Decision Making With Spreadsheet
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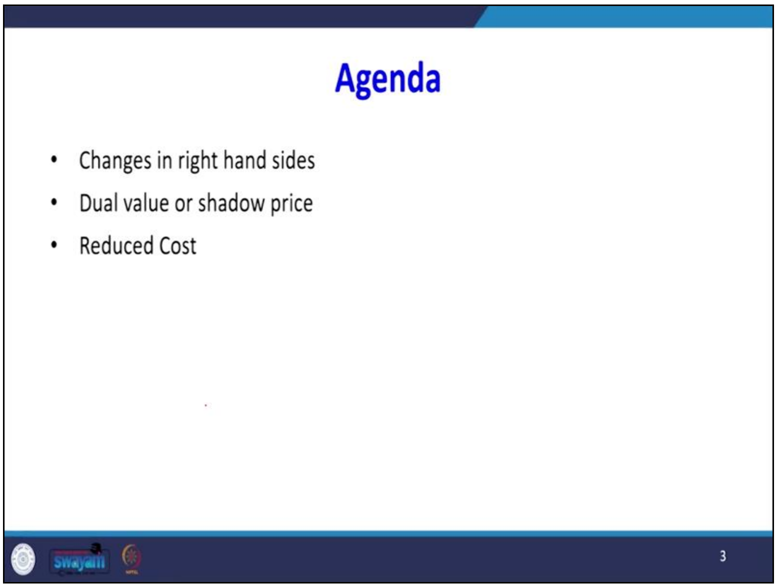
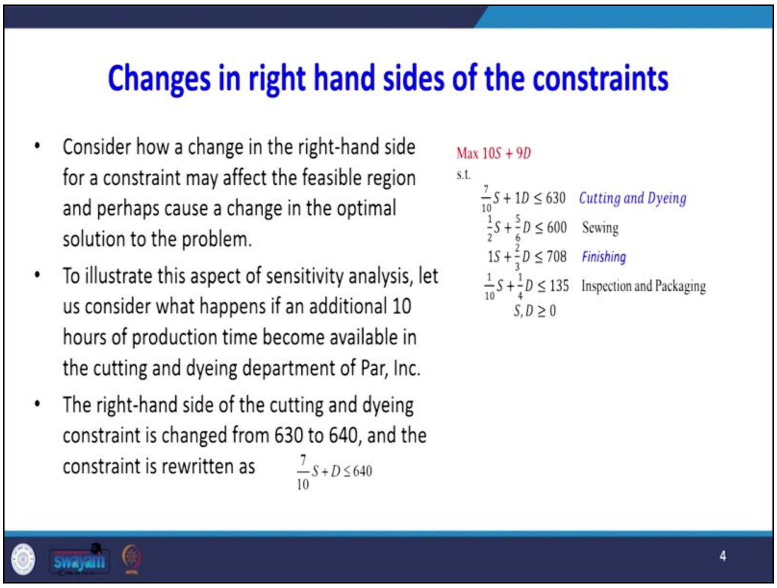
Lecture-07
Sensitivity Analysis - 2

Dear students, in the previous lecture, we started a topic called sensitivity analysis in which we discussed the effect of change in objective function coefficient on our optimal value. In this class, we will discuss the impact of change in the right-hand side value of the constraint on our problem.



The agenda for this lecture is what the effect is if any value changes on the right-hand side of the constraint. Then, we will discuss about what is the dual value. The other name is shadow price, and then there is another term called reduced cost.

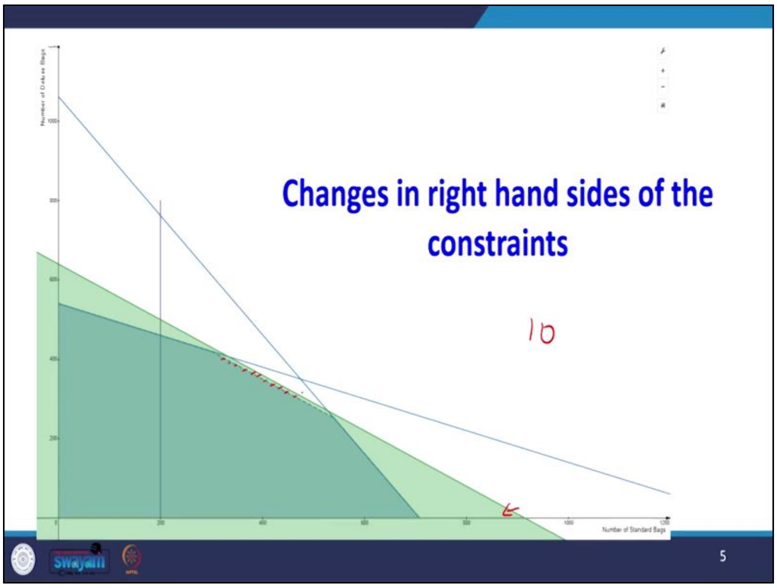


Now, first, we will see the changes on the right-hand side of the constraint. I have taken this problem, which we have been discussing since the beginning of this course. Consider how a change in the right-hand side of a constraint may affect the feasible region and perhaps cause a change in the optimal solution to the problem. So, we are going to discuss if there are any changes here on the right-hand side of the constraint and how that will affect our feasible region.

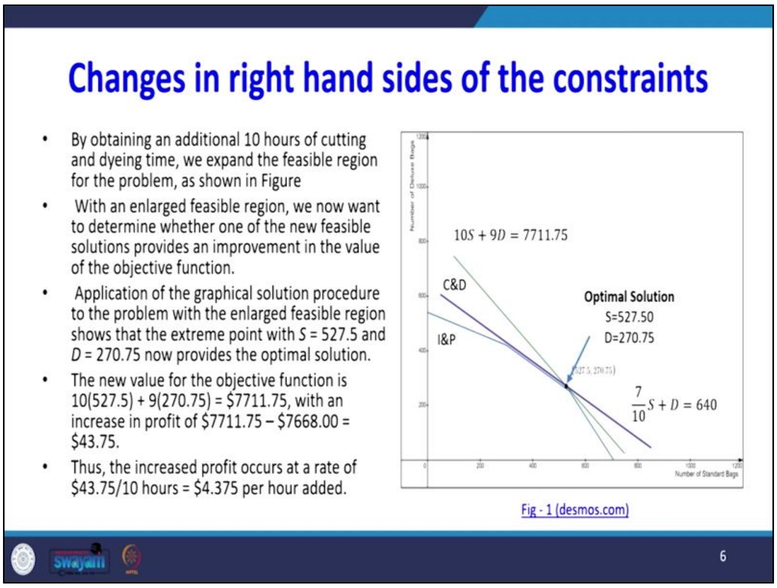
Thereby how that will affect our optimal solution, to illustrate this aspect of sensitivity analysis, let us consider what happens if an additional 10 hours of production time becomes available in the cutting and dyeing department. So, the right-hand side of the cutting and dyeing constraint is changed from 630 to 640 because we got 10 hours of additional resources.

Now, our constraint will become like this: (7/10)S + D <= 640.

Previously, it was 630. We have added another 10 units of resources. Now it become 640 by adding that 10 unit of these resources.



Now you see that there is there is a region that our feasibility region is extended. Now, this was after adding 10 units of additional resources; the initial one was in the dotted line because of this 10-unit increase on the right-hand side.



Now the feasible region is increased increasing that is shown in the shaded region shows that increase in the feasible region due to an additional 10 units on the right-hand side of the constraint by obtaining an additional 10 hours of cutting and dyeing time, we expand the feasible region for the problem as shown in figure here also there is a this much unit is additional area due to additional resources of 10 units with an enlarged feasible region we.

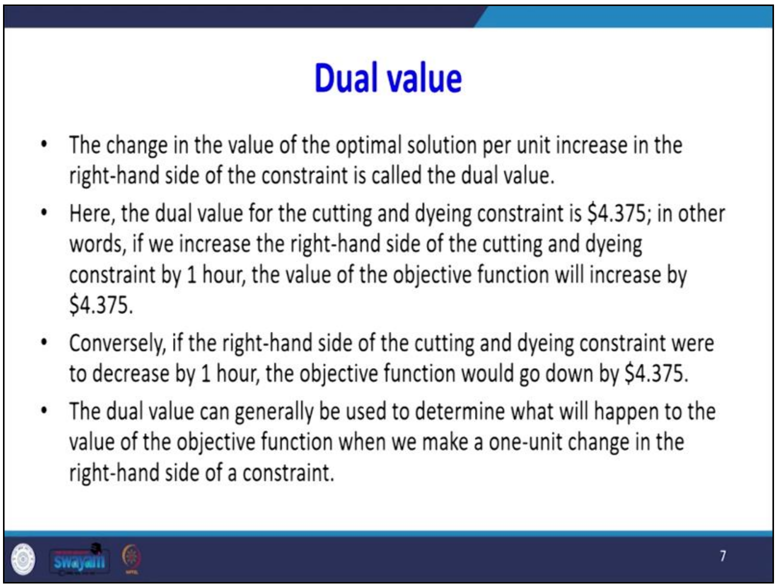
Now want to determine whether one of the new feasible solutions provides an improvement in the value of the objective function.

So, the application of graphical solution procedures to the problem with an enlarged feasible region shows that the extreme point with S = 527.5 and D = 270.5.

Now, provide the optimal solution. Now, what has become our previous solution solutions changed due to the addition of 10 units.

Now the new solution is this one 527.50 then another one is D = 270.75. The new value for the objective function is when you substitute this. So, ah 10 S plus 90 now the value of S is 527.5 value of D is 270.75. Now this is 7711.75 is the new value for our objective function. So, how much is increase has happened due to that 10 additional resources. So, when you our earlier solution was additional value of objective function is 76 by adding 10 unit resources.

Now our objective function values increase this much. So, the total increment for 10 additional units of the resources is 43.75. So, if you want to know per unit when you divide by 10. So, this is 4.375 dollar per hour added. So, what has happened now by adding one unit additional resources on the right hand side of that constraint ok increased value in our objective function is 4.375. This 4.375 is nothing but our dual value.



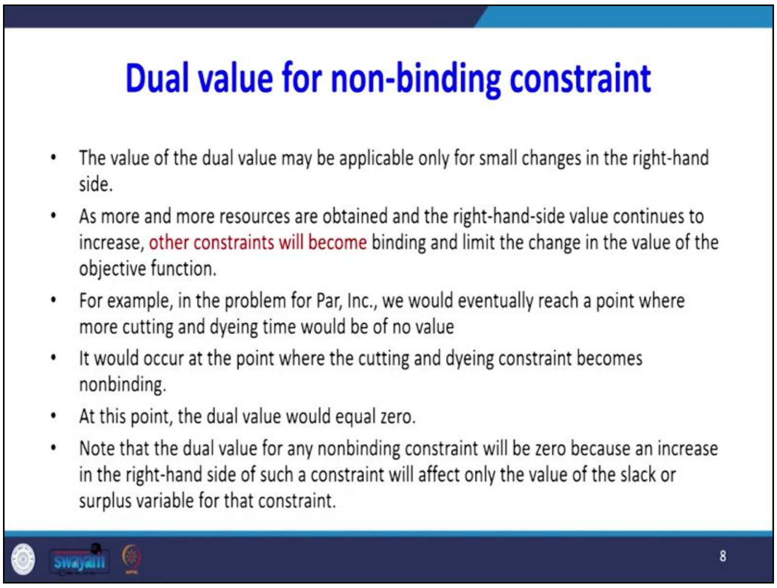
The change in the value of optimal solution per unit increase in the right hand side of the constraint is called dual value. Here the dual value for the cutting and dying constraint is 4.375 in other words if we increase the right hand side of the cutting and dying constraint by one hour the value of the objective function will increase by 4.375 dollar. Conversely if the right hand side of the cutting and dying constraint were decreased by one hour the objective function will go down by 4.375.

So, the dual value can generally be used to determine what will happen to the value of the objective function when we make one unit change in the right hand side of a constraint.

Dear students now I will explain with the help of excel what is the effect of the 10 unit change on the right hand side of the first constraint on our objective function. So, as usual what I have explained my previous class I have entered the value in excel. So, this is ah initially take it to 0 this is my decision values. Now right hand rate is 630 I have used this is the resources constraint as usual because for the this is the problem for which I have made the excel solver.

So, when you go to data solver yes when I solve it the objective function value is 7668. Now what I am going to do is I am going to increase in F6 instead of 630 I am going to increase this to 640. So, when I make 640 again, I am going to run it solver. Now this is 7711.75. So, the difference is 43.75 this is for 10 unit if I want to know for per unit. So, = this value divided by 10 is that is 4.375.

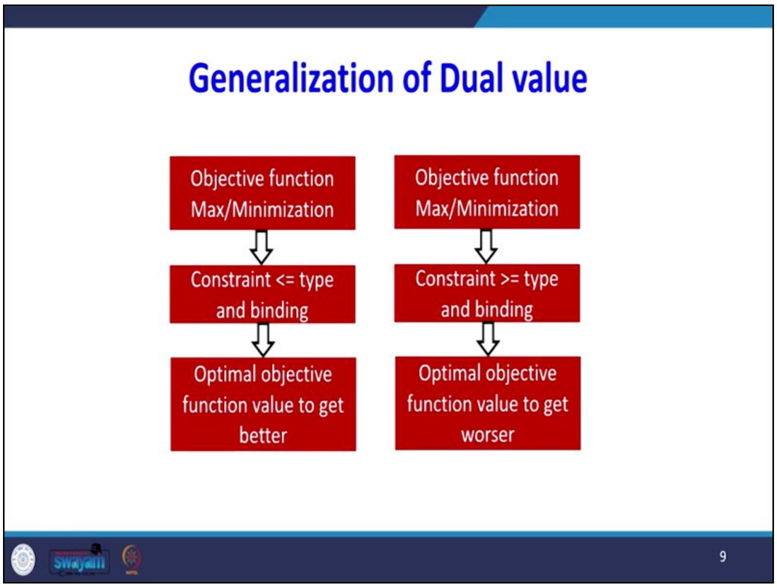
What is the meaning of this four three ah 4.375 if the right hand side is increased by one unit the increase increased value in the objective function is 4.375 this is nothing but our dual value.



The value of the dual value may be applicable only for small changes in the right hand side for example one unit increase or 10 unit increase. As more and more resources are obtained and the right hand side value continues to increase the other constraint will become binding and limit the change in the value of objective function. For example the problem which we are discussing we would eventually reach a point where more cutting and dying time would be no value.

It would occur at the point where the cutting and dying constraint become non-binding. At this point the dual value would = 0. Note that the dual value for any non binding constraint will be 0 because an increase in the right hand side of such constraint will affect only the value of the slack or surplus variable for the constraint. Because for the non binding constraint already there will be a slack or surplus variable.

We know that the effect of the slack and surplus variable on the objective function is 0. So, for non binding constraint adding any resources on the right hand side will not make any difference in our objective function value.



So, how we can generalize for the dual value. There are two possibility the objective function may be maximization or minimization if the constraint are less than or = type unbinding. So, in this situation the optimal objective function value get better. What is the meaning of to get better? If the objective function is maximization type if the constraint is less than or = type.

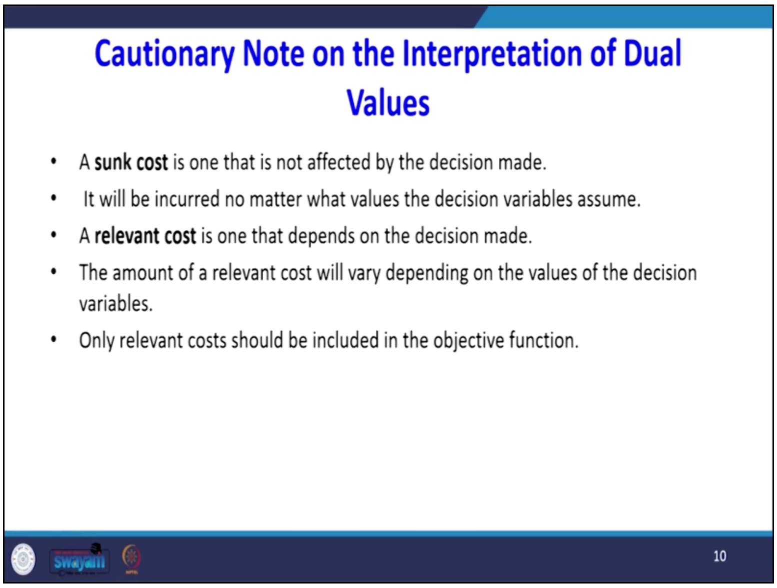
So, adding one resources on the right-hand side will increase your value of maximization function other way if the objective function is minimization type if the constraint is less than or = type by adding one resources will minimize your objective function that is the meaning of get better. So, because what is the meaning of adding any resources for the constraint which are less than or = type that means that you are providing extra resources.

So, when you for less than or = type when you are adding extra resources obviously the efficiency of the system will increase. What is the meaning efficiency if it is a maximization the problem will bring more the value of objective function increase if it is a minimization the value of objective function will decrease. On the other case if the objective function is maximization minimization but the constraint greater than or = type and binding.

Now exactly here what is happening here if you are tightening the constraint. So, when you are tightening the constraint the objective function value to get worser what is the meaning of worser? If it is maximization type if the objective function is greater than or = type if you are adding one resource on the right hand side your objective for the maximization type the value will decrease.

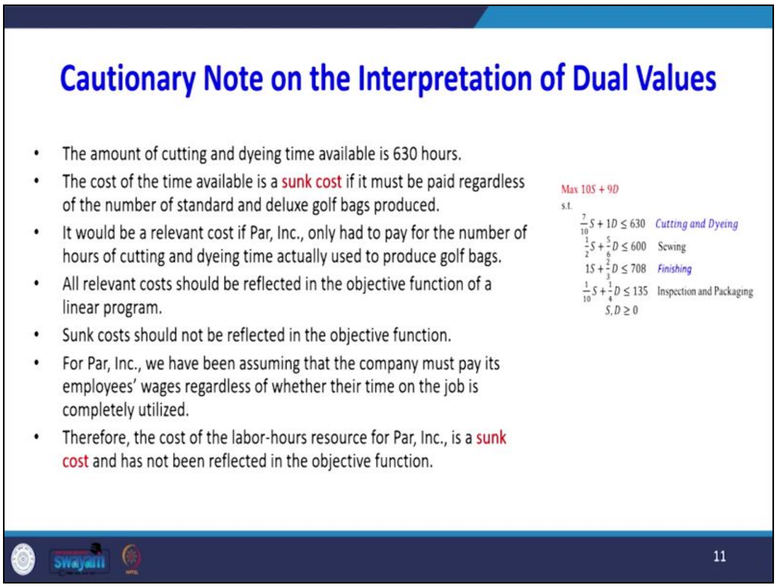
For the minimization type the value will increase because you are tightening tighten means you are not providing more freedom because the expectation is increases. For example yes say minimum should be 500. Suppose if you say this is and greater than or = sign assume that there is a binding constraint. Suppose if you are making this should be 501. So, what is happening that your expectation is increases when the expectation increases the efficiency of that system will decrease.

That means if it is a maximization your profit will decrease if it is the minimization your cost will increase that is the generalization of this dual value. Either for constraint which are less than or = type or constraint or constrained curtain or = type.



Cautionary note on the interpretation of dual values: Now there are two terms which you need to understand a sunk cost and relevant cost. What is the sunk cost a sunk cost is one that is not affected by the decision made it will be incurred no matter what values the decision variable assumes. For example if say constraint number 1 the amount which was sent amount which are spent for acquiring that constraint number one it is a sunk cast even though adding extra resources that will not be reflected on our objective function.

So, there is no point in adding extra resources when the cost is sunk cost. Then what is the relevant cost the relevant cost is one that depends on decision made. The amount of a relevant cost will vary depending on the value of the decision variables only relevant cost should be included in the objective function. So, when we interpret the meaning of dual values the meaning of dual value is applicable only for the relevant cost not for the sunk cost. What is the relevant cost by adding extra resources that will help to increase your objective function.



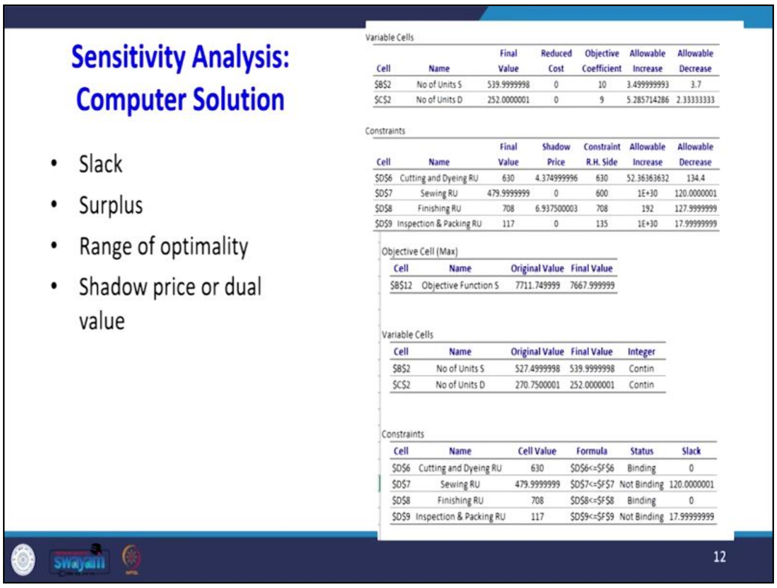
Now I will explain with the help of the problem which you have taken the amount of cutting and dyeing time available is 630 hours for this problem see 630 hours. The cost of the time available is a sunk cost if it must be paid regardless of a number of standard and deluxe golf bags produced. When it becomes sunk cost, see the salary is fixed for the employees. Whether you produce 631 or 640, the salary is fixed.

So, that time the salary becomes the sunk cost. So, it would be relevant if it would be a relevant cost per incorporation for the problem that you discussed only had to pay for a number of hours of cutting and dying time actually used to produce the golf bags. Many times in the industry context, if there is a contract employee, you pay the money based on how much bag has been produced. So, that money which you are spending on them is becoming a relevant cost.

Assume that it is a permanent employee. Whether he produces 630 or 640, the salary is fixed, and that cost is a sunk cost. So, the point that I am trying to make here is the interpretation of dual value is applicable only to the relevant cost. So, the point here is that all relevant costs should be reflected in the objective function of a linear programming problem. Sunk cost should not be reflected in the objective function.

For the problem which we are discussing we have been assuming that the company must pay its employees wages regardless of whether their time on the job is completely utilized. Therefore, the cost of the labor hours resources is a sunk cost and has not been reflected in the objective function. So, this is the caution that you have to note down at the time of interpreting the dual value.

What is that point which you have to remember the interpretation of dual value is only applicable for a relevant cost, not for the sunk cost.



Now we will go back to our initial problem. Now, we will discuss the Excel output solver output to interpret the sensitivity analysis. What are the terms we are going to interpret? What is the slack variable, what is the surplus variable? What is the range of optimality? What is the shadow price dual value, and what is its interpretation? I have brought the output of our solver.

Now I am going to explain how to get the sensitivity analysis report in the solver. So, I have entered the problem. We know that, as usual, this is the resources utilized. This is the return. The sign of this constraint is right-hand side values. So, go to data, and go to solver. So, when I am solving. So, I got this one. You see that there is an answer sensitivity limit, then press. So, now you have the answer report, is there where you can get the value of objective function 7668? Then we got the value of B2 cells, that is, yes, the value of S is 540. Can you make it center?

So, the value of ah S = 540 and D = 252. Similarly, you can also make it centered. Now there is a there is a slack I will explain what the meaning is of this slack. And when you go to the sensitivity report here also, there is a value of S and D is there, there is a term called reduced cost, and then there is an allowable increase and allowable decrease. So, I have taken a screenshot of this output, which I will explain in the presentation.

Similarly, in the constraint, I will center it here. There is a final value of constraint there is an important term called shadow price. Shadow price is nothing but dual value. For some books, they call it shadow price, and for some books, they call it dual value. So, can you see that this is a 4.37? That is, if this constraint cutting and dying on the right-hand side is increased from 630 to 631, the objective function will increase by 4.37 units.

And here also there is a Currently, the right-hand side is 630. There is an allowable increase and allowable decrease. Similarly, for other constraints, perceiving constraint for finishing constraint and the last constraint after that, there is another report called limit report. In the limit report here, there is also a lower limit, upper limit, and corresponding objective function value, for example, the value of S lower limit to 0.

So, in our objective function, if you substitute S = 0, the objective function value is 2268. Suppose in that objective function S = 0 instead of 0. When you put uh 539, you will get 7667. The second one says that the value of D if we put 0, the value objective function is 500 and, sorry, 5399. If you put 252, you will get 7668. I have taken a screenshot of this output, which I will explain here, and with the help of a screenshot, I will interpret this answer.

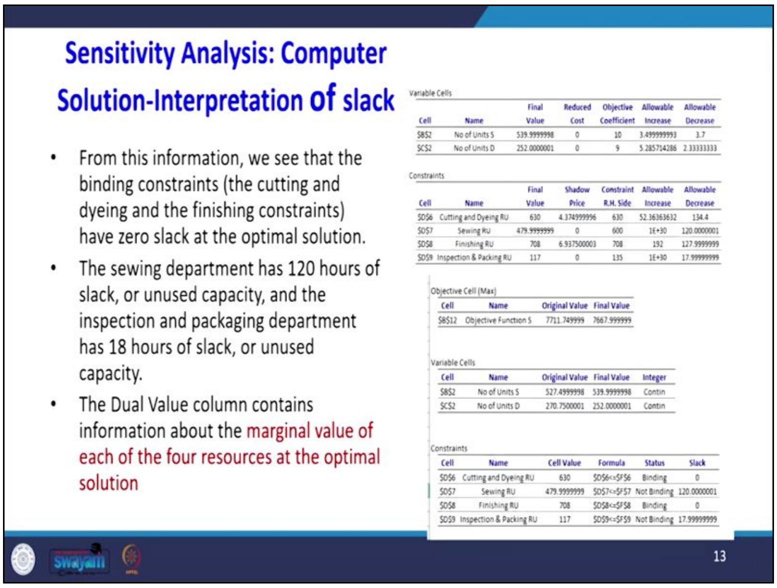
Now, from this, we can see the value of S is 540, and the value of D is 252. Now you see that there is an allowable increase. So, from 10, it can be increased to 3.4, which is a 13.4 allowable decrease of 3.7. Similarly, for our second addition variable, the value of D is 9. It can be increased by 5.2, and it can be decreased by 2.33. So, this portion says the range of optimality what is the meaning of the range of optimality even though the value of the objective function is increased by 3.4 and decreased by 3.7 for the first decision variable.

Our value, which is, 540 and 252, will remain the same, which is called the range of optimality. Second, you see that there is a shadow price. What is the shadow price for this constraint cutting and dying? If the right-hand side is increased by one unit, your objective function value will increase by 4.37. You see that this is a binding constraint which one cutting and dying. But see the second constraint, seeing constraint ok already there is a slack variable is there will come back where is the slack variable here for the sieving constraint already there is a slack I will interpret I will come back to the later this one.

So, for the value of shadow price is applicable only for binding constraint. For non binding constraint the value of slack variable value of dual value is 0. Here also the right hand side constraint can be increased 52.36 and decreased 134 without affecting the shadow price. This is called range of feasibility the first one is called range of optimality this is called range of feasibility.

At the bottom there is a slack variable. For example, for the first constraint, cutting and dying, the slack variable is 0. What is the meaning of slack here? If it is all are since all the constraints are less than or = type, these are unutilized resources, which means we are fully utilized. We have fully utilized cutting and dying department resources. Similarly, for the finishing department also, we are fully utilized, which is why the slack is 0.

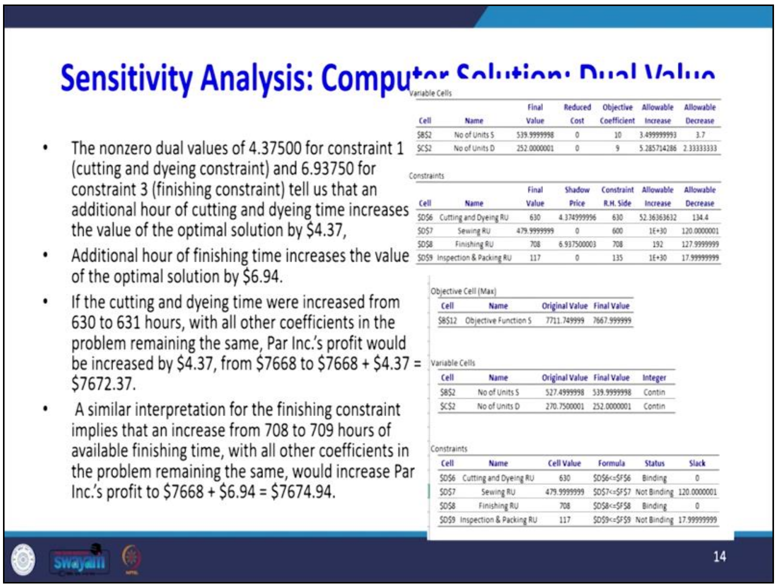
For another constraint, for example, receiving an inspection, we have a positive slack, which means that 120 units in the sieving department is 120 units of resources. I think hours are not utilized, and here, 18 hours are not utilized in the inspection and packaging department.



Now, we will go into detail about interpreting this output. So, from this information, which is given on the right-hand side, we see that binding constraints, such as the cutting, dying, and finishing constraints, have 0 slack at the optimal solution. Where are they here? There is this. The sieving department has 120 hours of slack in unused capacity, and the inspection packaging department has 18 hours of slack in unused capacity. This is 17.99, 18.

So, the dual value column contains information about the marginal value of each of the four resources at the optimal solution. So, the dual value here is the shadow price of this portion.

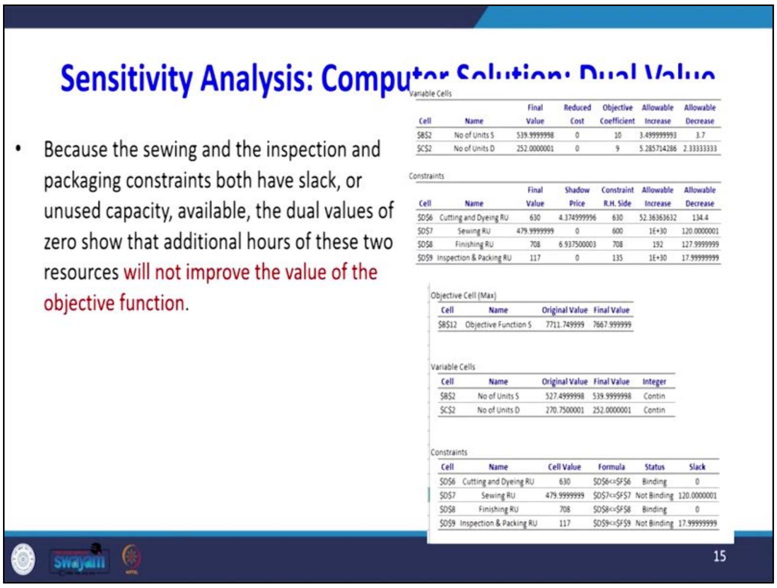
If the resources are increased by one unit, the corresponding increase in the objective function is called the marginal value of each of the resources at the optimal solution.



The interpretation of this dual value: The nonzero dual values of 4.37 for a constraint 1 is for the cutting and dying constraint and 6.93 where,is 6.93 and 6.93 for the finishing constraint, tells us that an additional hour of cutting and dyeing time increases the value of the optimal solution by 4.37 and additional hours of finishing time increase the value of the optimal solution by 6.94 dollars.

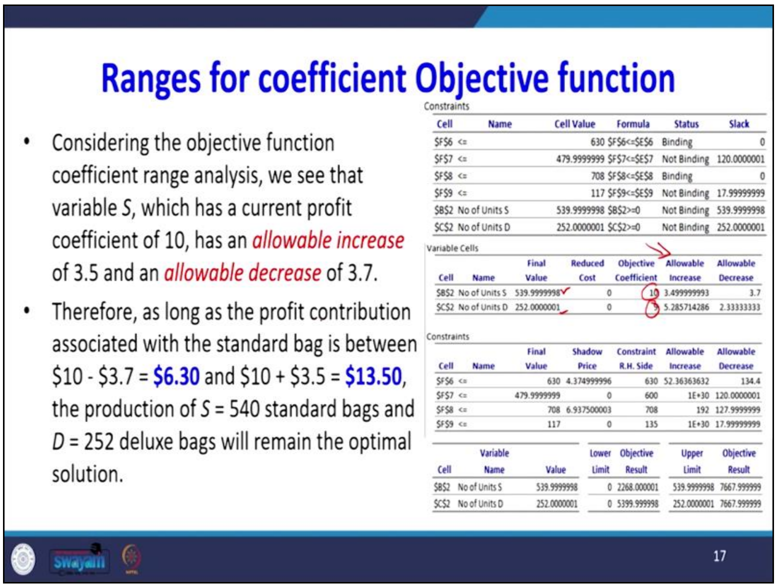
That is, if the cutting and dying time were increased from 630 to 631 hours with all other coefficients in the problem remaining the same, that is an assumption. We are making one change at a time. Okay, then the profit would be increased by 4.37, which is 7668, 7668 + 4.37, which is from 7668 to 7672.37 dollars. A similar interpretation for the finishing constraint implies that an increase in from 708 to 709. So, this one 708 if you add one additional unit in that department.

What is the condition with all other coefficients in the problem remaining the same? So, the increase in profit is 7674.94 this is an interpretation of dual value.



Because sewing an inspection and packaging constraints both have slack or unused capacity available. The dual value of 0 shows that an additional hour of these resources will not improve the value of the objective function. Because these two departments, which one is this department, already have unutilized resources, by adding extra resources again in those two departments, there is no use. So, that is the meaning of this one.

For a non-binding constraint where there is a positive slack is there adding any extra resources for that constraint will not improve our objective function value.



The other term we are going to say that it is called reduced cost. This is also a kind of dual value. So, the reduced cost associated with the variable is = the dual value for the nonnegativity constraint. This is also kind of a dual value, but this is for nonnegativity constraints associated with the variable. Where is the reduced cost is there here. So, there are two. These are nonnegative deconstructions.

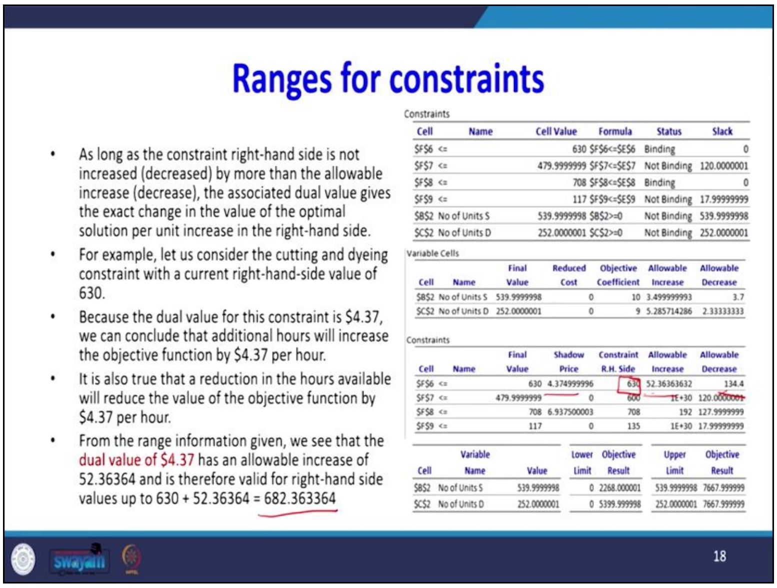
Because we are assuming that the decision values are greater than or = 0, what is the nonnegativity constraint? S > = 0, and D > = 0. So, the dual value corresponding to these two nonnegativity constraints is nothing but your reduced cost. Why it is called reduced cost, I will explain. From the figure, we see that the reduced cost of variable S on variable D makes sense considering variable S.

The nonnegativity constraint is > = 0, but the current value of S is already 540.
So, changing the nonnegative constraint 0 to 1 has no effect on the optimal solution. Already, we have got the value of S, which is 540, but adding one unit on the right-hand side nonnegative constraint will not make any result. That is why the reduced cost is 0 because increasing the right-hand side by one unit has no effect on the optimal objective function value. That is, the dual value of this nonnegativity constraint is 0.

Another name for the dual value for the nonnegativity constraint is reduced cost. Then we will talk about ranges for the coefficient of objective function where is there this one currently for the value of S it is 10, and for the value of D, the coefficient of S is 10, and the coefficient of D is 9. So, we will discuss ranges for the coefficient of objective functions is called the range of optimality. Consider an objective function coefficient range analysis here.

We have to concentrate here to see that the variable S has the current profit coefficient of 10 one and as an allowable increase is 3.5 and allowable decrease is 3.7; therefore, as long as the profit contribution associated with the standard bag is between so, allowable decreases 3.7. So, it is 6.30 dollars, and the allowable increase is 3.5, 13.50 dollars. The production of S is 540 standard bags and D = 252 deluxe bags will remain optimal.

So, what here is that the value of an objective function can be increased to the level of allowable increase and can be decreased to the level of allowable decrease quantity. So, that will not affect our final solution, which is the meaning of a range of optimality.



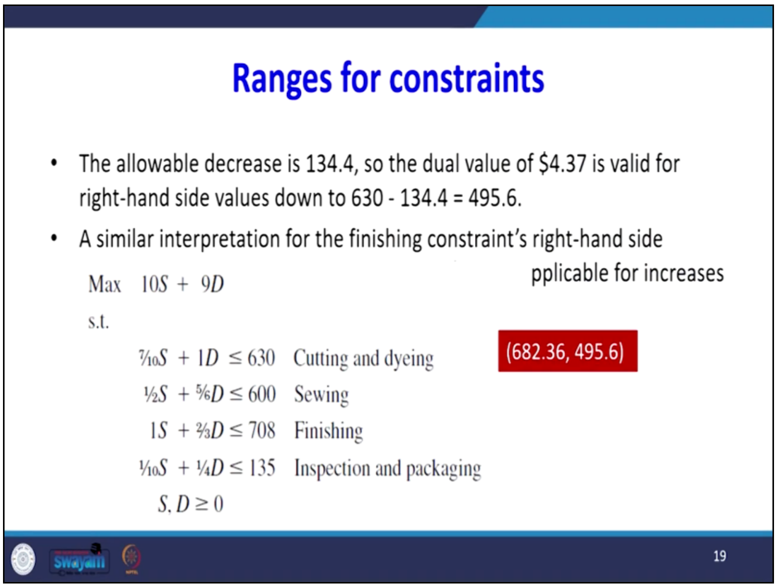
Then, we will go for the range of constraints. This is called the range of feasibility. So, here where is the range of cons. You see this one; for example, in the first constraint, the final value is ah, the right-hand value is 630, the allowable increase is 52, and the allowable decrease is 134.4. As long as the constraint on the right-hand side is not increased or decreased by more than the allowable increase, the associated dual value gives the exact change in the value of optimal solution per unit increase on the right-hand side.

What is the meaning of this one? If you increase the right-hand side value by 630+ 52 and decrease 630 - 134, your dual value will remain the same.
What if the shadow price remains the same? That is the meaning of this range of constraints. Actually, what is happening here when you add additional resources means the area of the feasibility region increases if the constraint is less than or = type.

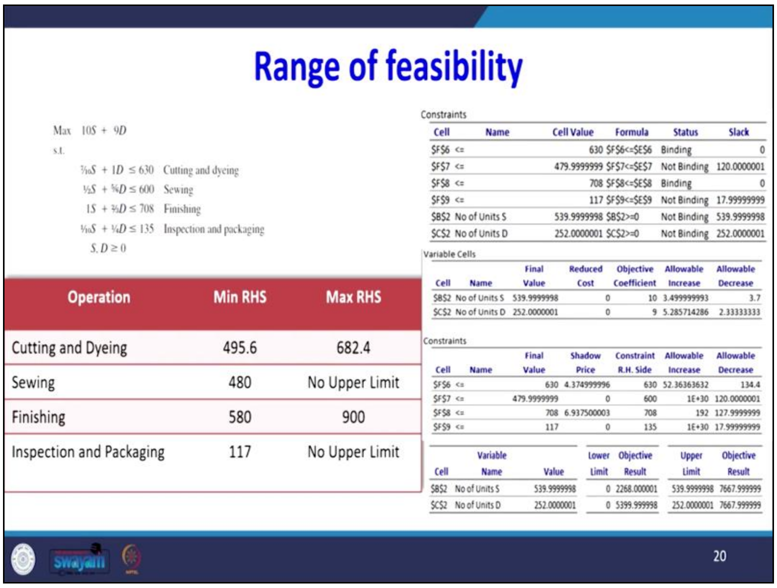
So, when the feasibility region increases up to a certain point where the shadow price will remain constant beyond certain points, the shadow fries shadow price will be affected. So, when as long as this increases within this range, that is 630 + 52 and 630 - 134, the shadow price will remain constant, which is the meaning of this range of constraint that is called a range of feasibility.

For example, let us consider the cutting and dying constraint with the current right-hand side value of 630 because the dual value for this constraint is 4.37 dollars we can conclude that the additional hours will increase the objective function by 4.37 dollars per hour is what we studied about the dual value it is also true that the reduction in hours available will reduce the value of the objective function by 4.37 dollar per hour from the range information which is given here we see that the dual value of 4.37 dollar has an allowable increase of 52.36.

This 4.37 is valid for right-hand side values up to 630 + 52 up to 682.36. Even though from 630, if you add more resources up to 682.3, your dual value will remain the same, but if you go for, say, 683, your dual value will change. That is an interpretation of this.



Like this, for each constraint, we can see allowable increases. For example, in the first constraint, the upper limit is 682.36, and the lower limit is 495.6. Within this range, your dual value will remain the same. When you go beyond 1682 and less than 495, your dual value will change. That is the meaning of this range of constraints.

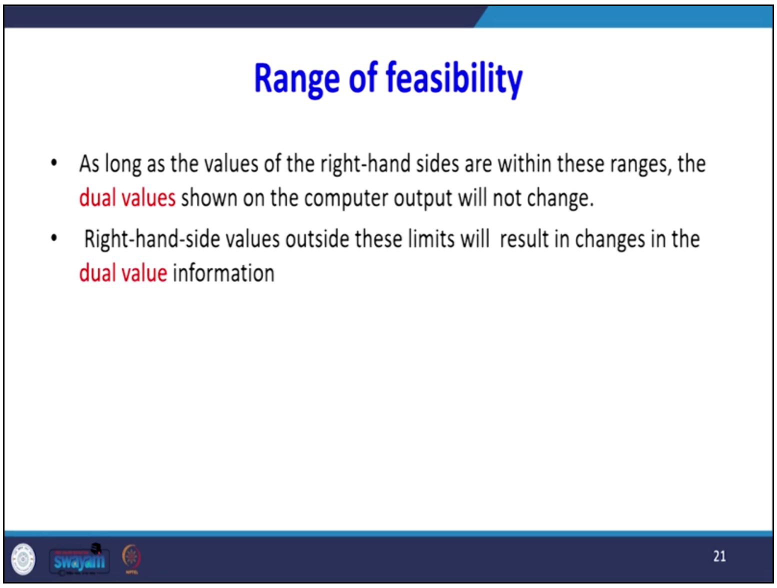


So, what I have brought for each constraint, is the minimum value and maximum value. For example, in the sieving department, there is a minimum value, there is no maximum, no upper limit. That is what we are learning from here because allowable increases this. Similarly, for finishing, the minimum value is 580, and the maximum value is 900. How did we get this one?

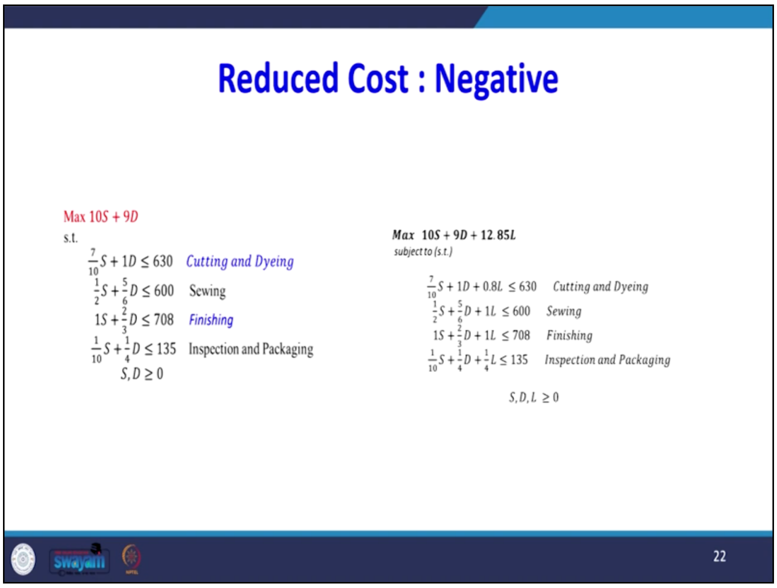
So, 708 + 192 = 900,

and 708 - 128 = 580 for the inspection package; currently, this is 135.

So, the allowable increase is 10 to the power 30. So, there is no upper limit but there is a lower limit of 17.3. So, this small amount on the right-hand side can be increased or decreased without affecting the dual value, which is nothing but your range of feasibility.



So, as long as the value of right-hand sides are within these ranges, the dual value shown on the computer output will not change. The right-hand side values outside these limits will result in a change in the dual value information sometime.



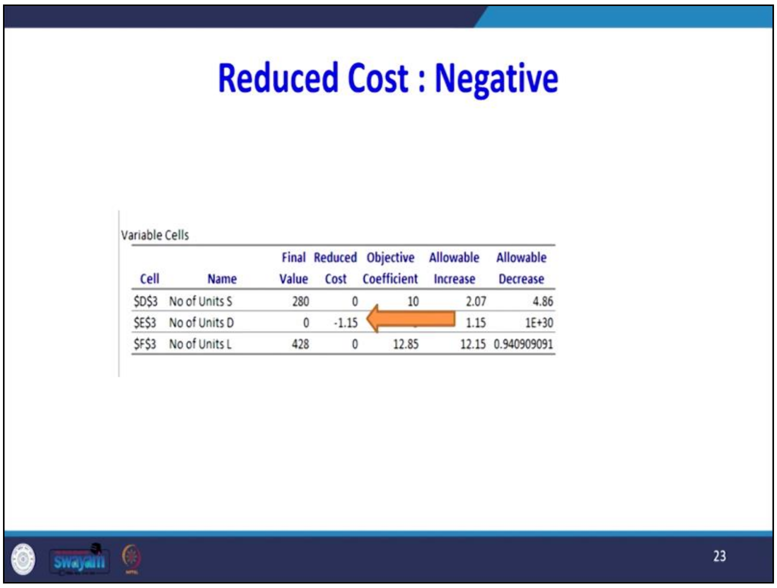
The value of reduced cost will become negative let us see that kind of situation, for example, this was our the problem which has been discussed for every lecture suppose this is a modified problem what is that the company is trying to introduce another type of bag say that has lightweight we know this is a standard bag deluxe bag lightweight bag. The profit contribution of that is 12.85, which also consumes this much resources in the cutting and dyeing department, sewing department, finishing department, and inspection packaging.

So, now this problem I am going to solve this with the help of a solver, I am going to show what is the value of reduced cost. This problem the reduced cost will become negative. So, I will explain what is the meaning of that negative reduced cost?

Students, now I have brought this modified our problem. What is it modified problem? We have introduced another product apart from standard bags and deluxe bags. There is a lightweight bag this is 0. So, the coefficient we know already is 10 years 90, which is 12.85, but these are the resources that are utilized by the lightweight bag as usual. So, here I am writing RU resources utilized. So, the objective function is here, and then the resources are utilized.

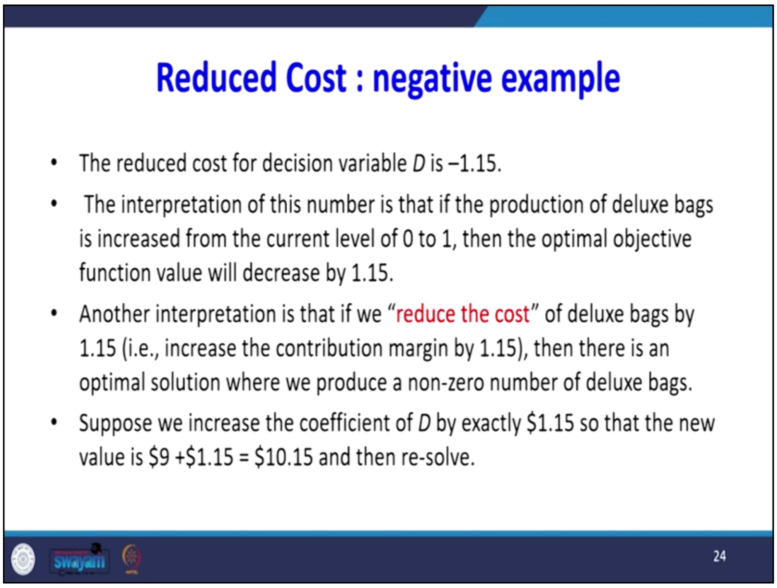
So, now I am going to solve this one data solver. So, when I solve this, I am going to see the other reports. Now look at this sensitivity report you see that the value of S is 280 and the value of D is 0 it says that you need not produce deluxe bags and the value of l which is a lightweight bag is 428 look at here this is the reduced cost is negative -1.15 what is this interpretation of this -1.15 that means that if the cost of producing this deluxe bags if it is reduced by 1.15 dollar then you will get a positive value here that is for we will get a positive value for the D.

So, what is the meaning of this? When you reduce the cost of manufacturing deluxe bags, you will get a positive D value that I will explain. So, what am I going to do?



Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D3	No of Units S	280	0	10	2.07	4.86
E3	No of Units D	0	-1.15		1.15	1E+30
F3	No of Units L	428	0	12.85	12.15	0.940909091

Now I brought the output of our excel solver. So, here we got the reduced cost for the D is -1.15. Will you explain what is the meaning of this -1.15.



The interpretation of this number is that if the production of deluxe bags is increased from the current level of 0 to 1, then the optimal objective function value will decrease by 1.15. As I told you, what is the reduced cost? Reduced cost is the dual value of your nonnegativity constraint. The dual value is minus 1.15. So, what does that say? So, we know that the constraint is like this D is greater than = 0.

Suppose you want to get some positive D value, that is, if you are making D greater than one. So, this is the binding constraint of the decision values. So, what will happen the optimal value will decrease by this much because this is an objective function. So, the optimal value decreases by -1.15. Another interpretation is that if we reduce the cost of deluxe bags by 1.15, that is, increase the contribution margin by 1.15, then there is an optimal solution where we produce a nonzero number of deluxe bags.

So, what is the meaning suppose if we increase the coefficient of D by exactly 1.15 dollars? So, the new value will be 10.15 dollars, and we can resolve it. So, when we resolve it.

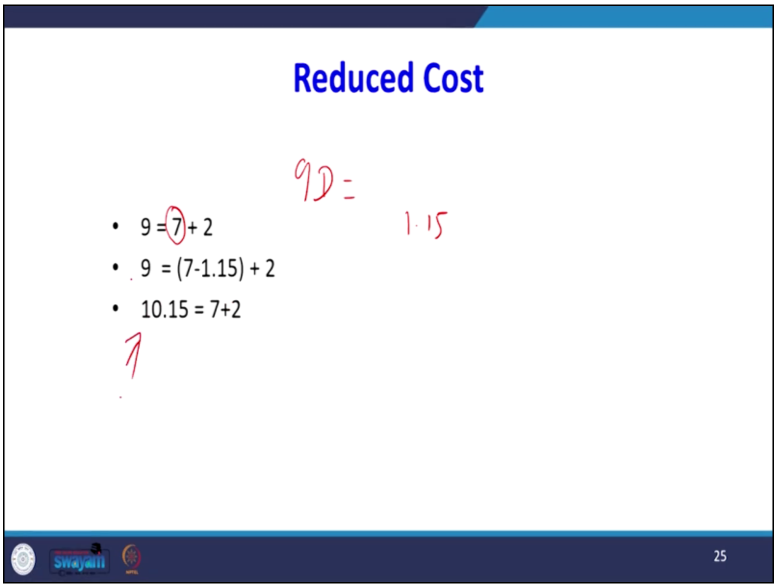
Now, when you look at this Excel, this was our previous problem. So, there are three products: standard deluxe and lightweight bags. There is a 10, 9, and 12.85. When you are doing this, we have seen the reduced cost is minus 1.15 dollars. Now, what are we going to do? So, we are going to add that much-reduced cost value.
So, 9 + 1.15 = 10.15. Again, I am going to solve it. So, before I solve you, remember this: when the coefficient of the deluxe back is 9, our objective function was 8299, approximately 8300.

So, now my coefficient of the objective function is 10.15. Why is it 10.5 in the previous 9? Because we have to reduce the cost? So, what is the meaning of reduced cost reducing cost means increasing the profit contribution. So, minus 1.15 when you reduce the cost. So, that means our profit contribution needs to be increased. So, if I solve it again, you should be very careful with these values of objective function also.

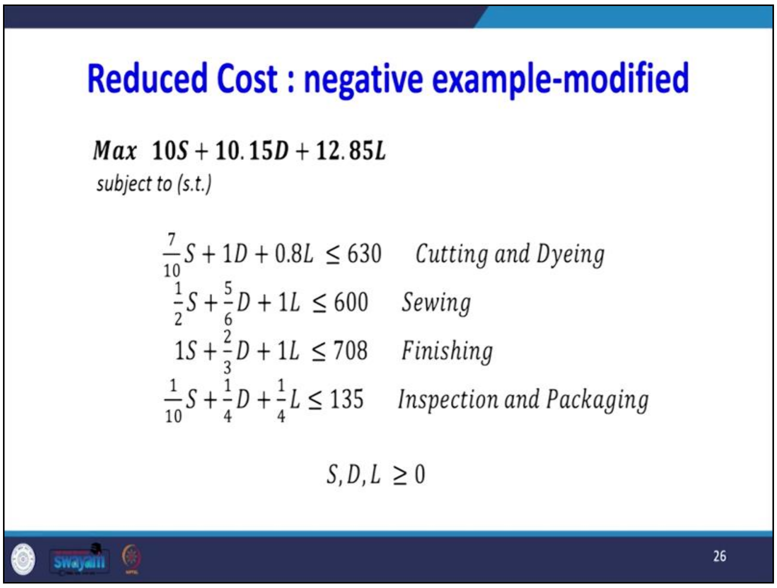
So, when you solve it again. So, I want to see how you see this: even though the coefficient is 10.15 again, our objective function value is the same that is 8299.8, but when you look at the reduced cost value, you see that the value of reduced cost is minus 8.10 to the power minus 16 at almost 0. So, here we are not getting positive value. The reason is this problem is having multiple optimal solutions.

One limitation of using Excel for solving linear programming problems is if the problem has multiple optimal solutions, we can see any one value, we cannot see all possible multiple optimal values. How can we verify this problem has multiple optimal solutions? So, previously when the coefficient of D was 9 at that time also our objective function was 8299 when I changed the subjective function to 10.15 coefficient of the deluxe bag is 10.15 also the value of the objective function remained the same.

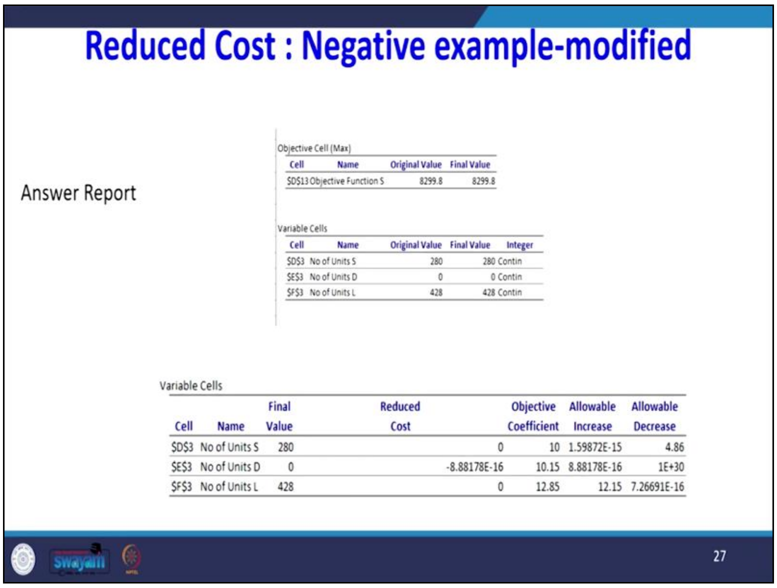
So, like this, since this problem has multiple optimal solutions. So, there may be some possibility the value of D is some positive value that we will discuss there.



Now I will explain what is the meaning of this reduced cost? Assume that say already, this is 9D. Ok, 9D is some quantity plus two. Suppose the reduced cost is 1.15 dollars. So, that means from 9, if you are reducing 1.5 dollars, that is equivalent to adding 1.15 on the left-hand side, that is increasing the profit contribution to 10.15. So, reducing 1.15 dollars that is reducing cost is equivalent to increasing the profit contribution.



So, now the coefficient of D is 10.15. So, when I solve this problem, 10 years plus 10.15 plus 12.85 l for this problem when I am solving, I got this output.



Objective Cell (Max)

Cell	Name	Original Value	Final Value
D13	Objective Function S	8299.8	8299.8

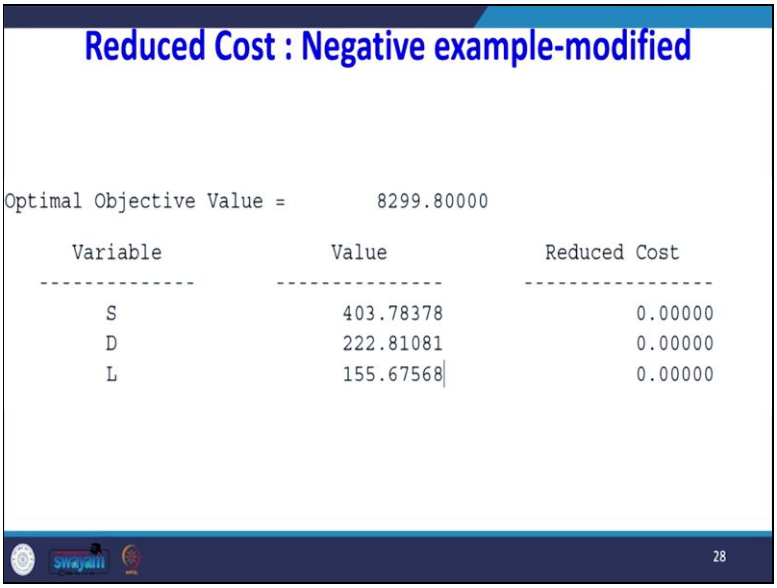
Variable Cells

Cell	Name	Original Value	Final Value	Integer
D3	No of Units S	280	280	Contin
E3	No of Units D	0	0	Contin
F3	No of Units L	428	428	Contin

Variable Cells

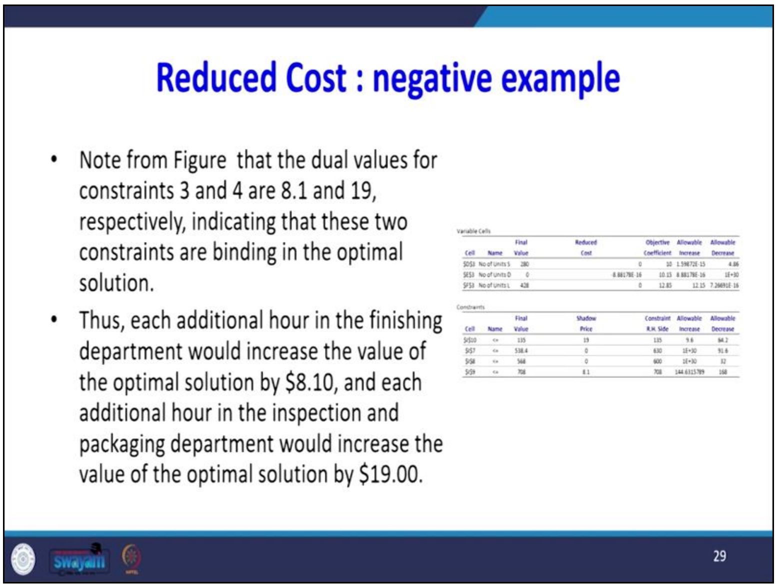
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D3	No of Units S	280	0	10	1.59872E-15	4.86
E3	No of Units D	0	-8.88178E-16	10.15	8.88178E-16	1E+30
F3	No of Units L	428	0	12.85	12.15	7.26691E-16

So, what this output says is the value of S is 280 and the value of l is 428, but you see that the objective function value is the same when it was 9 at that time, also 8299 when it was 10.15 at that time also it was 8299 that, means this problem has multiple optimal solutions. But look at the value of reduced cost. So, the value of reduced cost is 0 here because this is minus 8.8 into 10 to the power minus 16 it is 0.



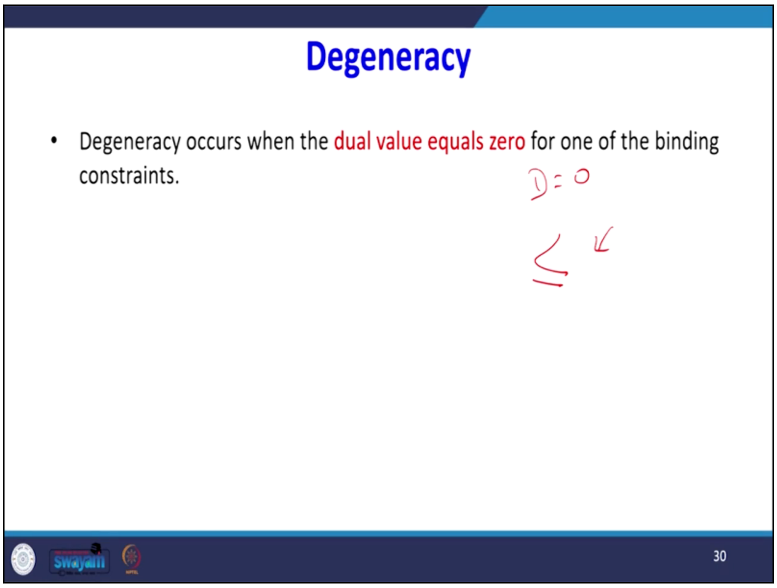
Now you see that there may be another because since I told you it is a multiple optimal solution, the value of S is 403.7, the value of D = 222.881, and the value of L is 155. So, when you substitute this value in the objective function, that time also, you are getting the objective function values. So, what is the meaning of this? This problem has multiple optimal solutions. One of the solutions is 403, 222, 155. In that solution, we have a positive D value.

One limitation of this Excel solver is that if the problem has multiple optimal solutions, we cannot see all possible optimal solutions with the help of Excel. But there may be some other software in the book that I am following they have used some other software we got different values for S, D, and L that also provide the same value of the objective function.



Note that from the figure, the dual value for constraints 3 and 4 is 8.1 and 19, 8.1 and 19, respectively, indicating these two constraints are binding in the optimal solution. Thus, each additional hour in the finishing department would increase the value of the optimal solution by 8.10, and each additional hour in the inspection packaging department would increase the value of the optimal solution by 19.0.

Because the shadow price is 19 for this constraint, if the right hands are increased by one unit, our objective function will increase by 19 dollars. So, similarly for this also because these two constraints are binding constraints, the other two constraints are non-binding. So, adding any resources will not help you.



The next term is degeneracy what is the degeneracy? Degeneracy occurs when the dual value equals 0 for one of the binding constraints. Generally, if the constraint is binding, adding any resources should increase your objective function value. But that means it should increase. There should be a positive dual value, but here you are adding any extra resources. For example, if the constraint is less than or = type, you are adding any resources.

But the dual value is 0. That situation is degeneracy, which means that adding any resources is not helping to achieve your not helping to improve your objective function. So, that is such but it is a binding constraint. So, that situation is degeneracy. Conclusion, Dear students, in this lecture, we have discussed the effect of change in the right-hand side value of the constraint. Then we discussed the meaning of dual values for binding and non-binding constraints, and then we discussed reduced cost and its interpretation. In the next lecture we will discuss about limitations of classical sensitivity analysis, thank you.