Course Name - Operations and Revenue Analytics

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Lecture - 13

Welcome, friends. In our previous sessions, we discussed how we have to take different types of decisions in uncertainty. Uncertainty is a very common issue in various operations management cases, right from forecasting, where we deal with a lot of uncertainties. We discussed different types of possibilities of uncertainties, and we also discussed how we can best use analytics to help us deal with those uncertainties. Then, we also talked about inventory. In the last three or four sessions, we have seen how different types of uncertainties—from the demand side and the supply side—are possible, and you have to consider those uncertainties in light of the customer service you want to provide.

So, uncertainties are inevitable, and you have to handle your case in this uncertain environment. In the last session, we introduced the concept of a decision tree—that when there are multiple possibilities, you can use them to make a good prediction. For that purpose, we also introduced the concept of expected value, and we saw how, in a very systematic manner, we can calculate the impact of our decisions. You have decisions, and then there are different possibilities that may arise from those decisions. Now, in this particular session, we are going to discuss, with the help of a detailed example, how a decision tree is used in decision-making.

The name itself is 'decision tree.' So, you can understand that it is meant for decision-making purposes only. So, we are going to discuss this with the help of a simple example. Because the theory of decision trees has already been covered in our previous session. Now, in this case, a company is going to open a particular factory.

So, you can say that the case is related to facility decision and these decisions facility decisions are very common in our supply chain management in network management. Because generally you have to take two three different types of decisions whenever you are deciding about the facility. One is location, second is size and nowadays one more decision is related to facility decision that is ownership. So, these types of decisions are very common when we are talking of facility decisions. Where to locate your facility?

Facility should be of big size, medium size, small size. Whether I should take the facility from the spot market or I should go for a lease agreement or I should construct my own facility. So, all these are the issues related to ownership also. So, these are the three very critical issues and then there may be few also few more also, but these are beyond our scope of decision tree. For example, whether you want to have a general purpose facility or a special purpose facility or general purpose facility, general purpose facility gives you more flexibility while special purpose facility gives you minimum flexibility.

But, the economies of scale, operating cost, these are more in case of special purpose facility because you have one type of equipment, one type of stacking arrangement that is very efficiently used for all the items which you are stocking in that facility or all the items which you are producing from that facility. When you have general purpose facility, you may go for frequent changeovers and frequent changeovers may give you flexibility, but the operating cost may be relatively higher. So, efficiency will be low in case of general purpose. So, depending upon your business interest, objective, the kind of SKUs you have in your offering you may decide whether to have general purpose or special purpose facility. Location, size, ownership, these are the issues which we are going to discuss with the help of decision tree.

Now, in this particular example, which I am discussing, the current demand for a particular product in the market is 500 units, which can go up and down by 20 percent. Now, if you remember, we discussed the case of binomial distribution while preparing the decision tree. So, 500 is the current demand, and it can go up and down by 20 percent plus and 20 percent down with equal probabilities. That means 0.5, 0.5, and 0.5; it can go up by 20 percent or reduce by 20 percent also. Then it says the retail price of the product is 4 dollars, and they can set up the factory that will cost them 2 dollars per product.

So, the current price is 4 dollars, and the cost is 2 dollars. They can also subcontract at the rate of 3 dollars, which can also go up and down by 20 percent. Now, it is possible that either the cost is 3 dollars for subcontracting, which is also liable to change by 20 percent plus or 20 percent minus, with equal probabilities. For the coming two years, they have to decide which option will be best suited for them, and the discount rate over the period is 10 percent. So, let us see what the different options available to them are for making this choice. So, three options are suggested: get all the demand from the spot market as needed.

So, you can get all the items; you have a 500-unit requirement and buy all 500 from the spot market, and that is one option. Option number two: you set up your own factory and get additional because your factory will have a limited size. Right now, the factory is, let us say, of 500-unit size. In the next period, if demand increases by 20 percent, that means the next period demand becomes 600. So, you have 500 from the factory plus 100 from the spot market.

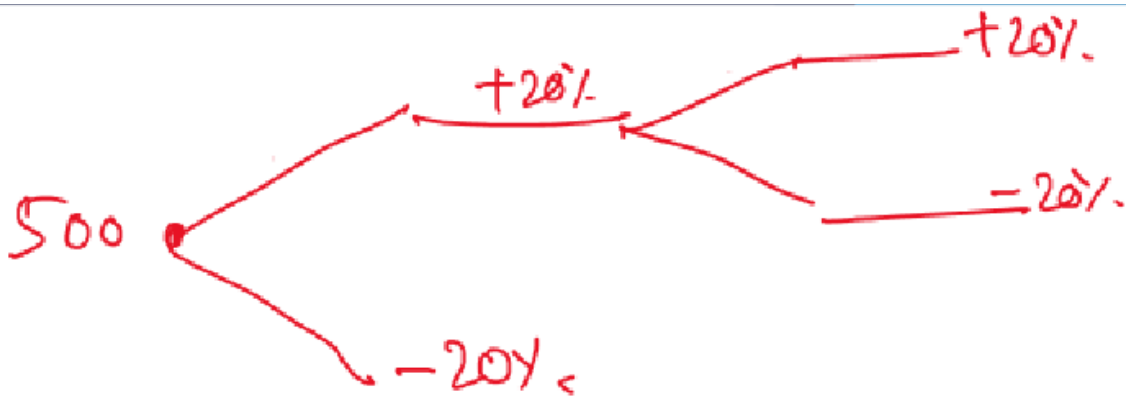
So, this is option number two. Option number one is to source all 500 from the spot market. The third option is entering a contract with a party where you have a lease arrangement with a minimum charge that allows variable uses of subcontracting with additional requirements from the spot market. So, you have the lease agreement where you are required to use a minimum value of the facility, and if you need extra requirements, you will source them from the spot market. So, these are the three options: one is to have your own factory and then source additional requirements from the spot market; another is to source 100 percent of the requirements from the spot market; and the third is to have a contract facility, but if you need more items, you can go to the spot market.

So, options two and three are more or less similar; the only difference is that in option two, you have ownership, while in option three, you take a lease agreement. Now, with these inputs, let us see the data given to us: current demand is 500 units per year, as I already mentioned, and this follows binomial uncertainty. Demand fluctuates up and down by 20 percent—plus 20 percent or minus 20 percent—with equal probabilities for both scenarios. The lease price is $2 per unit, and the spot market price is $3 per unit.

Spot prices are also subject to change by plus or minus 20 percent, with equal probability for increases and decreases. You earn a revenue of $4 per unit, and the discount rate is 10 percent.

A discount rate of 10 percent means—I hope most of you are aware of the time value of money— So, that means whatever is 100 rupees today will be worth 90 rupees tomorrow. What is 90 rupees tomorrow will become 81 rupees the day after. So, the discount factor means the value is depleting over time. We all know that the purchasing power of money decreases with time. So, the rate at which the purchasing power of money is reducing is denoted by the discount rate.

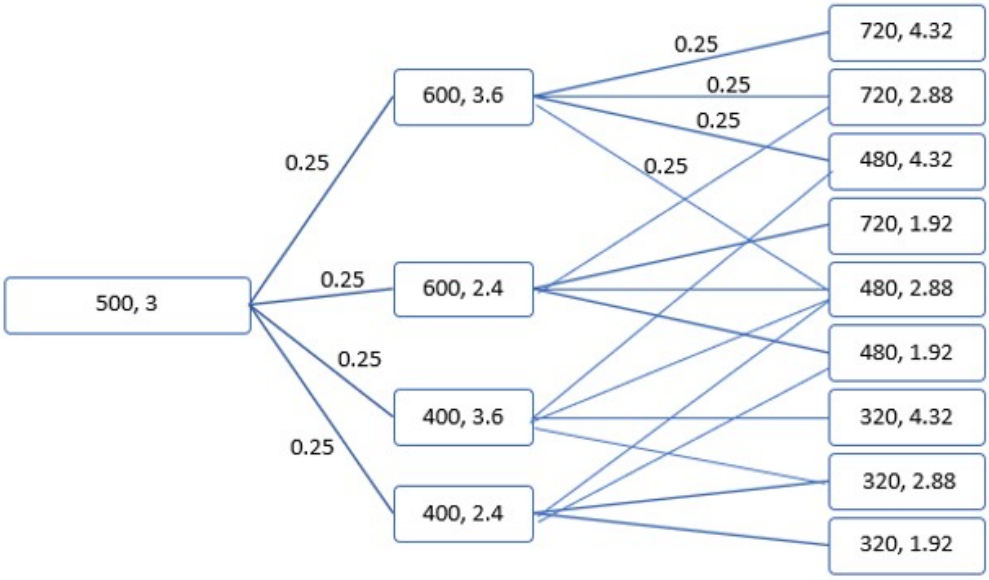
So, as we are now able to see that there are two important things. One is that two changes are happening. One change is in the demand, and another change is in the spot prices. One factor, two factors. And these two factors can be represented as a decision tree starting point: 500 demand can increase, demand can decrease, and then again from here, demand can increase or decrease.



Like that, and every time things are happening with equal probability. Now, the same thing is happening with the subcontracting price also. Right now, the subcontract price is 3 rupees. Prices may increase by 20 percent. Prices may decrease by 20 percent with equal probability, and so on.

Now, we need to put all this information in a single decision tree diagram and let us see what the shape of that decision tree diagram is that comes here in front of you. We have two variables: one is this 500, another is this 3. So, we are starting with this 500 and 3 in the beginning; 500 is demand, and 3 is the price of the spot market. Then there are possibilities that demand is increasing and price is also increasing. So, now you see there are four possibilities: demand increase and price increase, demand decrease and price decrease, demand increase and price decrease, and demand decrease and price increase.

Now, probability of each phenomena is 0.5, 0.5, 0.5, 0.5, 0.5, 0.5 that both these things will happen therefore, combined probability will be 0.5 into 0.5, 0.25 and same 0.25, 0.25, 0.25. So, there are four possible scenarios two where demand is increasing and two where demand is decreasing and that is where you have four scenarios 0.25, 0.25, 0.25 these are the probabilities.



You please remember that you can see or you can check your decision tree that the sum of all these branches the probability associated with all these branches should be equal to 1. Sigma probability should be equal to 1. Now, when demand is increasing, price is also increasing it becomes 600, 3.6 when demand is increasing, but price is decreasing 600,

2.4 then demand is decreasing, price increasing 400, 3.6 and demand is also decreasing, price is also decreasing that is 400, 2.4.

So, this is let us say our T_0 this becomes T_1 and this becomes T_2 and similarly I request all of you that you can make T_3 also. Now, here you see that because four different branches are coming from T_0 to T_1. So, you have four alternatives. Now, every node in T_1 is giving 4 you can say branches. So, ideally 4 into 4, 16 branches are there.

But, when you see this T_2 situation, only 9 exclusive, 9 exclusive alternatives are there. You can count 1, 2, 3, 4, 5, 6, 7, 8, 9. So, actually nine alternatives are there which are nine unique alternatives are there. So, on the same lines both can increase, both can decrease, one can increase, another can decrease, one will decrease, another will increase these are the alternatives which are available. So, in this way in this way more branches of this decision tree are possible.

People can expand it for T10 or even more periods. Now, after understanding this initial construction of decision tree, let us see how are we calculating profit related to decision tree using this decision tree activity. Like for the period number 2, if we calculate the profit at each node, now period number 2 here, this is the period 2. In this case, demand and price are all these different combinations. Demand is 720, price is 4.32.

So, total cost associated with this 720 and 4.32 is 720 is this and the price of not signing a lease and obtaining all the warehouses space from the spot market. So, the spot market price is 4.32 you are using option 1 where you are taking all the requirement from the spot market and therefore, 720 multiplied by 4.32 the total cost is this much. The total revenue will be the selling price is 4 rupees yes selling price is 4 rupees 4 dollars. So, our revenue will be 720 into 4, 2880. So, infact, it is a case of loss.

It is a case of loss that you are getting a loss of 0.230. So, if impact I want to calculate for is this, if this decision is coming for all spot market, So, here the impact is minus 230.4.

- Start with Period 2 and calculate the profit at each node
- For D = 720 p=$4.32, in Period 2:
- TC(D = 720 p=$4.32,2)= 720 * 4.32 = $3110.4
- TR(D = 720 p=$4.32,2)= 720 * 4 =$2880
- profit(D = 720 p=$4.32,2) = 2880 - 3110.4 = -$230.4

And similarly you have to calculate, you have to calculate for all the spot market related issues, it is possible that you have at the market demand increases, but the price decreases, it remains 2.88 for some reason I do not know but, it will be 720 into 2.88 but, our revenue is this much 2880. So, in this case it is easy to understand that there will be a positive profit will be positive in this case. So, we have to calculate the impact that if we are going to buy all the space in the spot market, what will be the impact of this?

Now, that impact is coming with a particular probability, this it is not that you are directly getting this impact because 25 percent probability that demand will increase price will also increase and then again 25 percent probability of getting a impact of minus 230.4. So, these probabilities will multiply and then you will get this expected impact. So, expected profit is therefore, calculated for all such you can say possibilities then we are going to get the expected impact that what is the total expected impact for our this particular decision. Now, another situation you can consider that ok, I am going to take a limited amount of space from the lease market and then I will see whether that is profitable or not, whether the expected impact of that particular situation is better or not. So, in this way we are calculating profit or loss for each of these nodes and we can continue this calculation for all these nodes.

Now, these are happening for period 2 and these are in reference of this decision of period 1. The decision of or the situation of 603.6 this is giving you 4 options and then you have calculated the calculations for option 1 2 3 and 4. We will discuss how to summarize these calculations in few minutes. Now, we are doing this discussion when we are having all the requirement fulfilled from our spot market. But, as I discussed in the beginning that we have three options and if let us say I want to compare between these options that whether.

So, our decision tree will come like this you have a present situation you take spot market. Then you take lease and then you take ownership, whether you are doing spot, lease or ownership demand may increase demand may go this way, this way, there are four options which are possible. Same four options are possible here also and same four options are possible here also. This is happening in T1 and same T1 will result into T2

four options like this and so on. So, now, the calculation which we were showing you are only for this part.

That is what we are showing, and when you have to compare, you need to do the entire calculation of spot versus lease. Okay, if I am doing the spot calculation, my expected profit or expected impact—I will say because profit is possible, and loss is also possible. So, in that case, what is your expected impact in option 1? Expected impact for option 2 and option 3, and all these impacts are going to happen in the future—after the second period, after the third period, after the fourth period, as many periods as you want to discuss, depending on you. So, we have to do a backward calculation also—what is the current impact of these future cash flows which are going to come to us—because the decision I am going to make today. Therefore, I consider the concept of the time value of money also, and in the time value of money, we are considering how a future cash flow, which is going to come to us, will have some value at the present time.

Because you have to do some investment today—whether you are building your own factory or you are doing some lease agreement. So, you need to pay some amount today and see whether it is profitable or less profitable. Accordingly, you will make a decision —whether to go for the spot market, whether to go for the lease market, or whether to take ownership. Now, we discussed one type of case where all the requirements are going to be fulfilled from the spot market. Take another consideration for comparison purposes —like we are going to make a plan for a capacity of 500, and the rest would have been bought at the revised prices.

Now, the same kind of evaluation process we are going to follow—what we did for the spot market case. Now, in this case, take a node of 720, 4.32. Now, when the demand is 720, you are having a capacity of 500 on your own. So, you have to buy only this additional capacity of 220 at the rate of 4.32, and the remaining you have on your own. But, for developing this capacity, there will be some initial capital investment you have to do, and therefore, the concept of the time value of money comes into the picture. For example, if I am going to buy everything from the spot market, there will be no need for the time value of money.

But, you may consider one option in respect of other option. Therefore, time value of money is coming because in one case you are investing something. In another case, there is no investment upfront. So, therefore, because some of you may be thinking why we are talking of time value of money we can do all this comparison in future only. But, there are some investments happening today therefore, we need to make the decision in today's context only.

And if for example, because if you see our decision tree these are the possible scenarios 320, 480 etc. Where you have less than 500 demand and because of less than 500 demand these additional capacity you have will remain unused. So, that is a risk also you have built extra capacity and that will not be used. So, now after so you have to do both these calculations as part of your homework also that develop a model like this itself. And here you have to calculate the expected value like in this case if you see we are in case 2, it will be let us say 720, 4.32.

So, here you will be calculating 220 at the rate of 4.32, while there is a case of 384.32 there will be no cost 0 cost. Because, you are not going to use any capacity from the spot market there are cases like 482.88 again it is 0 whenever it is more than 500 only in that case there will be a cost applicable in option number 2. So, now once we have calculated the impacts or all the branches for period number 2. Let us go back to the time machine and see what is the net present value of all these future cash flows? So, let us see how are we going to do that.

For this node 603.6, there are four branches coming. So, the first important thing is we need to calculate the expected profit in period 2. Over all these four possible states which are coming from this particular node of period number 1, and that is the expected profit for this particular node for period number 1. This comma 1 stands for period. So, because each of them has equal probability. So, I have taken this 0.25, which is the probability outside it, but for example, if they come with different probabilities.

So, you have to multiply these impacts with their respective probabilities. Incidentally, these are equal in my case. So, I can take them as a common factor outside the bracket. So, the expected profit for this particular node after this calculation comes to be 240, and

then we have to see because this profit will come after 1 year. After 1 year, I am expecting this profit of 240 dollars.

So, we have to go for the present value calculation of this. So, the expected profit at each node in period 1 is the profit during period 1 plus the present value of the expected profit in period 2. So, now at node 1, if I go back to this particular node. So, during period 1, whatever profit I am getting, plus whatever future profits I am going to get, their present value. So, that is what I am going to calculate.

Now, so expected profit of a particular period when D is sum P is sum at node 1 is the expected profit all 4 nodes in period 2 that may result from this particular node. And present value of expected profit PVEP stands for present value of expected profit is the present value of this expected profit and PDP1 and the total expected profit is the sum of the profit in period 1 and the present value of the expected profit in period 2 which I have just explained. So, this calculation is the present value of this expected value in period 1 is PVEP for this particular scenario expected profit for this divided by discounted rate factor. So, the net present value, we know that whatever cash flow I am getting, cash flow in future divided by 1 plus k into the power of n. So, if I am getting in next one period, n will become 1. What I am getting two period ahead, n will become square if I am getting three period ahead the value of n will change to 3.

So, since I am talking only one period ahead this is 1 plus k where k is the discount factor and in this case we have considered k's value as 10 percent. So, the present value of expected profit is 218. The profit which you are going in the year 2 that is 240 but its present value is equivalent to 218 and therefore, the total expected profit at this particular node where D is this P is this in is the sum of the profit in period 1 plus this 218.2 and that is 600 into 4 minus 600 minus 3.6. Because this is the revenue, this is the cost and this so 240 plus 218.2, 458.2 that is the total expected profit at this particular node and similarly you need to go backward to time T_0 also.

- The present value of this expected value in Period 1 is

PVEP(D = 600, p=$3.6, 1)

= EP(D = 600, p=$3.6,1)/ (1 + k)

=$ 240 / (1 + 0.1) = $218.2

- The total expected profit P(D = 600, p=$3.6, 1) at node D = 600, p = 3.6 in Period 1 is the sum of the profit in Period 1 at this node, plus the present value of future expected profits possible from this node

P(D = 600, p=$3.6, 1)

= [(600*4)-(600*3.6)]+ PVEP(D = 600, p=$3.6, 1)

= 240 + 218.2 = $458.2

Now, if you see whatever calculation I did for T1 the same calculation will go for T naught also and taking this calculation forward after this you can go for period T naught also where you can see that the initial demand was 500 initial price was 3.

And then expected profit at this particular level is whatever the expected profits at all the nodes of year 1 plus the profit you are getting at that particular level. So, all these are the four different terms 25 percent of this, 25 percent of this situation, 25 percent of this situation and 25 percent of this situation because four possible situations are emerging. So, the expected profit in year 1 is 954 and then you have to calculate the present value of this at the rate of 10 percent that becomes the present value 867.

- For Period 0, the total profit P(D = 500, p = 3, 0) is the sum of the profit in Period 0 and the present value of the expected profit over the four nodes in Period 1

EP(D = 500, p = 3, 0)

= 0.25P(D = 600, p = 3.6, 1) + 0.25P(D = 600, p = 2.4, 1) + 0.25P(D = 400, p = 3.6, 1) + 0.25P(D = 400, p = 2.4, 1)

=0.25x(458.2)+0.25x1832.73+0.25x305.45+0.25x1221.8

= $954.54

PVEP(D=500,p=3,0)=EP(D=500,p=3,0)/ (1+k)

=$954.54 / (1 + 0.1) =$867.77

So, your present value of profit is 867 and the profit which is coming because of this particular requirement in this period only that is 500 is the demand 3 is the price 4 is the revenue, so 500 rupees directly you can consider. So, 500 plus 867, 1367 is the total expected profit using the net present value concept for this spot market.

$$P(D=500,p=3,0)$$
$$= 500x4- 500x3 +PVEP(D=500,p=3,0)$$
$$= \$500 + \$867.78$$
$$= \$1367.78$$

We have option 1, giving us 1367.78 as the expected profit's present value. This is what I write for this. Now, another option we have considered in this problem is where 500 are owned and the rest are on the spot market. Now, let us say for building these 500 on your own, there is a cost of, let us say, 1000 dollars. And option 3: 500 leased, the rest on the spot market. This 500 lease is coming to you because you are making an agreement at the beginning itself for the next 2 years. So, this is available to you at the rate of two per unit.

So, you have a fixed rate which is applicable for the next two years. So, the question is whether this option is better, that option is better, or the other option is better. We have explained the process of calculating how we are going to determine the expected profit with the net present value. I request all of you to do the same calculations for option 2 and option 3 as well, and then you can compare which option is the best one and take a decision accordingly. So, this is also a very interesting use case of our predictive analytics, where you can predict the expected profit and accordingly decide which is the more profitable option for you.

With this, we come to the end of this particular session. Thank you very much.