Course Name - Operations and Revenue Analytics

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Week - 02

Lecture - 10

Welcome, friends. So, we are seeing the use of analytics in different areas of decision-making in operations management. And as we started in our last class about the use of analytics for inventory management, which is another very important area after forecasting. We discussed a very simple case of EOQ. However, at the end of the session, we discussed that in practical life, the scenario is not so simple. There are different types of uncertainties that are very common in any scenario. We discussed that uncertainties may come from the supply side or from the demand side as well.

So, in this particular session, we will focus on how we are going to manage those uncertainties and make better decisions to fulfill the objectives of the organization. You remember that with inventory management, the important things we are looking at are service level. I want to keep inventory so that whenever a customer requires a product, I should be able to fulfill that customer's requirement. I want to achieve a particular service level—higher and higher service level. And on the other side, the cost of keeping inventory should be as low as possible. These are my two objectives: improving the service level.

If you want to maintain a very good service level, you should have an infinite amount of inventory. Mathematically, that is the answer. But no company in the world can have an infinite amount of inventory of any product. And as we all know, in the current environment, SKUs are increasing. SKU stands for stock-keeping units.

So, these SKUs are continuously on the rise, and therefore, there will always be pressure on the company to optimize your physical space, as inventory takes physical space. So, you have to optimize your physical space while also needing to achieve higher service levels and, at the same time, keep the cost as low as possible. So, in this particular session, we will be talking about demand uncertainty. What are the different kinds of measures of service level? How are we going to determine the safety level, cycle service level, and calculations of safety stock for particular service levels? So, let us first understand how we are going to measure demand uncertainty. For measuring demand uncertainty, we generally assume—and you will see that it is not just an assumption—that demand is considered to be normally distributed.

And when I say demand is normally distributed, I hope all of you are very familiar with this normal distribution. In this normal distribution, as you are seeing, there is a bell-shaped curve, and this bell-shaped curve has a total area under the curve. This total area under the curve equals 1, and this center line represents the mean of the data. As you see, the curve is spread around the mean line. So, this spread is basically due to its standard deviation, which is sigma. So, here you see that I have written the 68-95-99.5 rule. This means that when you move around your mean, the area under the curve will increase as your sigma levels increase.

If you are just within plus-minus 1 sigma around the mean value, you have 68 percent of the area. If you add another 1 sigma on either side, then it becomes 95 percent of the area. And then, if you add 1 more sigma, this becomes 99.5 percent of the area. So, we have tried to explain this to you in the form of this diagram.

$$D = Average\ demand\ per\ period = 10\ units/day$$

$$\sigma_D = Standard\ deviation\ of\ demand\ per\ period = 2\ units$$

$$L = Lead\ time\ (time\ between\ order\ placement\ and\ recieved = 10\ days$$

Also, in our last session, we became familiar with this sawtooth pattern. Now, the consumption rate, which we described as a smooth line in our last class, is actually not a smooth line. It can very well be represented as a normally distributed line, where this line we are drawing is the mean line, and it actually has variation around it—that is what I am trying to show in this diagram.

So, these are the variations, and then also you have the 68, 95, and 99.5 rule applicable in this case also. Where capital D represents the average demand per period, and sigma D is the standard deviation of demand per period. We are assuming L as the time between when an order is placed and when it is received, that is the lead time.

There may be a variation in lead time also, which we will be discussing. Now, if we consider the average demand per period is 10 units per day and the standard deviation of demand per period is 2 units, the lead time is 10 days.

So, here the demand during the lead time, DL, becomes 100 units, that is the demand during the lead time, which is 100 units. Now, the per-day demand is 10 units, and in those 10 units, the standard deviation is 2 units. Like this way, 10 is the average, and then if you draw, it may go up to 1 sigma, 2 sigma, 3 sigma, 1 sigma, 2 sigma, 3 sigma. So, this is 2, 4, 6, this is also 2, 4, 6.

That means demand can vary on any day from 8 units to 12 units, with a probability of 68%. Demand can vary from 6 units to 14 units, with a probability of 95%.

And demand can vary from 4 to 16 units, with a probability of 99.5%. That is how you will read this rule of 68, 95, 99.5 and the meaning of this normal distribution. Now, the average demand, the demand during the lead time, that is the average demand during the lead time, is now 100 units. And the standard deviation of demand during the lead time is not possible to add like this or multiply like this. You have to first calculate the variance of daily demand.

If the standard deviation is 2, then the variance of daily demand is equal to the square of the standard deviation. So, here this is the standard deviation of demand during the lead time: square root of 10 multiplied by 2 equals 6.32 units, which is our first value you need to calculate, D.

$$\sigma_L = \sqrt{L}\sigma_D = \sqrt{10} \times 2 = 6.32\ units$$

So, the formula for sigma L for the period of L days is to multiply sigma D squared by L, and it can be simplified as the square root of L multiplied by sigma D, which is what we have shown at the bottom of the slide.

$$\sigma_L = \sqrt{({\sigma_D}^2 + {\sigma_D}^2 + {\sigma_D}^2 + {\sigma_D}^2 \ldots .)L}\ times$$

So, this is how we have calculated the amount of uncertainty. Now, in other words, this is for daily demand; this normal distribution is for daily demand. Let me create the normal distribution for the demand over the entire lead time, the normal distribution for lead time.

So, the shape will remain the same, but the values of the average and sigma will change. The value of the average will be 100 instead of 10 here, and the sigma is now 6.32. So, 1 sigma is 6.32, plus 2 sigma is 12.64, 3 sigma is 18.96, and that applies in the negative direction as well: -6.32, -12.64, and -18.96. And you can appropriately find what the 68 percent probability is, what the 98 percent is, what the range for 95 percent probability is, and what the range for 99.5 percent probability is. After understanding the measure of uncertainty, let us also understand the measures of product availability.

Product availability can be described in different ways. One very common way of describing product availability is the product fill rate, fr, the product fill rate. Now, this product fill rate means that you are coming to a particular outlet, and a product is supplied to you from the available inventory. That means you already have the stock, and from that available inventory, you are fulfilling the customer's requirement. So, that means if I have a stock of, let us say, Q items, I am having the stock of Q items.

So, the probability that the demand in any period is less than Q is my product fill rate. So, these product fill rates, etc., are actually measured in terms of probabilities that a customer is going to get the product or not. So, that is the general way of expressing the fill rates. So, here this product fill rate is the probability that demand is less than or equal to Q. So, as long as our demand is less than Q, a customer is going to receive the product from the available stock. Another important measure is the order fill rate.

Because it is quite possible that in one order you may require different types of products, and let us say there are five different types of products in a particular order. It is quite possible that out of five, four are available in the stock, but one is not available in the stock. So, I will say that the order is not fulfilled. If all the items which are in the order are readily available in my inventory, then the order is completed. So, whenever you have more items in a single order—if only one item is in your order—then in that case, the product fill rate and order fill rate are the same.

But generally, we all know that one single order may consist of different types of items, different types of products. So, therefore, order fill rate and product fill rate are discussed separately. So, the fraction of orders that were fulfilled entirely over a certain number of orders is known as the order fill rate. And then another important measure is the Cycle Service Level (CSL). This is, in fact, the most common one, the most used cycle service level. We have so many order cycles in this sawtooth pattern; we see that there are so many orders, so many cycles happening continuously. So, in this cycle, whether you have fulfilled all the customer demands from the available stock, in the next cycle also, in the next cycle also.

So, out of a certain number of cycles, in how many cycles have you fulfilled the demand of the customer from the available inventory? That is the cycle service level. So, cycle service level is generally more common because if you have the safety inventory available in every cycle, you will be able to fulfill the demand of the product from that safety inventory. Whenever the demand of the product crosses the safety inventory limits, then in that cycle, you will not be able to fulfill the demand from the available stock. So, therefore, cycle service level is very common and most used in our discussions. Now, let us see how we are going to do the calculations related to safety stock.

$$Expected\ Demand\ during\ the\ lead\ time\ =\ D \times L$$

$$Where\ D = weekly\ demand\ and$$

$$L = Lead\ time\ (in\ weeks)$$

$$Safety\ Stock\ (SS)\ = ROP -\ D \times L$$

Like we have just discussed, we are going to calculate the expected demand during the lead time. If you have the daily demand or weekly demand, D is demand per period. It is not good to write weekly demand; it means demand in any period. That period may be a month, daily, or weekly. And this is the lead time. So, D multiplied by L is our requirement of the mean requirement of the items.

Safety stock is ROP minus D multiplied by L. Let us understand how this formula has come about. As we already see this sawtooth pattern, and below this sawtooth pattern, we have safety stock. Now, here you see that I am placing the order here, I am placing the order here at ROP. Now, when this ROP is placed, this is Q divided by 2 plus this safety stock. This Q divided by 2—no, sorry, this is not Q divided by 2—this is ROP, which is fulfilling the requirement for this lead time period. This is the lead time.

So, generally, whatever inventory is required for the fulfillment of this lead time is D multiplied by L. So, this is basically D multiplied by L plus this safety stock. So, our ROP (reorder point) is D multiplied by L plus safety stock, or in other words, you can say that safety stock is ROP minus D multiplied by L. So, this becomes an important relation for determining the calculations of our safety stock. So, with this example, you can understand how we are calculating these things. Our average demand is 2,500 units per week, the standard deviation of demand is 500 units, and the lead time is given as 2 weeks.

So, the standard deviation given is the standard deviation for weekly demand. So, for 2 weeks, we need to calculate the standard deviation. In this way, the standard deviation for lead time is the square root of 2 multiplied by 500, which is our standard deviation for the lead time duration.

Now, the order quantity Q is 10,000 units, and the ROP is 6,000 units. So, you can see the diagram again, which will help us in easy understanding. So, our ROP is 6,000, this Q is 10,000, and this is my ROP, which is at 6,000.

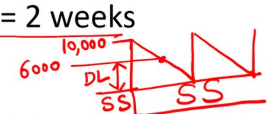
And this is basically D multiplied by L, and this is the safety stock. So, our ROP of 6,000 equals DL plus safety stock, and DL is 2,500 units for 2 weeks. So, the safety stock becomes 1,000 units, which is what we have calculated. Now, in this case, if you want to know what my average inventory is, it is the average cycle inventory plus safety stock. So, the average cycle inventory is the order quantity divided by 2: 5,000 plus 1,000, making 6,000 my average inventory level.

Average demand per week D = 2500 units

$\sigma_{LT} = \sqrt{2} \times 500$

Std dev of demand σ = 500 units Lead time = 2 weeks

Quantity ordered (Q) = 10000 units at ROP of 6000 Units.



$$\textbf{Safety stock} = \textbf{ROP} - \textbf{DL} = \textbf{6000} - \textbf{2500} \times \textbf{2} = \textbf{1000 units}$$

$6000 = DL + SS$
$= 2500 \times 2 + SS$
$SS = 1000$ Units

$$average\ inventory = average\ cycle\ inventory + SS$$

$$= 10000/2 + 1000 = 6000$$

And then, if you need to calculate your holding cost, this holding cost will always be calculated not on average cycle inventory but on this average inventory. We can also calculate average flow time in this particular case for how much duration the inventory is there in my stocks because if the inventory duration is higher, I have to block my capital for a longer duration. I need more working capital if my flow time is more. If my flow time is less, I am able to generate cash at a much faster rate, which is also very important for a business—how fast you are able to generate your cash. If your cash generation is happening at a slow pace, you need more working capital to run your operating expenses. So, average flow time in our supply chain discussions we have a very simple formula known as Little’s Law. And using that Little’s Law, you have this I equals to D into T, where average inventory equals demand multiplied by flow time.

I = D x T

I = Avg Inventory, D = Demand, T = Flow Time

So, we can calculate the flow time as average inventory divided by demand rate. So, the demand rate is 2500 units, 6000 is my average inventory divided by the demand—2.4 weeks—that is the average flow time that a particular item remains 2.4 weeks in our stocks.

The holding cost will be calculated for 6000 units

Average flow time = Avg. Inventory / Demand = 6000/2500

= 2.4 weeks

So, you can understand that either by reducing the average inventory or by increasing the demand, I can reduce the average flow time so that I can generate cash at a much faster rate. Now, after discussions of the safety stock calculations, another very important area we need to discuss is CSL—cycle service level—out of various measures of product availability. CSL is the most important one, which is basically the probability of not stocking out in a cycle, which is possible only when the demand during the lead time is less than ROP.

You see here you are with your safety stock also. This is your ROP, and at this ROP level, whatever you are requiring is available in the form of DL plus SS. Now, demand during this lead time period—from here to here—if this demand is less than ROP, you are able to fulfill the customers' requirement, but if demand becomes more than this ROP level, there may be some customers who are not getting their products. So, demand during lead time should be less than or equal to ROP for fulfilling the customers' requirement. So, CSL is basically the probability that demand during lead time is less than or equal to ROP, and if you see this diagram on the lower side of your screen, this is

$$CSL = \ Prob.(demand\ during\ lead\ time\ \leq ROP)$$

simply a normal distribution curve where we have mean demand as DL.

Standard deviation over lead time: this is the standard deviation over lead time, which is sigma L. Using Excel analytics, we can simply calculate CSL as the normal distribution because this entire demand curve is a normal distribution curve. So, for a normal distribution, ROP, DL, and sigma L1—if you input these into Excel, you will get the value of the cycle service level with respect to a particular inventory policy you are following. Now, using different combinations, you can see what CSL you are achieving, or you can do the reverse as well. We can use Excel to find that. If you want to achieve a particular CSL level, what should the ROP be? So, those calculations are also possible.

Now, if you see this simple calculation of CSL, the same data—standard deviation of demand: 500 units. First, as I explained, we will calculate sigma L, the standard deviation for the lead time period, which is the square root of L multiplied by sigma D, resulting in 707 units.

$$\sigma_L = \sqrt{L}\sigma_D = \sqrt{2} \times 500 = 707 \ units$$

$$\underline{CSL} = F(\underset{ROP}{6000}, \underset{DL}{5000}, \underset{\sigma_L}{707})$$

Now, CSL is a function of ROP, DL, and sigma L.

This is put into an Excel format where the normal distribution probabilities for this expression are calculated, giving you a CSL equal to 92 percent.

$$CSL = NORM.DIST(6000,5000,707,1) = 0.92$$

That means 92 percent of the demand will be fulfilled from our stock in this case, and around 8 percent of the demand will not be fulfilled from our inventory level. So, in this way, CSL calculations can be used to determine the customer satisfaction level.

A higher CSL means you are able to fulfill the demand of more customers. Now, using this knowledge of CSL and uncertainty, we have this equation: ROP equals DL plus SS (Equation 1).

We have calculated CSL like this, and for this CSL, we often have a reverse calculation as well. If we want a particular CSL—say, 90 percent—what should the ROP be? That can also be a question, and based on that ROP calculation, I can determine what the safety stock should be.

So, then I will go for normal inverse calculation, then it becomes an inverse function, and that is also possible to get easily in your Excel tables. DL plus SS is equal to this ROP, and then SS can be ROP minus DL. So, all these equations are simply our used equations for

CSL and ROP safety stock calculations in our uncertainty cases.

$$ROP = D_L + SS$$

$$CSL = NORM.DIST(ROP, D_L, \sigma_L, 1)$$

$$ROP = NORM.INV(CSL, D_L, \sigma_L)$$

$$DL + SS = NORM.INV(CSL, D_L, \sigma_L)$$

$$SS = NORM.INV(CSL, D_L, \sigma_L) - DL$$

Now, using this set of formulas, we can have this illustration where again the same data lead time is this, and our sigma L will be under root 2 into 500, which becomes 707. Now, we want to achieve a CSL of 90 percent, and when we want to use a CSL of 90 percent, let me say that first I will use this ROP formula.

So, first I will calculate ROP using the normal inverse function in Excel with this CSL, our DL, then sigma L1, which I have substituted here, and then this is with available data; it will come somewhere around 5906. So, the safety stock will be ROP minus DL, and that becomes 5906 minus 5000, which equals 906. So, that is how you can calculate the safety stock for a given CSL level, which is a more common calculation in our lessons of uncertainty. So, with this, we come to the end of this session. Thank you very much.