Course Name - Operations and Revenue Analytics

Professor Name - Prof. Rajat Agrawal

Department Name - Department of Management Studies

Institute Name - IIT, Roorkee

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Lecture - 11

Welcome, friends. In our earlier class, we were talking about how to handle uncertainty in our inventory management and uncertainties related to inventory management. We discussed that largely, uncertainties are on the demand side. When we have some kind of uncertainty in the consumption rate, that needs to be handled. For handling that uncertainty, we have a very important tool known as safety stock. We discussed how we are going to calculate the safety stock. We discussed some of the measures in the field of uncertainty, like product fill rate, order fill rate, and more importantly, cycle service levels.

We discussed how cycle service levels can be calculated with the help of the normal distribution function in Excel. Now, you can calculate cycle service levels. You can also calculate what the reorder point should be based on these CSL calculations. All those things we covered in our last session. Going further, it is also possible that there may be uncertainties related to the supply side as well. There is a demand side, and there is a supply side.

Supply-side uncertainties more commonly occur in two ways. In all our previous classes, we have assumed that whatever we order, the supplier delivers the same quantity. You order 500 units, and the supplier gives you 500 units. But you can always assume that the supplier can also provide short supplies. If you order 500 units, the supplier may give you just 400 because the supplier has commitments to many customers and only a limited quantity is given to you. That is quite possible.

Second, the supplier is telling you, 'I will deliver in 10 days if you give me the order. I will deliver in 10 days.' But the supplier may take 12 days to deliver. So, there are uncertainties related to the supply side as well. Then, in many kinds of inventory issues when procuring items, there are possibilities that your supplier may offer you a deal. For example, if you order 500 units, I will charge you 5 rupees per product.

But if you order a minimum of 1,000 units, I will charge 4 rupees 50 paisa, and if you order a minimum of 2,500 units, I will charge only 4 rupees per unit. So, these types of situations are very common in our day-to-day lives, even in your personal life. When you go to vegetable markets, you see these kinds of quantity discount situations, where a vegetable seller or food seller may charge different rates for bulk buying. So, if you encounter a quantity discount situation, how will it change your decision-making? And then there are situations where you are producing on one side, receiving orders on another, and simultaneously consuming them as well.

Like in our EOQ model, if you remember, all replenishments happened instantly. A single straight vertical line represented the replenishment. But it is possible that replenishment may happen over time, and during this period, consumption also occurs simultaneously. So, these types of models are known as production lot sizing models or economic production models. So, how do we decide the economic order quantity, which in this case we call the economic production quantity (EPQ)? That is what we will discuss in this class.

So, as I said, the first thing we are discussing is the supply side uncertainty. So, if you see this model, here we have this ROP; this is the ROP. Now, here we have this fixed lead time, but it is also possible that this lead time is variable. There may be a mean lead time and then a standard deviation of lead time. So, you may have a very complex situation: you already have a mean demand, as we discussed in our last class, and a standard deviation of demand, and you also have another variable—the mean lead time and standard deviation of lead time.

So, if you want to calculate the overall demand during the lead time, then you simply multiply this with this: mean lead time with mean demand.

D_L = Mean L_T X Mean Demand

But if I want to calculate the sigma of demand during lead time—sigma of demand during lead time—this is calculated with this formula.

$$\sigma_{DDLT} = \sqrt{L\sigma_D{}^2 + D^2 S_L{}^2}$$

If you see this formula, we have two terms: one takes care of the variations in demand, and another takes care of the variations in lead time—L into sigma D square plus D square Sl square, where Sl represents sigma Lt, which equals Sl. So, this is how we calculate the standard deviation of demand during lead time (DDLT stands for demand during lead time).

So, these are the calculations, and once you have DL and sigma DDLT, you may recall what we discussed in our previous class—that the normal distribution curve will come into the picture. This is DL, and then you have 1 standard deviation, 2 standard deviations, and 3 standard deviations. These are sigma DDLT; this is plus 2 sigma DDLT, and this is plus 3 sigma DDLT, and so on, on the left side of DL as well. So, here is a simple illustration to explain how these calculations will take place: we have a weekly demand of 2500 units, a standard deviation of this demand of 500, a lead time of 7 days (L is lead time), and a standard deviation of lead time of also 7 days—the mean lead time is 7 days. This is, in fact, the mean lead time or average lead time, and this is the standard deviation of lead time.

So, our average demand is 2500, which is the per-day demand multiplied by 7 days, totaling 17,500 units. Then, we calculate sigma DDLT using this formula. The sigma DDLT also comes out to be 17,550, and we are considering a CSL of 90 percent for this particular situation. So, if you remember, we can use the normal inverse function in Excel, and multiplying that function by sigma DDLT will give you the safety stock of 22,491.

D = 2500 units/day

σ_D = 500

L = 7 days

$S_L = 7$ days

$D_L = 2500 \times 7 = 17500$

σ_{DDLT} = 17550

At a CSL of 90%

$ss = NORM.INV(CSL) \times \sigma_{DDLT} = 22491$

This SS is approximately equal to 9 days of demand

Now, I have a question for all of you: What should be the use of this safety stock? You can see that in the bottom line, we have mentioned this point: The daily demand is around 2,500 units.

If I divide 22,491 by 2,500, it is approximately nine days. So, the meaning is I am keeping a safety stock to handle the situation for up to nine days. It is possible that there may be some delay on the supply side or an increase in demand. The cumulative effect, if I may say, is possible to handle for let's say nine days if there are only variations in lead time or an increase in daily demand, since daily demand can also have a standard deviation of 500 units. So, in that cumulative manner, if 22,491 units of safety stock are kept, I will be able to achieve, on average, a CSL of 90 percent—meaning 90 percent of orders will be fulfilled from available inventory. Ten percent of customers may not be able to get orders from the available stock due to supply-side and demand-side uncertainties.

Now, another situation we want to discuss, as I mentioned earlier, is quantity discount. In quantity discount situations, you are given different price ranges—P1, P2, P3, P4—where prices decrease as your order quantity increases. So, your order quantity Q is inversely proportional to the price. That is the idea behind quantity discount models: higher prices lead to lower order quantities. Lower prices may lead to higher order quantities. Now, the question is: Sometimes, when we want to take advantage of quantity discounts, we may not order the EOQ quantities because we want an EOQ that minimizes total inventory cost.

This price benefit is not coming from the inventory cost. The price benefit is coming from the material cost, and if you remember our first class on inventory discussions, our EOQ calculation is basically based on holding cost and ordering cost. So, in this case, if some discounts are available for larger purchases, how do we incorporate this benefit? Ultimately, please remember one more thing: though we are discussing many analytical decision-making aspects. When we started these inventory management discussions, we said that we want to minimize the total inventory cost.

But that is not the complete picture. We want to minimize the overall cost of our operational activities. Inventory is just one subset of that. But if I am getting a bigger benefit because of a larger purchase quantity, then I may sacrifice the benefit of inventory cost. So, here I have a more holistic model where I will include the material cost as well as the inventory cost to see whether the benefits are really significant.

If the benefit from quantity discounts or bigger purchases can give me some advantage at the cost of increased inventory cost, I will still prefer to go for those kinds of benefits. But if by increasing the lot size or purchase quantity, the benefits are lost. If there is no benefit—in fact, if my total cost is increasing—I will not opt for quantity-discounted purchases. So, in this way, I am going to discuss how we are going to handle these quantity discount cases. In the quantity discount case, we have a very systematic process for handling the inventory calculations.

Here, you are having, let us say, two or three prices. We have a requirement of a product which is coming at the rate of, let us say, 2000 units per year—that is the R. I am using the same symbols which I started in every session. So, I am not repeating what the meaning of R is. It is 2000 units per year, which means our annual demand. Now, the supplier is ready to give me three different prices. One price is ₹2. So, you can note them down as ₹2 for less than 1000, ₹1.90 for 1000 to 1999.

And ₹1.86 for more than or equal to 2000.

So, now I am getting quantity-related prices. If I am ordering more quantity, the supplier is ready to give me a discounted price. Now, first, the cost of carrying the inventory is given as 16 percent of the average inventory value. This is the data. Here, I have assumed ₹20 as the ordering cost.

P1: 2 for Less than 1000

P2: 1.90 for 1000 – 1999

P3: 1.86 for equal to or more than 2000.

Ordering Cost (Assumption): 20

Now, as the first step of this particular process, I have to calculate different EOQs. This is step 1.

So, I am calculating in this case because three prices are given: P1, P2, P3. So, I can say that EOQ1 is basically corresponding to P1, EOQ2 is corresponding to P2, and EOQ3 is corresponding to P3. With these three prices, I am calculating EOQs using the same formula: $\sqrt{(2RC_p/C_H)}$. You see, in the numerator, there is no change: $\sqrt{(2R \times C_p/C_H)}$. So, in the numerator, there will be no change. R is constant—the annual requirement. The ordering cost is also constant: ₹20 per order. C_H will vary. Because one is ₹2, another C_H will be calculated for ₹1.90, and the third C_H will be calculated at ₹1.86.

C_H is 16 percent of the material price. So, 0.16 into 2, 0.16 into 1.90, 0.16 into 1.86—these will be the values in my denominators. So, accordingly, the calculation tells me 500, 513, and 519—these are the three EOQs.

$$EOQ1 = \sqrt{\frac{2 \times 2000 \times 20}{0.16 \times 2}} = 500\ units \qquad EOQ2 = \sqrt{\frac{2 \times 2000 \times 20}{0.16 \times 1.90}} = 513\ units$$

$$EOQ3 = \sqrt{\frac{2 \times 2000 \times 20}{0.16 \times 1.86}} = 519\ units$$

But now you can see, if I am charging 1 rupee 90 paisa, the minimum order quantity expected is 1000. But here, my EOQ is coming to 513, which is less than 1000.

So, the supplier will say, 'Hold, you will not be getting the benefit of 1 rupee 90 paisa.' 'If you are giving me an order of 513, I will charge you 2 rupees.' So, he will say, 'I have nothing to do with your EOQ calculations; I will charge as per my policy.' So, according to the supplier, this is an invalid EOQ because it does not fall in the prescribed range for which the price of 1 rupee 90 paisa is applicable. And similarly, our third EOQ is also invalid.

This is also an invalid EOQ. So, both these are invalid EOQs—513 and 519—and only one valid EOQ I could find, which is 500 units. Now, based on these things, I will calculate my total inventory cost at three particular levels. One is at 500, another at 1000 where I am getting a price break, and the third is at 2000. The rule we see is that we have to calculate the total cost at all valid EOQs and at all price breaks.

So, in this case, we have three EOQs: 500, 513, and 519. These two are invalid; only 500 is valid. The others are invalid because of the price ranges on which that particular price is applicable. So, identify only the valid EOQ, which is 500. And then we have to calculate the total cost at all valid EOQs, which is 500 only—one valid EOQ. We have two price breaks in this particular case: one price break is at 1000, and another price break is at 2000. So, in this way, we have to calculate the total cost at 500, TC at 1000, and total cost at 2000. These calculations are already available to us on this particular slide at the 500 level, the 1000 level, and the 2000 level.

And here we see that these are coming to 4160, 3992, and 4038.

I can request all of you to try these calculations on your own. You can use Excel if you avoid calculators. So, you can use Excel and see that by putting the formula, you can get the values of these total costs. Out of these three total costs, the minimum cost is 3992, corresponding to the quantity level of 1000. So, it becomes our order quantity. This is our

Now calculating TC at 500, 1000 and 2000

Where: $TC_{Q=500} = \frac{500}{2} \times 0.32 + \frac{2000}{500} \times 20 + 2000 \times 2 = 4160$

Smilarly $TC_{Q=1000} = \mathbf{3992}\ and\ TC_{Q=2000} = 4038$

order quantity, which is actually giving the benefit of the lowest total cost. Here, we are enjoying some benefits—not complete—some benefits of EOQ and the benefit of quantity discount.

So, we have compromised. When I say some benefit of quantity discount, I will request all of you to please explore why it is some benefit. You may recall our discussions in class, where we discussed that at the EOQ level, the holding cost and ordering cost are equal. But at this level, TCQ equals 3992. If you calculate the holding cost and ordering cost, they are not equal. When we take the effect of quantity discount, this becomes the lowest cost, and it is the order quantity for this particular case. Moving further, we have another important case: production lot sizing.

As I said in the beginning, there may be situations where, in our traditional models of EOQ, these vertical lines are the replenishment lines and these slanted lines are the consumption lines. But in some cases, it is possible that our replenishment lines are like this, and then the consumption lines are like this, and then the consumption line. And when we have these kinds of angular replenishment lines, it means the replenishment and consumption are happening simultaneously. The name indicates that only in the manufacturing facility, when we have a production facility, we have to discuss production lot sizing. But any place, any place where we have replenishment and consumption simultaneously, in that case, this production lot sizing is applicable.

Now, the rate of production is P and the rate of consumption is R. So, the inventory—that means you are producing, let us say, 10 units per day and you are consuming at the rate of 3 units per day. So, I can say that inventory is getting developed, inventory is getting developed at the rate of P minus R, 7 units per day—that is the rate of inventory accumulation. So, for some time, I am running this facility for T days, and then I am using the inventory—whatever we have built in these T days—over D days, and T is smaller than D. T is smaller than D generally because you are running your facility for 3 or 4 days and then you are consuming these over a 10-day period or 20-day period. And one more thing: P, the production rate, is generally not only greater but much, much greater than R because if the production rate is equal to or less than R, no inventory accumulation will take place.

So, these are the two important conditions for developing the production lot sizing or EPQ models. Now, here, since the entire discussion is similar to the EOQ model, there is only one important change, which you can directly see from the formula—the denominator term here.

$$Q_p = \sqrt{\frac{2R\,C_p}{C_h(1-\frac{r}{p})}}$$

We have a multiplying factor of 1 minus r upon p—that is the only change. We are having that—what is the rate of accumulation of inventory—that is deciding the peak level. You see, my production quantity Qp is the one lot; I am going to produce Qp items, but my average inventory is not going to be Qp level. If I am producing Qp, so Qp will be the item I am consuming; Qp will be the item I am consuming.

So, average inventory in this case is Qp by 2. But, if Qp is not produced over a day, Qp is produced over some number of days, then what is going to happen? I am going to produce same Qp, but these Qp are consumed like this. So, you are producing Qp, but you are not touching these peaks. So, the height of this peak will be, this height of the peak will be Qp into P minus R upon P.

That is the height of the peak and therefore, the average inventory in the case of APQ is QP into P minus R upon 2P that is the height. And therefore, this is we already know is the annual ordering cost this will be our because of this discussion it will be our annual holding cost and that is our Qp cost. So, with this different type of models which are possible in our inventory discussions and we discussed three important models in this particular class where we discussed that how you are going to handle the supply side uncertainty and then with the help of example we saw that a complex case where supply side as well as demand side uncertainties were there. We also discussed quantity discount case which are very very common in bigger purchases and then we also discuss that where your replenishment is not instant, replenishment is happening over a period of time and during the period of replenishment consumption is also happening simultaneously what should be the order quantity and what will be the maximum order level in these situations? So, with this we come to end of this session.

Thank you very much.