Course Name - Operations and Revenue Analytics

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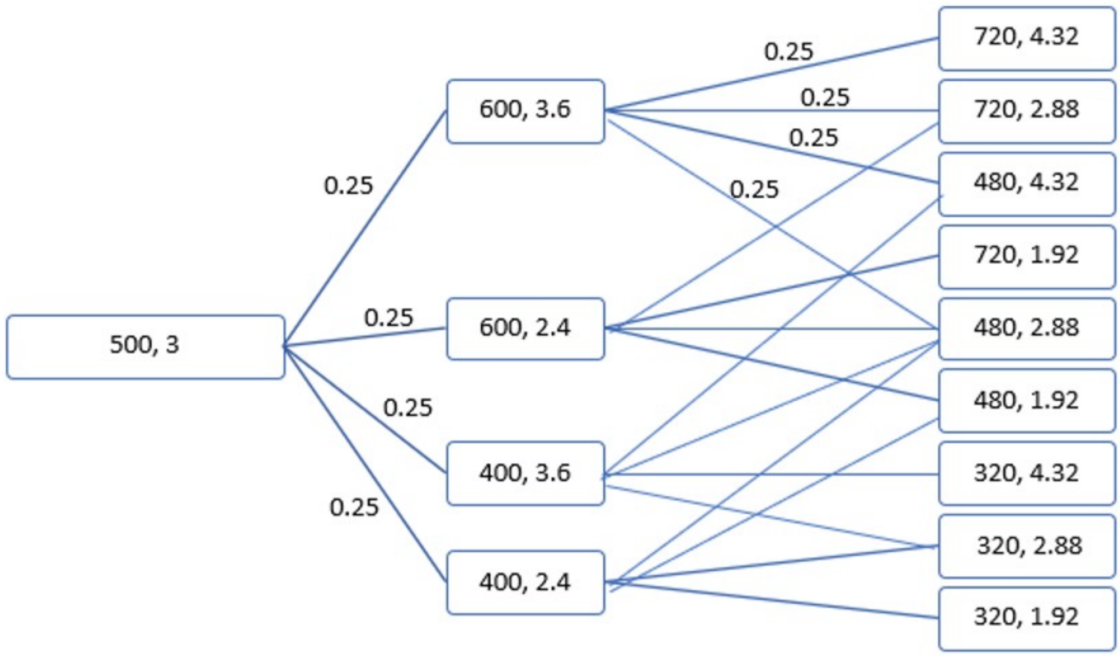
Lecture - 14

Welcome friends. In our last session, we discussed a very interesting prescriptive analytics tool: the decision tree. We discussed how, if you have a situation with alternatives, you must decide which alternative is the best. In that case, a decision tree is also a possible solution. Similarly, in many situations, you may not consider all possible outcomes, lack confidence in your historical data, or find it difficult to create a mathematical model.

In that case, another interesting tool we will discuss in this class—simulation activities—can also help in decision-making. First, we will revisit the problem we discussed in our last class about the decision tree. You may recall the decision tree we used, which was also presented in the previous class. If you recall, we had two or three options, including the stock market. Another option was building capacity—for example, captive capacity plus additional supply from the spot market.



We previously discussed spot market calculations, and I assigned a task: planning to maintain a captive capacity of 500 units, with the remaining demand (e.g., 100 units for a 600-unit requirement) sourced from the spot market. So, 500 units will come from captive capacity, and the remaining 100 will come from the spot market. In cases where demand reaches 720, 500 units will come from captive capacity, and 220 will come from the spot market. However, the downside is that if demand is only 320 or 480, I may not fully utilize the captive capacity. The capacity remains, but there will be underutilization—20 or 180 units, depending on demand—since captive capacity exceeds actual demand.

So, we need to see which option is better, and for that purpose, we were planning to have a decision tree. How did we do the calculation? We discussed the concept of expected profit. For this, our time was T0, this was time T1, and this was our time T2. So, we calculated the expected profit for T2 first. And then we calculated the present value of expected profit for T1 plus the expected profit for T1, which became the total profit of T1. Similarly, we did the backward calculation for T0 also.

And there may be some initial investment, and then the NPV of all cash flows at the beginning. So, initial investment and NPV of all these are the two things. The difference of that will give you an estimation of profit or loss for a particular option. So, that calculation we have completed for the spot market in our last class. Now, as I said, we have considered a situation of captive capacity of 500. And the remaining capacity comes from the spot market. I have given you this task, but to help you, we are now coming with this ready-made calculation. Here, though you see that only nine unique combinations of demand and prices are there—this is demand, this is price.

So, only nine unique combinations are there in T2, but for the purpose of calculating present value and net present value, we have done that each of these nodes is emanating four different branches. Branch 1, branch 2, branch 3, branch 4—it is a different matter that most of these branches are leading to a common impact. So, now we are going to have this calculation to show you how we are going to proceed with this option number 2, where we are keeping the captive capacity of 500, and the additional requirement will come from the spot market. So, as you see, in the case when it is 720, 500 will come from the captive capacity, and the remaining 220 will come from the spot market. And though we have nine unique combinations of demand and price, you see that for the sake of our calculations, we have divided this table into 16 groups.

A group consists of 4 items like this. So, if you see this, let us say node 1, node 2, node 3, and node 4. So, these calculations are for node 1, this is for node 2, this is for node 3 because you may remember we have to calculate the present value of future cash flows, the present value of future cash flows. So, to calculate the total expected profit in year 1 at node 1, I require the total expected profit from year 2 from all the possibilities and then their present value for node 1 of year 1. You will understand all these things in a minute.

Demand	Available	Spot	Price	Cost	Revenue	Profit	
720	500	220	4.32	950.4	2880	1929.6	
720	500	220	2.88	633.6	2880	2246.4	
480	500	0	4.32	0	1920	1920	
480	500	0	2.88	0	1920	1920	2004
720	500	220	2.88	633.6	2880	2246.4	
720	500	220	1.92	422.4	2880	2457.6	
480	500	0	2.88	0	1280	1280	
480	500	0	1.92	0	1280	1280	1816
480	500	0	4.32	0	1920	1920	
480	500	0	2.88	0	1920	1920	
320	500	0	4.32	0	1280	1280	
320	500	0	2.88	0	1280	1280	1600
480	500	0	2.88	0	1920	1920	
480	500	0	1.92	0	1920	1920	
320	500	0	2.88	0	1280	1280	
320	500	0	1.92	0	1280	1280	1600

Now, if I see the first part of this table, out of 720, 500 comes from the available capacity, 220 comes from the spot market. The spot market price is 4.32, the revenue, if you remember, comes at the rate of 4, the sale price. So, the demand is 720, this is 720 multiplied by 4, so that is 2880, and the cost comes because of these 220 units which use the spot market, available to you at 4.32. Now, in all those situations where your demand is less, like here 480, there is no need for spot markets. So, therefore, if these spot markets are 0, the cost will also be 0. You can check the entire table.

Only when there is a requirement for the spot market will there be a cost term. The revenue comes from 2880, 2880, 480 multiplied by 4, and so on; this entire column is derived. The profit is revenue minus cost. Revenue minus cost, revenue minus cost, revenue minus cost—that is the profit. Now, to get this, this is the expected profit, and this expected profit 2004 is basically 1929.6 multiplied by the appropriate probability, which is 0.25.

And similarly, for all these, the probability is the same. So, you can actually do the sigma of this multiplied by this to the 0.25; your expected profit is coming to be 2004. Now, this expected profit of 2004 is coming in year 2, in year 2. To calculate its present value, we have to present value of year 2 profit in year 1, which will be 2004 divided by 1 plus k, that is the discount rate, and this k is 10 percent, as we remember. So, this 2004 divided by 1.1 comes out to be around 1821.82, and similarly, for this also, if you calculate in the same way for all these, the present value in year 1 will be 1650.91.

Decision Tree case 2

S.P. = \$4   K = 10%

Leasing the annual capacity of 500 at Rs 2 and serving additional demand at spot price.

Period 2 Table:

220×4.32   720×4

	Demand	Available	Spot	Price	Cost	Revenue	Profit	Expected Profit
Node 1	720	500	220	4.32	950.4	2880	1929.6	
	720	500	220	2.88	633.6	2880	2246.4	
	480	500	0	4.32	0	1920	1920	
	480	500	0	2.88	0	1920	1920	2004
Node 2	720	500	220	2.88	633.6	2880	2246.4	
	720	500	220	1.92	422.4	2880	2457.6	
	480	500	0	2.88	0	1280	1280	
	480	500	0	1.92	0	1280	1280	1816
Node 3	480	500	0	4.32	0	1920	1920	
	480	500	0	2.88	0	1920	1920	
	320	500	0	4.32	0	1280	1280	
	320	500	0	2.88	0	1280	1280	1600 → 1454
Node 4	480	500	0	2.88	0	1920	1920	
	480	500	0	1.92	0	1920	1920	
	320	500	0	2.88	0	1280	1280	
	320	500	0	1.92	0	1280	1280	1600 → 1454

$(1929.6 + 2246.4 + 1920 + 1920) \times .25 = 2004$

PV of Y2 Profit in Y1 $= \frac{2004}{1+K} = \frac{2004}{1.1} = 1821.8$

PV in Y1 = 1650.91

Its present value will go to 1454 point something, and this is also 1454. So, as we have seen, these are the expected profits which we will be using for calculation in our year number 1.

Demand	Available	Spot	Price	Cost	Revenue	Profit	Profit from 2	Profit (1+2)	
600	500	100	3.6	360	2400	2040	1821.818	3861.818	
600	500	100	2.4	240	2400	2160	1650.909	3810.909	
400	500	0	3.6	0	1600	1600	1454.545	3054.545	
400	500	0	2.4	0	1600	1600	1454.545	3054.545	3445.454

Now, for year number 1, we have this table where we have taken these profits using their present values. If I try to summarize how we are proceeding here: T2, then T1, and then T0. So, every year has some profit coming in; we are using its present value plus local profit, you can say, and then its present value plus local profit.

Local means the profit for that particular period only, and in this order, for year 1, when I am calculating, these are the demands which are available to us from this demand tree, and the capacities which I will be using from the spot market are 100 and 100 whenever the demands are going to 600. The spot market price will be like this. So, you have 100 into 3.6 and 100 into 2.4 as the cost for these two situations. Revenue will come from the total demand, and the selling price is 4 rupees, like this. So, your profit will be 2400 minus 360 and so on, and this table is coming; this particular column is coming from our previous calculation, and this is the sigma of 2040 and 1821.

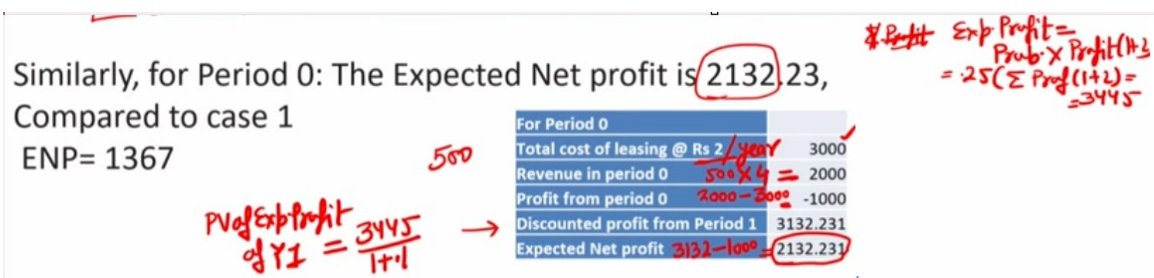
Period 1:

Demand	Available	Spot	Price	Cost	Revenue	Profit	Profit from 2	Profit (1+2)	
600	500	100	3.6	100x3.6 = 360	600x4 = 2400	2400 − 360 = 2040	1821.818	2040 + 1821 = 3861.818	
600	500	100	2.4	100x2.4 = 240	2400	2160	1650.909	3810.909	
400	500	0	3.6	0	400x4 = 1600	1600	1454.545	3054.545	
400	500	0	2.4	0	1600	1600	1454.545	3054.545	3445.454

Now, the expected profit in year 1 is basically the sum of profit multiplied by—rather, it is not appropriate to say 'sigma' because each of these demand possibilities and the price possibility have a probability. All these combinations have a probability of 25 percent. So, for the expected profit, we are actually going to multiply the probability column with the profit column—the final profit column. Incidentally, all the probabilities are of equal nature. So, you can summarize it as 0.25 multiplied by the sum of profit, which comes out to be 3445, since we have done this calculation in Excel. So, there are some digits after the decimal also, but for the sake of our convenience, we can very easily ignore these points after the decimals. Now, let us go to this particular node, T0, for period 0, because we are talking about a three-period scenario—T0, T1, and T2—and there can be a captive capacity decision also. We have done a lease agreement for the captive capacity, okay?

At the beginning of this particular process, I am shaking hands with you for a price of 2 rupees and 500 capacity. So, for one year, this is per year. So, for one year, the cost is 1000 rupees, and for three years, the cost will be 3000 rupees. The revenue—the demand in the first year is 500. So, 500 multiplied by 4—the revenue will be 2000.

So, my revenue is less, and I am incurring more cost in the beginning. So, my profit is 2000—that is, the revenue minus the cost is minus 1000—and whatever discounted profit I am getting from period 1, that is the expected profit's present value—the present value of the expected profit of year 1. That will come out to be 3445 divided by 1 plus 0.1, and that is around 3132. So, the expected net profit—the expected net profit will be 3132 minus 1000—that is my expected net profit: around 2132.



For Period 0	
Total cost of leasing @ Rs 2 / year	3000
Revenue in period 0 500 x 4 =	2000
Profit from period 0 2000 − 3000	-1000
Discounted profit from Period 1	3132.231
Expected Net profit 3132 − 1000 =	2132.231

And if you can recall our previous class, the expected net profit when we were going to buy all the units from the spot market was 1367.

So, you can very conveniently make a decision that it is better to have a deal where you are having a lease agreement of captive capacity for 3 years rather going for spot market purchase every year. Because here the profit was 1367 here it is 2132 and more scenarios may give you better opportunity that okay we have done this calculation with 500 as a lease agreement and remaining from the market you can try and that will make slightly more complex calculation where you take a decision of a captive capacity. You can say this is a homework captive capacity of 400 rest from spot market and you can see that please check what will be the net expected profit and that will give you more idea about the calculation process also and better decision making. After completing this decision tree example then another very popular approach is Simulation.

I hope most of us are aware what do we mean by simulation and there are different types of simulations which are possible. In this particular class, I will be focusing on Monte Carlo simulation which is very very commonly used across the problems for decision making. Inventory problems can be handled with Monte Carlo. Service problems can also be handled. If you go to operation research, queuing problems can also be handled with the help of this Monte Carlo system.

So, there are wide variety of uses of Monte Carlo system where we are going to use a particular kind of a system where every new incidence, every new event is happening in a random manner. One customer is coming after how much difference another customer will come I do not have any idea. Generally, when you see in a petrol pump may be for 15 minutes there is no customer but all of a sudden three customers come and this is purely a random behaviour. So, trying to model that random behaviour is our Monte Carlo simulation which is very powerful, very useful and we will see in this particular session that how this Monte Carlo simulation is possible in variety of situations. Where you have to done lot of approximation, stochastic models are there where you are having some idea that how customers are going to come, but it is very difficult to develop a mathematical model for these things.

For example, we are creating a situation so that we can explain to you the use and the working of the Monte Carlo system. Now, in this particular case, there are three types of demands for a particular product, and since we are talking about a small case, I am considering only three levels of demand: 3, 4, and 5. Now, the probabilities of these are 20 percent, 50 percent, and 30 percent. We assign these probabilities based on our past experience. And in this case, for every type of situation, you have to see that the next event—the next customer—will have a demand of 3 units, 4 units, or 5 units. We follow a system known as the system of random numbers, and depending upon it, you can take a number of random digits—2 digits, 3 digits, or 4 digits.

How much spread of probability do you have? Like in this case, we have 20 percent, 50 percent, and 30 percent, or you can say 0.20, 0.50, and 0.30, and then we are also making a cumulative probability column: 0.2, 0.7, and 1.00. So, here we are using only two digits after the decimal. So, we can have only two-digit random numbers in this particular situation. So, since the probability is up to 20.

So, I need 20 random numbers—any 20 random numbers—which are coming for this particular first level. So, these 20 numbers are between 0 to 19. Then, 70 is the probability—sorry, 50 is the probability of 4. So, I need 50 random numbers for designating the probability of 4, and these 50 numbers are taken between 20 to 69. Then, 5 is 30 random numbers, which are required, and these 30 random numbers are coming between 70 to 99. You see, since we do not have practically all the possibilities of randomness, we have given a particular block of random numbers to a particular level of demand.

Theoretically speaking, in the most ideal situation, these 20 random number should also be 20 different numbers. Now, I am considering only up to 0 to 19, a particular block of random number. So, whenever we are going to use Monte Carlo simulation, we actually do not have real random numbers, it is quasi random numbers with which we are going to operate but that is our practical limitation. Otherwise, it is quite possible that I may have one random number 1 and 79, 89, 50, 3, 4, 20 such random numbers I put under one bin and that will make my 3. And similarly, but that will take huge amount of time and practically it is very difficult to do that.

So, therefore, we take these kind of bin system where we have a block of random numbers representing a particular level of event. Whether the demand will be 3, 4, 5 these are the events. So, next event will be a demand of 3 or 4 or 5, it is decided on the basis of random number which I can generate either using our simple calculator or from the tables which are available in books for random numbers using the computers also. So, there are variety of ways through which you can generate the random number. Even you see when you are throwing your dice whether 1 will come or 4 will come or 6 will come that is also a kind of random number, but there you have only 6 random numbers.

Now, in this particular case when we have the theory of this particular Monte Carlo system. We have to run this system of inviting the events in a sequential manner may be some number of time. So, that the output starts stabilizing and for that purpose I will take you to the excel demonstration which will help us that how the output stabilizes and we are deciding what is going to happen with each new random number. So, here we are having a small situation for which we are going to do this problem in the excel sheet. We have two situations.

One is the demand because it is an inventory problem. So, demand can be 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12, and these are the associated probabilities. The supplier may take either 2 days, 3 days, 4 days, or 5 days. In all our inventory models, we generally used to consider fixed lead time, but practically, it is not going to be a fixed lead time. There may be different lead times. So, the probabilities are 20 percent, 30 percent, 35 percent, and 15 percent.

These are some of the costs available for the data we have. The ordering cost is 80 rupees per order. The holding cost is 2 rupees per unit per day. Generally, you remember the data for holding cost, which we used in our previous classes, is mostly on a per-year basis, but here it is per unit per day. The shortage cost, if the order is not fulfilled—if demand is there and you are unable to fulfill it—is 20 rupees per unit.

The reorder point is 20 units. Whenever your inventory reduces to 20 units, you place a new order. The beginning inventory is 30 units, and the order quantity every time you place the order is Q. Let us see the performance of this particular system and how you will model this situation.

Problem

Firm XYZ is trying to estimate the total inventory cost, where, based on the historical data, they have the following distribution of demand and lead time based on historical data.

Ordering cost: 80 Rs /order

Holding cost: 2 Rs/unit per day

Shortage cost: 20 Rs/unit per day

Reorder point: 20 Units

Beginning Inventory: 30 units

Order Quantity: 40 Units = Q

Demand	Probability
3	0.02
4	0.08
5	0.11
6	0.16
7	0.19
8	0.13
9	0.1
10	0.08
11	0.07
12	0.06

Lead time	Prob.
2	0.20
3	0.30
4	0.35
5	0.15

You may find some very interesting outcomes with the help of our simulation running. So, we will go to Excel now.

So, now we are coming to this simulation Excel sheet, and we have generated all the required information needed to develop this Monte Carlo simulation. For that purpose, as you remember, there can be different types of demands.

So, if I go to the demand table, demand can vary from 3 to 12. These are the probabilities which we already know. We converted these probabilities into cumulative probabilities, and then you have the random number ranges. These are the random number ranges. So, in fact, we will be needing this information when we work on the simulation worksheet.

Similarly, for the lead time also. 2, 3, 4, 5 these are the possible lead times, and similarly, random numbers are generated for different lead times also. Now, going further to our simulation worksheet, we have started working on the simulation for this. Day 0, our beginning inventory is 30 units. Beginning inventory is 30 units on day 0.

That is given to us in the problem. Day 1, I generate a random number, maybe using my calculator. The first random number—now, when you generate a random number, the random number may be very long. For example, the first random number which I press in my calculator now shows me something like 5, 3, 7, 8, 9, like this. Now, depending upon my requirement, I may take 2 digits, 3 digits, 4 digits, anything.

So, you can say that the random number for me is 5. I am taking the first digit of this. Now, when this 5 is there, we can see that the demand is this: 5 is falling in the range of 2 to 9. So, the demand is correspondingly 4 units. So, we go to this and we put the demand as 4. When the demand is 4, you will have 26 units left with you, and you will fulfil the demand of 4 units. Now, the ordering cost and the holding cost is 2 rupees per unit per day.

So, at the end of the first day, you are left with 26 units. So, you will incur a holding cost of 52 rupees, 52 rupees at the rate of 26 multiplied by 2. The next day, you again generate a random number between 0 to 100. The next random number is 5, sorry, 0, and this 0 corresponds to a demand of 3 units. So, 3 is the demand. 26 minus 3 leaves you with 23, and then 23 multiplied by 2 gives 46 as your holding cost. You will keep decreasing your inventory until you reach the reorder level of 20.

So, we will generate another random number. Then, this random number corresponds to a demand of 10. When this corresponds to a demand of 10, 23 minus 10 becomes 13, and 13 is less than 20. So, you have to trigger one order here as per your policy, and the order will be of 40 units, which you also know. So, here you need to know after how many days this order will come to you. So, you need to generate one more random number.

Now, it is quite possible we have generated separate random numbers for the lead time. But, I may work because it is a random number so there is no mistake in that. I may work with this 79 also. I may work with this separate random number I have generated for lead time that is 91 also. So if 91 is my number, 91 will fall in 5 days lead time.

So this 5 day lead time means after 5 days I will come. If this would have been 79, if I take the 79 as my random number for lead time, this 79 will come under 4 days lead time. So, in that case, after 4 days, I will get the supply. Now, since I have accepted 91 as my random number, I will get after 5 days. So, if it is 79, this would have been 4 days.

Right now, I am talking with 91, it is 5 days. And the so, right now my 13 units are there and the stock will get replenished after 5 days, where I will get a supply of 40 units and now when you have placed the order at this point you will incur a cost of ordering also and only 13 units were there. So, cost of holding will be 26. Similarly, next day comes. Random number is 60, demand is 8, 13 minus 8, 5 are left, I am running a holding cost of 10 units.

Another day comes, I have a demand of 3 units, perfectly fine, I am able to serve. Then, on the sixth day, random number comes to be 10, sorry, 84 and corresponding demand is 10 and now I have only 2 units available with me. So, I can fulfill the demand of 2 but I will not be able to fulfill the demand of 8. So, in fact, I have a balance of minus 8 here and for that purpose I will pay a shortage cost of 8 into 20 rupees 160 rupees. This is my shortage cost I am able to, I have to pay on this particular day.

Because, I do not have any item available with me and I have to pay the shortage cost for this particular item. Then, next day comes. Here, I have another random number of 80 and the corresponding demand is 5. Since, I do not have any item, again for this demand of 5, I have to pay a shortage cost of 100 rupees. Now, on day 8th.

Now I am getting the fifth day after placing my order the supply of forty items come to me but the demand on this particular day is seven. So, immediately seven items are lost means sold from this so only thirty three items are left with me. And in this way continuously, I am updating for as many rounds as I want like we have done this particular solution if I go to the bottom of this table for 40 particular days. So, 40 is not a enough number we can do it for let us say 100 days, 200 days because this is just to be done on a paper. So, there is no cost associated for running the simulation and after 80 let us say n number of days you can calculate that what is your total cost of this particular model.

So, the total cost may include the cost of holding, ordering, plus shortage. So, all these are the different types of cost; the sum total of that is this particular thing. Now, in this case, since the model is there, you can try this simulation model like our decision tree model by changing the policies. Here, my reorder point is considered as 20 units. You see that right now, my shortage cost, ordering cost, and holding cost.

Should I decrease my reorder point from 20 to 15, or should I increase it from 20 to 30? You can just play this simulation game again and again, and with those new alternative scenarios, you will be able to make a decision. So, that is also a good way of making a decision, as we have created just one policy in this example. But you can create multiple policies and compare your simulation results to see which particular policy gives the minimum cost of inventory—minimum cost of ordering, holding, and shortage—that is a good policy for my system. So, with this, I hope you can continue solving this particular problem using your own systems by creating multiple ordering policies and accordingly see which policy is more suitable or gives you the minimum cost of inventory.

With this, we come to the end of this particular session. Thank you very much.