Course Name - Operations and Revenue Analytics

Professor Name - Prof. Rajat Agrawal

Department Name - Department of Management Studies

Institute Name - IIT, Roorkee

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Lecture - 15

Welcome friends. So, as we were discussing with the help of different types of tools like we discussed in our last few sessions about the use of a very interesting tool of analytics that is decision tree. How there are multiple options which are possible and out of those multiple options we are able to decide which is the best possible outcome on the basis of various probabilities which are there for different types of alternatives. With the help of simulation also, we discuss that how you can have a good decision for your future. And all these are the nature of our prescriptive analytics where we are giving a kind of a suggestion that out of various alternatives, which alternative is going to give you a better result.

Moving into the use of these types of cases, we have another set of problems where we can use these types of knowledge in our real life situations. News vendor problems are very often discussed in our inventory management discussions where the nomenclature of news vendor comes that you have a particular period, where you have only single opportunity to procure and on the basis of what decision you are taking you have to survive for that entire period. So, the nomenclature comes from the philosophy that there is a newsboy who is there at a bus stand in the morning he has only single opportunity to procure the newspapers that what should be the optimal stocking quantity of the newspapers with that boy. So, that in the day whatever demand comes he is able to maximize his profit. So, he has to see, he should not overstock, if he overstocks and demand is less, he may incur some kind of losses because of some unsold newspapers are left with him at the end of the day.

On the other hand, it is also possible that he stocked less, demand is more and he is unable to fulfill that extra demand and in this case he is not able to get the more profit or the profit which is possible he is deprived of that particular profit. So, both these scenarios are possible and in this scenario we need to decide what is the more suitable quantity which can take care both these types of cost. So, in this particular session we are going to discuss this particular case of inventory management which is very popularly known as News Boy Problem or News Vendor Problem. So, here as I have already explained you the nature of the problem then we will have some numericals, some data through which we will try to explain you that how our knowledge of analytics will help us in solving such type of problems. So, here I have some data with me.

Where you see that, based on my past experience, we have a possibility of different levels of demand. If you see on the screen, I know that my demand can vary from 15 to 29 numbers on a particular day. There can be a possibility that on a particular day, only 15 units are sold, and it is also possible that on some other day, 29 units are sold. So, demand can be 15, 16, 17, and so on up to 29. The probabilities for different levels of demand are also given to me.

This is again based on my past experience. So, these data are coming from past knowledge. I am able to assign—or rather, I am able to know—what can be the demand levels and, for each demand level, what can be the possible probability. So, the probabilities are moving from 1 percent, 2 percent, 3 percent, 5 percent, 10 percent, 12 percent, 11 percent, and so on. These are the probabilities for all these different levels of demand, and the sum total of these probabilities is equal to 1.

That is the important thing: the sum of these probabilities should be equal to 1. Now, based on this particular information, let us see what the possibilities are here. You see simply this data. So, without going into any kind of analytics, I can see that 22 has the maximum probability—12 percent. 22 has the maximum probability. So, a layman will say that, okay, we should keep 22 newspapers or 22 units of this product because the maximum possibility is of 22 units on any particular day. So, it looks quite logical that the highest probability is with 22 units.

So, we should stock 22 units. Now, I have some more data with me, and using that data, let us calculate what will happen if I stock 22 units. Here, we are considering the cost price of this product as 1 rupee and the selling price as 1 rupee 50 paisa. So, you can say that the profit per unit is 50 paisa. So, here we are considering the cost price as 1 rupee and the selling price as 1 rupee 50 paisa.

So, the profit per unit is 50 paisa per unit. Now, as we just discussed, we should expect to procure 22 units. Now, let us see: if I procure 22 units, the cost price is 1 rupee, the selling price is 1 rupee 50 paisa, and the profit is 50 paisa. With this information, if I procure 22 units, let us see what the cost is. The cost is 22 Q equals to 22, as I am procuring 22 units.

So, whether the demand is 15, 16, or any level, the cost price is 20, 22, and then the revenue which I am going to get is also very interesting. If I sell 15 newspapers, then one paper gives me 1 rupee 50 paisa, so 15 newspapers will give me 22 rupees 50 paisa. And so on, as I go up to here: 16 into 1 rupee 50 paisa, 17 into 1 rupee 50 paisa, 18 into 1 rupee 50 paisa, 19 into 1 rupee 50 paisa. So, here: 20 into 1 rupee 50 paisa, 21 into 1 rupee 50 paisa, 22 into 1 rupee 50 paisa. Now, if the demand is 23, but since you have only 22 papers, from here onwards your revenue is 22 into 1 rupee 50 paisa; it is not going to increase with increasing demand—it gets stagnated.

And then you can calculate profit also for each of these possibilities, which will be revenue minus cost. So, you will have, like in this case, 22.50 minus 22, which is 50 paisa.

Demand (in units)	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Probability	0.01	0.02	0.03	0.05	0.07	0.1	0.12	0.12	0.11	0.09	0.09	0.08	0.06	0.04	0.01
Cost: Q=22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22
Revenue	22.50	16X1.50	17X1.50	18 X 1.50	19 X1.50	20 X 1.50	21 X 1.50	22 X 1.50	22 X1.50	22 X1.50	22 X1.50	22 X1.50	22 X1.50	22 X1.50	22 X1.50
Profit = Rev. - Cost	22.50 -22 =.50														

Now, here you see the possibility of this profit of 50 paisa happening is just 1 percent. So, we will calculate another term, which is known as expected profit, which is going to be the sigma of probability Pi multiplied by profit at i.

$$\text{Expected Profit} = \sum_{i=15}^{29} (p_i \times \text{Profit}_i)$$

And i equals 1 to, let us say, if these are from 15 to 29, like in this particular case.

So, for all the respective probabilities, you have to multiply their profits and then do the sigma, which is the calculation of this expected profit. For the sake of convenience and to save time, we have done this calculation, and this calculation gives us, for this particular data, the expected profit.

If you see this table here. The cost is 22 for all these entries; the revenue is changing from 22.5 up to 33. Up to this level, the revenue is changing. Why is it changing?

Demand	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Probability	0.01	0.02	0.03	0.05	0.07	0.1	0.12	0.12	0.11	0.09	0.09	0.08	0.06	0.04	0.01
Cost (@22)	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22
Revenue	22.5	24	25.5	27	28.5	30	31.5	33	33	33	33	33	33	33	33
Profit	0.5	2	3.5	5	6.5	8	9.5	11	11	11	11	11	11	11	11

I just explained to you, and from here onwards, it is the same: 33. So, whether the demand is more, but still you are getting only the same level of revenue, and therefore, the sum total of 0.5 multiplied by 0.01 plus 2 multiplied by 0.02, and so on, up to the last 11 multiplied by 0.01, comes out to be our expected profit. It is actually expected profit, not expected revenue, which is 9.395.

Expected Profit = 0.5 x 0.01 + 2 x .02 + -------- + 11 x 0.01 = 9.395

Now, I request that whether it is the maximum possible expected profit or not, we need to check, and that we can check. Now, you can do the same exercise by taking either 21 or 23, because for 21 also, the probability is 12 percent.

So, please see if I stock 21 units, what is going to be my expected profit? Maybe 21 may give me more expected profit. So, I will decide to procure 21 units maybe because the probabilities are very near: 22, 23, 21, 12 percent, 12 percent, 11 percent. Maybe 23 may give us more profit. So, we may do this by trial and error method or by doing all the computations again and again for these different levels. So, we have got a very direct method through which you need not go for all these iterative calculations, hit-and-trial methods. You need to see that there is a direct method which can help you in determining what is the optimal stocking level.

Now, for that purpose, we want to have, we have the calculation available with this 21 also. If I take this 21, our expected profit is this. So, now it is interesting for Q equals to 21, expected profit is 9.495, and when we did the calculation with Q equals to 22, the expected profit is what we just calculated: 9.395. So, you can clearly see that by reducing the order quantity from 22 to 21, our expected profit is slightly higher. So, it is always advisable that I should keep this level of quantity; that is a better decision.

But, can there be a direct method also? Now, somebody will say, sir, why have you stopped at 21 only? Why not try at 20 also? Maybe 20 will further increase your profit. So, we have to check on all these levels, and then you can understand how much huge amount of time and effort will be required for doing these calculations. So, for that purpose, we have a simpler method, and in that simpler method, as I said in the beginning of this particular session, that if you are stocking more and the demand is less. For example, in this case, if I stock 21 and on that particular day the demand remains to be 16 only, for example.

So, you are wasting 5 units of your item, there is no need of these 5 units and these are wasted. On the other hand, you are stocking 21, but on that particular day the demand happens to be 26. Now, there is a possibility of earning more profit but you are deprived of that particular extra profit. So, these are represented as cost of overstocking and cost of understocking. When you are stocking more and you have unsold items at the end of the day, these are contributing to your cost of overstocking.

Cost of understocking, there is a possibility of more profit and you are deprived of that particular opportunity for gaining more profit. So, ideally we want to minimize both these costs and that will give us highest level of expected profit. Now, for that purpose we are considering a simple situation where we have defined how mathematically we are going to calculate this cost of overstocking and cost of understocking. You may remember our example where the cost price was 1 rupee, selling price was 1 rupee 50 paisa. So, cost of overstocking CO and cost of understocking is CU.

Cost of overstocking is the cost price and cost of understocking is the profit which you are not able to earn, that is selling price minus cost price. Generally, this is the beginning of this particular model, but it is also possible that if there are some unsold items, if there are some unsold items or you can say there is a value in the scrap also, there is a salvage value also. So, if salvage value is there, in that case if salvage value is there, if salvage value is represented as small l. In that case, cost of overstocking will be C minus l,

C = Cost Price = Rs 1 per paper

S = Selling Price = Rs 1.50 " "

Cost Price ← 1. Cost of Overstocking – C_o

Profit per unit ← 2. Cost of Understocking – C_u

$C_o = C$ → Loss

$C_u = S - C$ → Profit

because some of your losses can be compensated by the salvage value which you are able to get at the end of the period if things remain unsold.

So, now how are we going to develop the mathematical expression for this particular case?

So, we are like to stock highest number of quantity you see there is a cost of overstocking another is cost of understocking. Now, you will like to stock as much as that by stocking this you are having more probability of increasing your expected profit than your expected loss with any additional unit, this additional unit can result into profit also, this additional unit can result into losses also. So, as long as this additional unit is helping you in increasing your expected profit you will keep increasing this number. But, there will be a point where this additional unit may hamper your profit and may result into more losses, more expected losses. So, that is the our optimal level of stocking quantity.

So, in this case the formula development works like this. That expected profit from stocking additional unit is more, expected profit is more than the expected loss from stocking the additional unit. So, profit, expected profit by the additional unit is more than expected loss from additional unit. C_U into probability demand is greater than equal to Q. Now, when the, when will this expected profit will happen? That if demand is more, I am having this much of quantity, demand is this much.

So, when demand is more than the quantity I am stocking, so by increasing the number of quantity as long as I am coming to this level by stocking additional number of units I am able to increase my profit. Now, I have if this is the level of demand and if I further stock then these stocking additional units which are beyond my demand level these are going to result into losses. So, I can express this particular concept, this is the concept and then into this mathematical expression as Cu into probability demand is greater than or equal to Q this is the expected profit. And the expected loss is Co into probability demand is less than Q and this can be represented as Cu into probability demand is greater than equals to Q is more than Co into probability demand is less than Q and then it can slightly be readjusted mathematically.

$$C_u \times \text{Prob. }(D \geq Q) > C_o \times \text{Prob. }(D < Q)$$

$$C_u \times \text{Prob. }(D \geq Q) > C_o\,[1 - \text{Prob. }(D \geq Q)]$$

And then finally, we can write that probability demand is greater than Q is Co upon Co plus Cu.

$$\text{Prob. } (D \geq Q) = \frac{C_o}{C_o + C_u}$$

Probability demand is greater than equals to Q, this is cost of overstocking ivided by cost of overstocking plus cost of understocking. Like in this particular case, we had cost price 1 rupee, selling price 1 rupee 50 paisa. So, our cost of overstocking is 1, cost of understocking is 50 paisa. So, our probability that demand is greater than equals to Q is equals to 0.67. Now, we got this probability label, but how to execute this probability label, let us discuss that aspect also.

So, for the execution purpose let us see that there are these are the different levels of demand where you have let us say. Now, what does it indicate? That these are the demand and these are the probability. That there is a probability of 10 percent that demand will be 10 units. Demand can be 20 also that again has a probability of 10 percent.

Demand can be 30 units, it can again have a probability of 10 percent. But, you understand that demand of 20, 30, 40, 50, 60 will only be possible when the demand of 10 has completed. Demand of 30, 40, 50, 60 will only be possible when you have completed the demand of 10 and 20 units and so on. Demand of 60 is only possible when you have completed up to 50 level. So, because in this situation we have to calculate cumulative probability and generally we calculate cumulative probability from top to bottom.

But considering this scenario, that cumulative probability will be highest for demand of 10 units, because in all these scenario only when you have completed 10 units you have sold 10 units, then you can go to the higher levels of demand. So, we will make the cumulative probability table from bottom to up like this. So, using this knowledge that how our cumulative probability tables are made, we are coming back to the example which we were discussing that from 15 to 29 the demands are given their probabilities are given. Now, in this case we will complete the table of cumulative probability which is the last column a last row, which is 0.01, 0.05 and so on going like this and finally, the cumulative probability is 1 for 15 units. Now, the probability which we are searching that is 0.67 the meaning is probability should be equal or more than 0.67.

So, as we are coming here, here, here for 22 the cumulative probability is 0.6 and the next highest is 0.72 which is just higher than 0.67. So, this 21 becomes our optimal stocking level 21 becomes our optimal stocking level and this is what is the answer for this particular situation that if I stock 21 units, I will be getting the highest you can say profit, expected profit and I recommend to stock 21 units. You can do now because we have already done the calculation that if our Q star that is the stocking quantity is 21, the expected profit that we have already calculated is 9.495. That is our highest expected profit. Yes, our quantities may change.

Our quantities may change. How? For example, there is a salvage value also given in this case. For example, our data says that we have 1 rupee cost price, selling price 1 rupee 50 paisa. Now, if salvage value L is let us say 25 paisa.

So, in this case, my cost of overstocking will become 1 minus 25 paisa, which is 75 paisa. The cost of understocking will remain the same, and in this case, my probability will be 0.75 upon 0.75 plus 0.50. And this will be 0.6.

$$l = 0.25$$

$$C_o = 1 - .25 = .75$$

$$C_u = 1.50 - 1.00 = .50$$

$$P = \frac{.75}{.75 + .50} = \frac{\not{75}\ 3}{\not{125}\ 5} = \boxed{.6}$$

Now, when I see 0.6, it is possible that you will have this probability level if the salvage value is 25 paisa per unit. I request all of you to please calculate what the expected profit will be. What will be the expected profit if I have a salvage value of 25 paisa? That is again going to be an interesting calculation, and you have to apply some extra effort for the calculation of expected profit. Why am I asking you to do this calculation? We already have in our class the calculation of expected profit for Q equals to 22, which is 9.395.

But since now the salvage value is also available, that expected profit calculation will change. So, that is what I want to know: if you have a salvage value, what will be the expected profit? Yes, I have given you the answer that the highest expected profit will be at Q equals to 22. But I have not given you the answer for what the expected profit will be at that particular level. So, please do that calculation, and in our next session, we will discuss the answer to that question and also what the concept of cycle service level is in association with this particular algorithm we are discussing. With this, we come to the end of this particular session.

Thank you very much.