)$ are posterior probabilities when the market research study report is unfavorable. We have to find out these values $P(s_1/F)$ and $P(s_2/F)$.



In addition, we must know the conditional probability of the market research outcomes given each state of nature. For example, we need to know the conditional probability of a favorable market research report given that the state of nature is in strong demand. So we need to know P(F) given s_1. We should know these values. From this, what is required is P(s_1) given F.

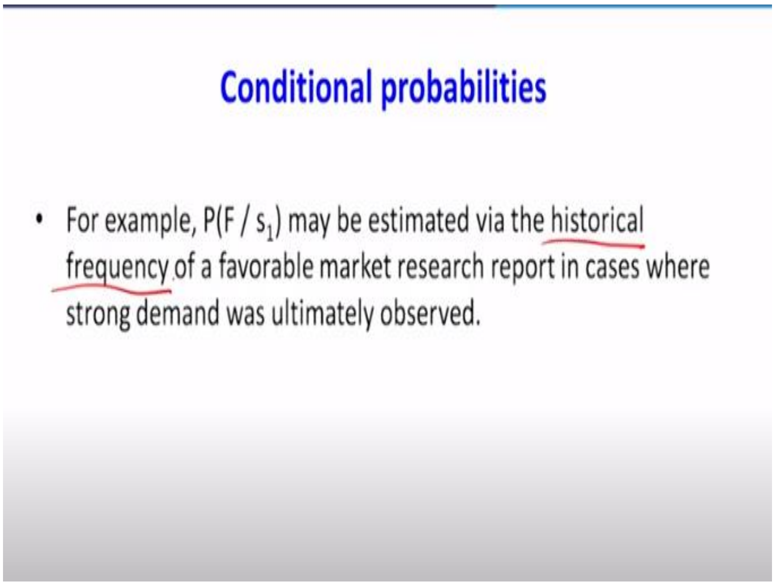
This is our branch's probabilities. But this probability has to be known to us. What does this say? If the demand is strong demand, what is the probability this market research company will provide a favorable report? So, this probability comes from the past data of the market research company or the credibility of the result of this market research company.



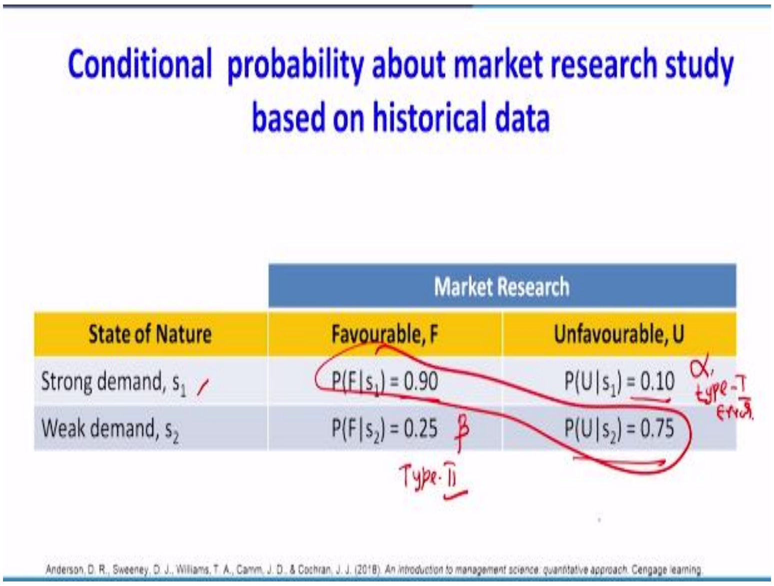
Note that this conditional probability of F given state of nature s1 is written P(F/s_1).

To carry out the probability calculations, we will need conditional probabilities for all sample outcomes given all states of nature, that is, $P(F/s_1)$, $P(F/s_2)$, $P(U/s_1)$, and $P(U/s_2)$.

It is these values that discuss the credibility of the report of this market research company.



For example, $P(F/s_1)$ may be estimated via the historical frequency of a favorable market research report in cases where strong demand was ultimately observed.

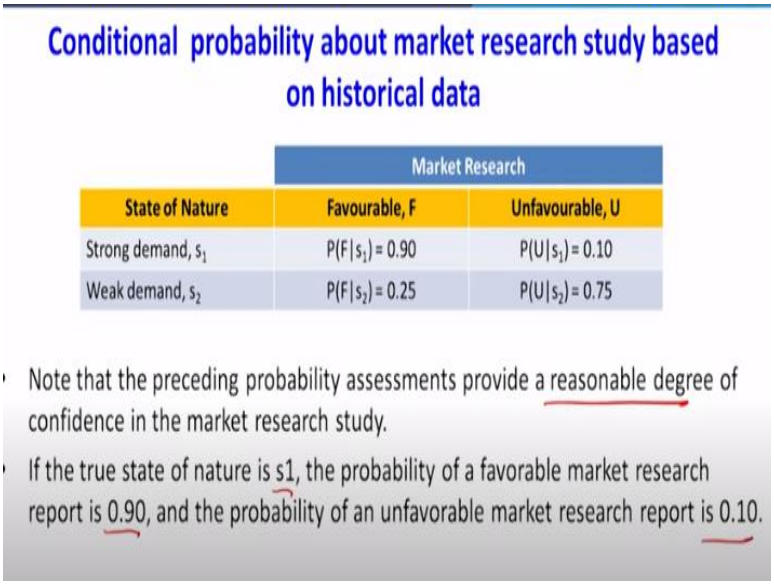


Now, the conditional probability of market research study is based on historical data. So this is given to us. What it says is that when the demand is strong demand, what is the probability this market research company will provide a favorable report? Say there is a 90% chance? Now, if the demand is strong demand but the company is providing an unfavorable report. For that, it is only a 10% chance, okay.

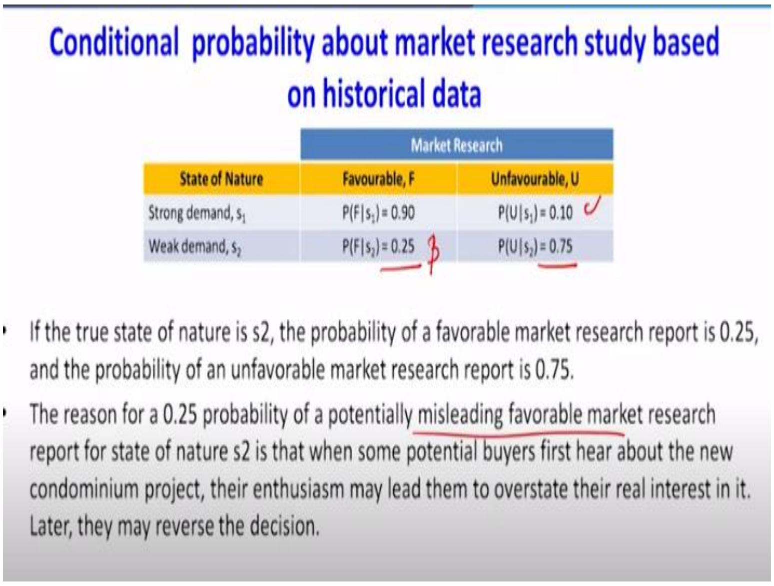
See another name for this one of your alphas, type I error. Look at the second row. When the demand is weak demand, what is the probability that the market research company will provide a favorable report? That is 0.25. So, this is called your beta. That is nothing but a type II error. So, something is weak, but the companies report that it is favorable.

But look at the next one. When the demand is weak, what is the probability that a market research company will provide unfavorable reports? So, 0.95. Since these diagonal values, right, are somewhat higher, it says that there is a reliability of the report provided by this market research company. Because when there is a strong demand, 90% of the time, it provides favorable reports.

If there is weak demand, 75% of the time, this company provides unfavorable reports. So, we can trust the results of this market research company.

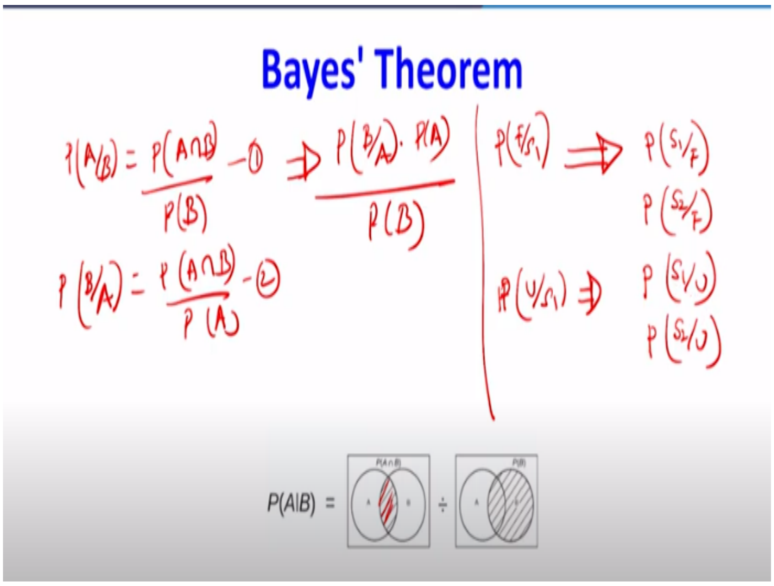


Note that the preceding probability assessment provides a reasonable degree of confidence in the market research study. How can we say there is a reasonable degree of confidence? If the true state of nature is s1, the probability of favorable market research in the research report is 0.9. The probability of an unfavorable market research report is 0.10.



Look at the table. If the true state of nature is s2, that is, weak demand, the probability of a favorable market research report is 0.25, and the probability of an unfavorable market research report is 0.75.

The reason for a 0.25 probability of a potentially misleading favorable market research report for the State of Nature s2 is when some potential buyers first hear about the new condominium project; their enthusiasm may lead them to overstate their real interest in it. Later, they may reverse the decision. That is why we are getting 0.20, sorry, 0.25. As I told you, this is our type II error beta. This is your alpha type I error.



Dear students. I am going to recollect your knowledge of conditional probability. After that, I will explain how Bayes' Theorem can be used to find the branch probability. So we know that P(A) given B can be written as P of A intersection B divided by P of B. Suppose I want to know if P(B) given A is okay. So this can be written as P of An intersection B divided by P(A).

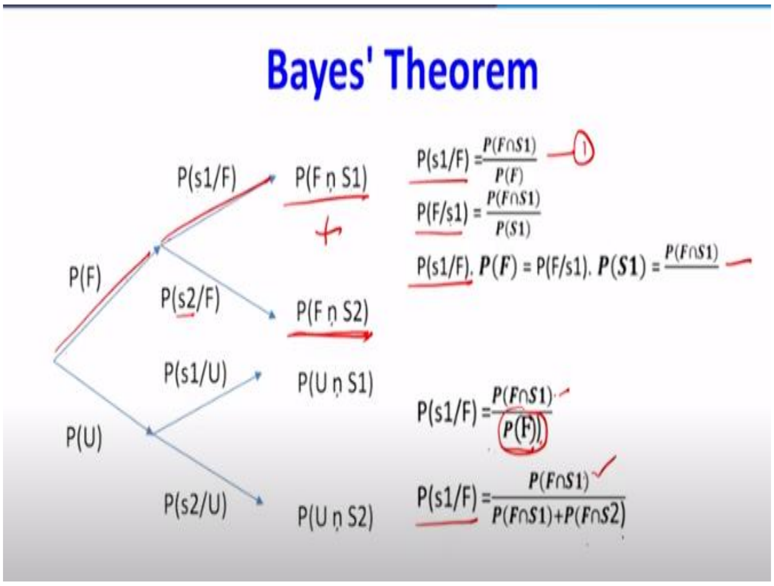
Look at numerators 1 and 2. The numerator is the same. So, what I can do is suppose in this expression, instead of this numerator A intersection B, I can write P(B) given A multiplied by P(A) upon P(B). So, this expression is nothing but your Bayes'—conditional probability. So, P(A) given B is equal to P(B) given A multiplied by P(A) divided by P(B).

If you want to represent this P(A) given B in a Venn diagram, first we have to find out the intersection. Look at this: A is there, and B is there. So, this shaded portion is an intersection B. That is our numerator divided by whole P(B). Okay, this is P(B). So, we are going to use this idea to find the branch probability. So, in our problem, what is given to us? Suppose I write here.

So, what do we need to find out? We have to find out the probability of s1 when the report is a favorable report. Similarly, we have to find out the probability of s2 when the report is favorable. Similarly, we must find out P(s1) when the report is unfavorable. Similarly, we must find out P(s2) when the report is unfavorable. But this information is not available to us.

What information is with us? We have this information P(F) given s1. So, from this information, we have to find out this information. What is the P(F) given s 1? This is a marketing research company; when the report is, when demand is strong, demand there is this small amount of probability that they will provide a favorable report. Here, we also have this information: P(U/ s1).

What is the meaning? This also shows the credibility of your market research company. That is, if the actual reality is a strong demand, the probability of giving an unfavorable report is P(U/s1) . But we want P(U/s1). So, this one, we are going to find out with the help of Bayes' Theorem.



Dear students, in the previous slide, I explained the concept of conditional probability. Now, I am going to explain how we are going to find this branch probability by extending the conditional probability, which is nothing but your Bayes' Theorem. We have seen that suppose P(s1) is given F; that is what we want to know. That is, if the report is favorable, what is the probability it will be in strong demand?

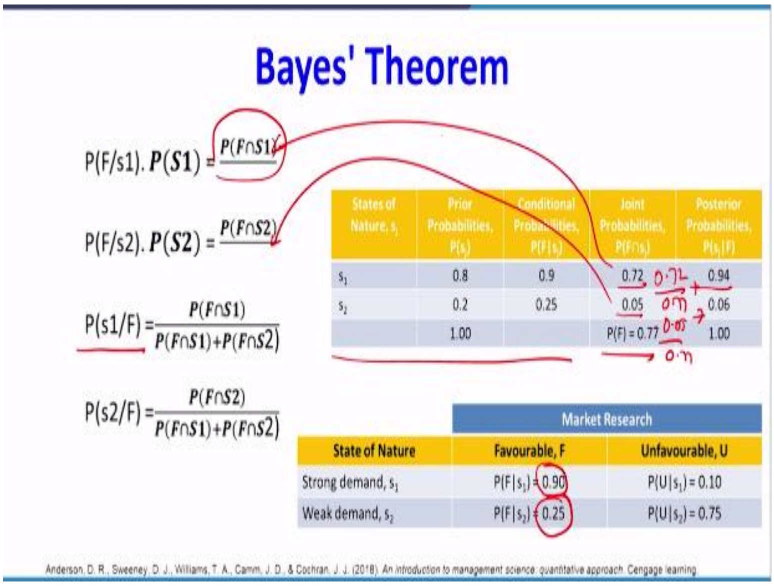
So what do we have to do? P of F intersection s1 will be P(F). This is our first expression. P(F) given s1. This information is available from the market research study report. What is the meaning of this one? If the actual demand is strong demand, what is the probability that the company will provide a favorable report? So, as usual, P(F), given s1, will be F (s1).

So when I cross multiply, what will happen is P(s1) given F multiply P(F) equal to P(F) given s1 multiplied by P(s1). That is equal to P(F) intersection s1. So, from this expression, what I wanted to know was this information: P(s1) given F. So, what I am doing is dividing this numerator P(F) given s1 by this P(F).

Now I am going to pictorially represent the meaning of this P of F intersection s 1 and the P(F). Look at the picture on the left-hand side. We assume that we know the probability of a favorable report. The next one says that P(s1) is given F. If the report is favorable, what is the probability of strong demand? So, when you multiply this, that is the intersection P of F intersection s1.

So, when the report is a favorable report, there is a chance the demand may be weak demand. So that is P(s2) given F. That is your P of F intersection s2. So now what I can do is this P(F) can be replaced by P(F) given s1 plus P(F) given s2. How can we replace it because only the P(F) is split into two branches? So instead of P(F), we can write P of F intersection s1 plus P of F intersection s2.

So, if you divide this P of F intersection s1 by the sum of this P of F intersection s1 plus P of F intersection 1, I can get P of s1 F. But this information P(F) given s1 and P(F) given s2 we are going to find out from our historical data. How?



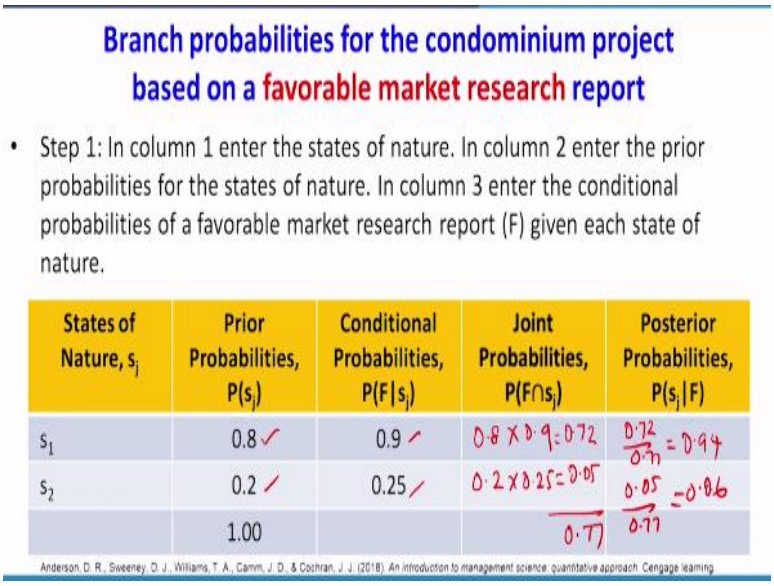
Look at this table. In this table, the prior probability known to us is s1 and s2. And P(F) given s1 is 0.9. How did we get this 0.9? We got this 0.9 from our value. This value is the market research company, which will provide a 0.9 percent probability if the actual demand is strong. That is why I have written P(F) given s1, so 0.9. We got 0.25 from this value, okay?

So, when I multiply this P(s) and P(F) given s 1, is it that I can get P(F) intersection s 1, right 0.72? Here, if I multiply 0.2 by 0.25, I can get P(F) intersection s2. Now, I am going to use these two values, P of F intersection s1 and P of F intersection s2; I am going to substitute this value here. We have seen this expression in the previous slide. The second one, 0.05, is nothing but this one P of F intersection s2.

So, in the previous slide, I have shown you that P of s1 given F is nothing but P of F intersection s1 divided by the sum of these intersections. So, what is that value? It is

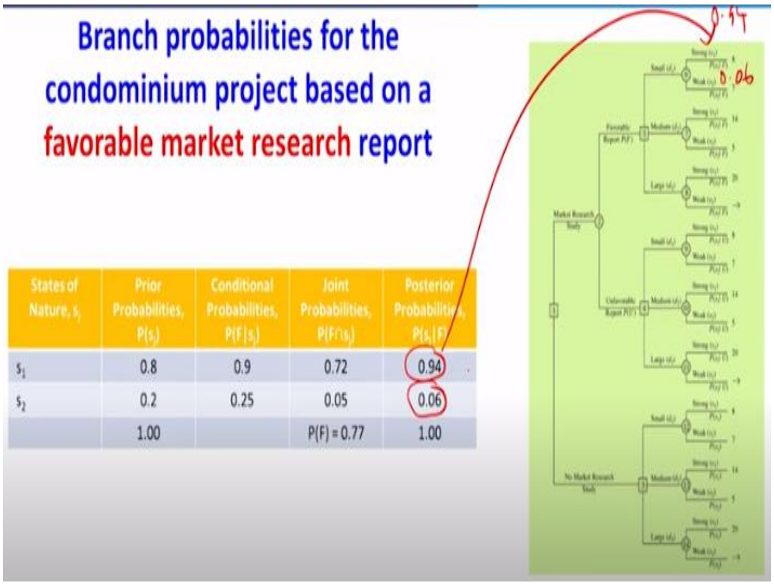
0.2 plus 0.05. So, how did we get 0.94? That is 0.72 sum of these two values P(F). That is 0.72 plus 0.75, we are getting P(F). So, 0.72 divided by 0.77 means I am getting 0.94. Here, 0.05 divided by 0.77, you are getting 0.06.

So, this is nothing but P(s1) given F. So now, I am getting this value from our past information, P of F intersection s1 and P of F intersection s2. That past information is used to find out P(s1) given F. So, this is the final value.

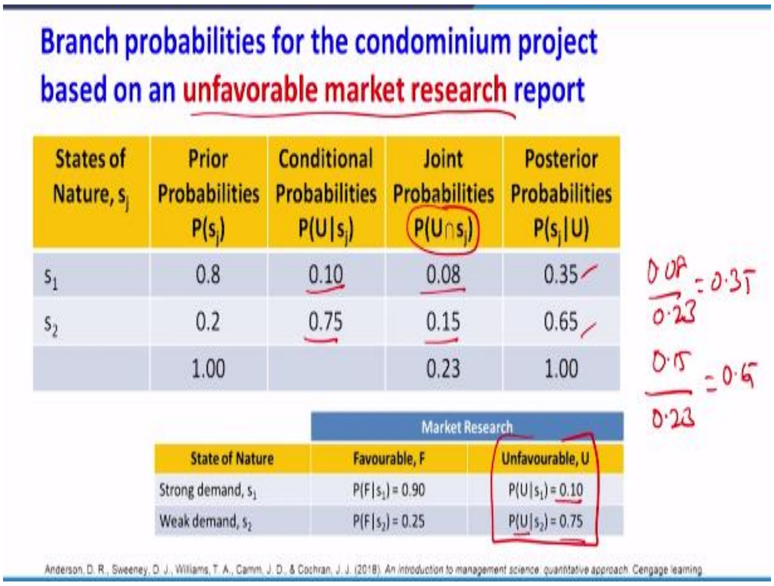


Dear students. Now, I am going to summarize the procedure for finding branch probabilities with the help of this tabular approach. So, what is step 1? In column 1, enter the states of nature. We have written s1 and s2. In column 2, enter the prior probabilities for the states of nature. This information 0.8, 0.2 is given to us. In column 3, enter the conditional probabilities of favorable market research report F given each state of nature.

So, 0.95, 0.9 and 0.25. Then you multiply this by 0.8 multiplied by 0.9. Then 0.2 multiplied by 0.25. This is the final value. when you add it to 0.77. To find out the posterior probability, so 0.72 upon 0.77. Then 0.05 divided by 0.77; 0.94, 0.06.



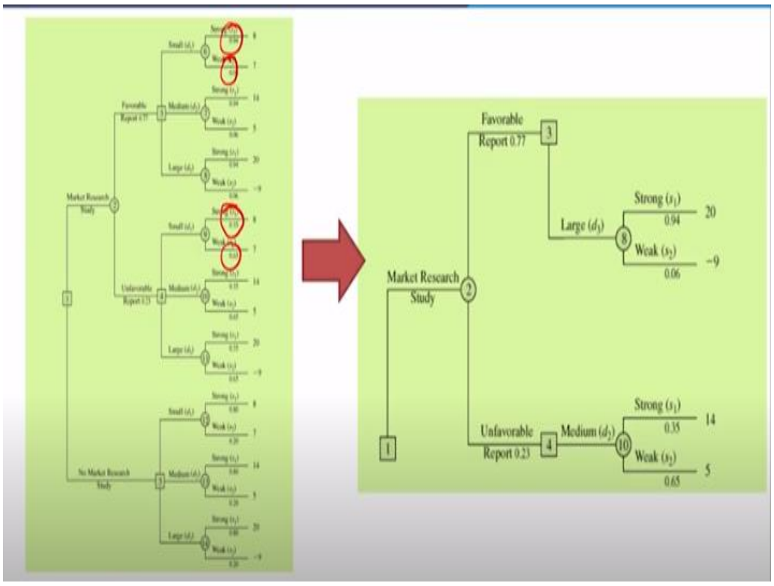
Now we have got the final result. What is that P(s1) given F is 0.94? P(s2) given F is 0.06. So, this value is nothing but this value. So, in our previous problem, this 0.94 is given. Similarly, 0.06 is given to you in advance. So, we have done our procedure. But now, I have explained how we got this value of 0.94 and 0.06. This is when the report is favorable. Now that the report is unfavorable, the same procedure will be followed.



Now, finding the branch probabilities for the condominium project based on an unfavorable market research report like what you have done for our favorable market research report, the same procedure has to be followed. First, you have to enter the states of nature. Then prior probability. Now conditional probability P(U) given sj. So this portion of the data. What is the meaning of this one?

When the actual demand is strong demand, what is the probability that the company will provide an unfavorable report, 0.10 that I have written here? When the actual demand is weak demand, what is the probability that the company will provide an unfavorable report, 0.75 that I have written? So now, the intersection of P(s) and P(U) is given sj. So when I multiply it by 0.08, 0.15.

Then I have to add this. So, 0.23. So, how did I get 0.35? So, 0.08 upon 0.23. You will be getting 0.35. The next one is 0.15 upon 0.23, and I am getting 0.65. Now I have branch probability of 0.35 and 0.65. Here, the important concept that we are learning is that with the help of these intersections, P of U intersection s1 or P of F intersection s2, we are getting our posterior probability by equating this joint probability.



So now we have this. I have explained to you how I got 0.94. Then I explained this value, as well, how I got 0.35 and 0.65, similarly 0.06. Now, once these values are known, we can get the final result. So, what we have done in this lecture is that I have explained how we got the branch probabilities. In the previous lecture, this branch property was given to you. Now, we have found this branch of probabilities using Bayes' Theorem.

Dear students, in this lecture, I have explained the risk profile. Then, I compared the risk profile with sample information and without sample information during decision-making. The next point I have explained is the expected value of sample information and what is the interpretation of this expected value of sample information. Finally, I have explained with the help of Bayes' Theorem, how to compute the branch probabilities. Thank you very much.