Course Name - Operations and Revenue Analytics

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Lecture - 16

Welcome friends! So, as we were discussing in our last class that there are situations where we have only single opportunity to procure in a particular season. We discussed this example with the help of a very popular newsboy problem. We discussed that how you have to assign cumulative probabilities and by checking a particular level of probability, you can decide that what is our optimal stocking quantity? Optimal stocking quantity which is giving us maximum expected profit, not profit, expected profit because it may change a lot based on your probabilities which you are assigning to different types of demand labels.

In the last of that particular class, we modified our example and we also said that there may be a case where you have a some kind of scrap value or salvage value. And we know that most of the items have salvage value and nowadays we all are moving into this environmental sustainability, circularity, circular economy. In these cases, this salvage value becomes very important. Every product may have some salvage value and therefore, if the products are not used you will not throw them like that without any economic value gaining from those unused products. You can have some kind of economic value in the leftover products also.

So, we were discussing that particular case where I gave you the situation and I am expecting that most of you have done the calculations that what will be the expected profit when our order quantity becomes 22 in light of salvage value. So, that example we are going to complete in this particular class and we will also see the concept of Cycle Service Label which is known as CSL. You may recall in our classes of inventory management, when we were talking of optimal product availability, we discussed three very important measures of product availability. One was product fill rate, another was order fill rate and third was cycle service level. We discussed that CSL is the most commonly used measure of our product availability.

So, how this entire discussion can also be linked with the concept of CSL and then using the direct excel functions, we can calculate various important things which we are supposed to calculate for running or executing the model of newsboy-boy, newsboy inventory management. So, as you may recall from our last class, the cost price was rupees 1, selling price was 1.50, the salvage value we consider to be 25 paisa. And therefore, C_O came to be 75, C_U was already 50 and then this probability P which is C_O upon C_U plus C_O is 0.60.

$$C_o = C - l$$

$$\text{In our example, } C_o = 0.75$$

$$C_u = 0.50$$

$$P = 0.75/1.25 = 0.60$$

We may remember that how this particular quantity became our order quantity in this particular case. Now, I gave you the task, that we need to find expected profit in this particular case.

I am expecting that most of you have done this particular example, but for the others those who could not complete or who are unable to understand for them, I am making the attempt for solving this particular situation. So, here if my let me add few more rows in this table to explain you the calculations. Now, here you see my stocking quantity is 22. So, my cost is 22. For all the days, it is 22 only.

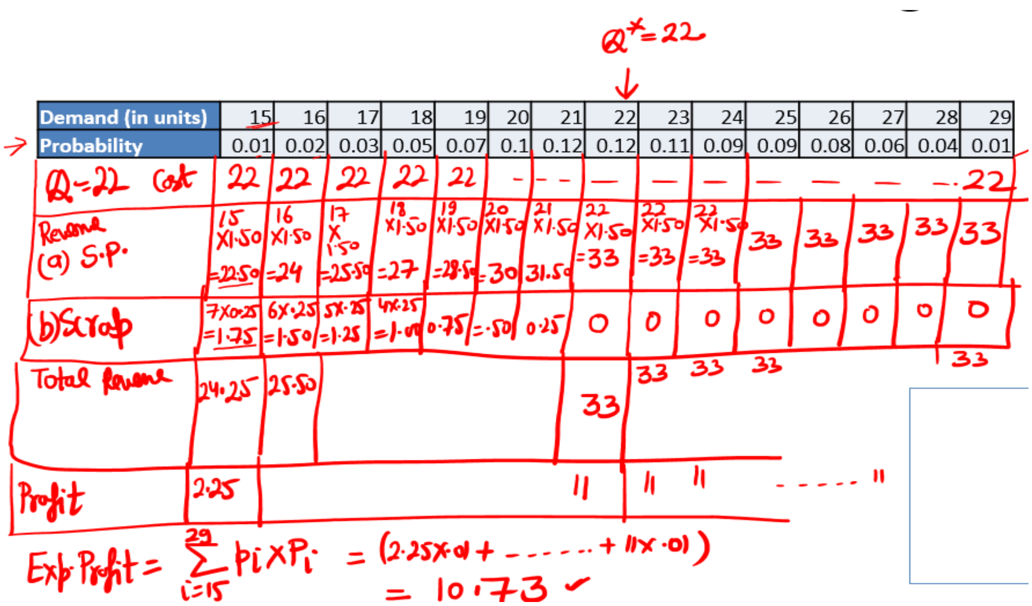
Let us talk of revenue. Now, there will be two sources of revenue. Please remember, in this case, there will be two sources of revenue. One is by the sale of the papers and another is from the scrap value of the unsold newspapers. So, we need to write revenue A.

Selling price, revenue B from scrap. So, in case of first 15 papers multiplied by 1.50, 16 multiplied by 1.50, 17 multiplied by 1.50, 18 multiplied by 1.50, 19 multiplied by 1.50. 20 multiplied by 1.50, 21 multiplied by 1.50, 22 multiplied by 1.50. Perfect but when it goes to 23, 24, 27, 28 or 29 it is only 22 into 1.50, That means 33, it will be 33 it will be 33, it will be 33, it will be 33 and 33. Now, the scrap is you have 22 papers, 15 was the demand. So, you have 7 newspapers which will be sold at the rate of 25 paisa, 1 rupees 75 paisa.

This is 22 rupees 50 paisa. Similarly, this is 24 and here you have 6 unsold which are going for the scrap at the rate of 25, 1 rupee 50 paisa and so on 25.50 here it is 5 into 0.25, 1.25. Here it is 27 here it is 4 into 0.25, 1 rupee, here it is 28.50, here it is 75 paisa, here it is 30, here it is 50 paisa, here it is 31.50, here it is 25 paisa. And now comes the level of 22 where you have no scrap because all the papers are sold. So, no scrap revenue and the same is for others also no scrap revenue.

So, my total revenue becomes I can have this calculation that is 22.50 plus 1 rupee 75 paisa that is around you can say 24.25. This is 25.50 and so on up to here it is going to increase to 33 and beyond that it is 33 33 33 and up to this level 33. Then you can calculate profit also. So, now you see that profit earlier for this case when you have only demand of 15 was 50 paisa and now the profit will be 24.25 minus 22 that means 2.25 and so on your profit will be increasing here up to 11 rupees and then 11, 11 and 11. Finally, we will calculate the expected profit which will be sigma product of probability into profit from i equals to 15 to 29.

And when I do this, 2.25 into 0.01 goes up to 11 into 0.01. If I do this, the expected profit, as I understand, will come around 10.73.



Demand (in units)	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Probability	0.01	0.02	0.03	0.05	0.07	0.1	0.12	0.12	0.11	0.09	0.09	0.08	0.06	0.04	0.01

Now, you remember when I did the calculation without salvage value from my previous class for Q equals to 22, the expected profit was 9.395 without scrap value. But now, with scrap value, this expected profit has increased to 10.73. That is why, generally, scrap value, if you have it, gives you the scope of stocking a higher number of units, and your cost of overstocking is reduced, so your risk-taking capacity increases. And that may help you in improving your expected profit, like in this particular case, we see that our expected profit has increased by around 1.40 rupees. So, that is a significant improvement from no salvage value to with salvage value.

After understanding this, I hope most of you got this answer, and those who could not do it now can redo the calculation with the help of this understanding. Now, we go to discuss a very related concept, which I mentioned at the beginning of today's session also—the concept of Cycle Service Level. In the cycle service level, if you remember, we have this EOQ model. So, let us say over a period of time there are a thousand cycles. Over a period of time, there are a thousand cycles. Out of a thousand cycles, how many cycles' requirement, how many cycles' requirement am I able to fulfill from the inventory which is in my stock? The inventory which is in my stock, am I able to fulfill? Now, imagine that you can consider one cycle. You can consider this one cycle as a single-period case also, provided two things are happening.

One, whatever is left over in this particular cycle is not carried forward for fulfilling the requirement of the other cycle or the subsequent cycle. And second, if there is extra demand in this particular cycle and you do not have that much of the item, that demand is lost. Demand is also not backlogged, and the items are also not carried forward. So, it more or less becomes a single-cycle case, which we were discussing with the help of a newsboy problem. Now, the same logic: cost of overstocking, cost of understocking, and here we have slightly changed our conventions. In place of L, we are writing salvage value as S. In place of selling price, we are simply writing it as price P. So, I hope you will not get confused with our changing conventions.

And the optimal cycle service level is CSL, and what should be the corresponding optimal order size that we are writing as O star? Now, please remember this very important definition. Which says, which basically defines what CSL is? It is the probability that the demand during a particular cycle season is below or at the O level. If demand is going more than O star, I will not be able to fulfill that demand.

So, CSL is basically the probability. For example, let me tell you, assume CSL is 90 percent. So, it means that if I have 1000 cycles or if I have 100 cycles, for example, 100 cycles of inventory replenishments, out of those 100 cycles, in 90 cycles the demand is able to be fulfilled from the available inventory, and available inventory is coming from my lot size, whatever is my order quantity, that is the maximum available inventory queue in a particular cycle, and that queue is sufficient for fulfilling the demand of that particular cycle. That is the meaning of CSL. So out of 100 cycles, in 90 cycles, the demand is less than the uppermost limit of inventory which is there in my stocks. There may be 10 cycles where demand is more than that particular queue level.

So, generally in our business, we decide this optimal cycle service level based on the industry practices, and we also decide based on how we are going to consider our customer. If our customer is ready to wait in a particular scenario, you may still operate with lower CSL because, though I am assuming that the lost demand is lost. But generally, if a customer is ready to wait, this lost demand is not lost; he may come in the next cycle. So, for practical purposes, you may keep low CSL in case of your higher inventory cost and customer is ready to wait. But if the customer is not ready to wait, there are very close substitutes which are available in the market.

Market is full of competition. In that case you need to have higher CSL. So, according to decision of CSL we need to decide our optimal order quantity. So, here already in our last session we have seen that how are we going to develop the formula for CSL and the formula for CSL is nothing but it is expressed in the same way which is cost of understocking divided by cost of understocking plus cost of overstocking.

$$CSL^* = Prob(Demand \leq O^*) = \frac{p-c}{p-s} = \frac{C_u}{C_u+C_o}$$

Now, CSL is probability that the demand during the season during a particular cycle is equal or less than O naught and same thing we discussed in our previous session that probability demand is more than, demand is more than a particular level of order quantity. For that purpose, if you recall, probability demand is more than Q is equals to CO upon CU plus CO, that we discussed in our previous session, previous class.

Prob (D>=Q) = $C_O/(C_U + C_O)$

Now, here, CSL is Cu upon Cu plus C0. So, I request that you should not get confused because of this formula of CSL versus the formula which we used in our previous session. Now, let us see with the help of one illustration that how these things will be operated.

So, CSL as just now we saw is cost of understocking divided by cost of understocking plus cost of overstocking. and this can very easily be calculated with the help of mean and standard deviations using our normal distribution functions. The demand in our all the sessions we are considering to be distributed as normal distribution and when the normal distribution has mean of mu and standard deviation of sigmas. So, O inverse O star becomes a inverse normal function of three characteristics that what CSL you are expecting and then mu and sigma.

$$O^* = F^{-1}(CSL^*, \mu, \sigma)$$

CSL is coming from here and mu and sigma are coming from our normal distribution of the demand.

This also helps us in extending our discussion to one more interesting level. Like the problem which we were discussing for this newsboy. In this, the different probability labels which we were considering were 0.01, 0.02, and up to 0.01. These probability labels were discrete probability labels. But I can also say that the demand for this newsboy was

normally distributed, not like this discrete distribution. The mean demand of the newsboy is, let us say, 21, and the standard deviation is, let us say, 1 unit. So, this also becomes a way of expressing the demand and its associated probability. Now, I request, if I change this question from this discrete to this kind of a continuous thing, can you determine what will be the probability of 15, what will be the probability of 16, and what will be the probability of 29? This is a very interesting question for all of you to determine these probabilities. So, using this knowledge that it is not necessary that demand has a discrete probability distribution, demand may also have a continuous probability distribution like the normal distribution I am talking about.

Now, using this understanding that it is possible to have a normal distribution. We can, because that is more commonly used, say that demand has a normal distribution with a mean of mu and standard deviation sigma. We can use this normal inverse function, which requires three important characteristics. One is the probability, the second is the mean, and the third is the standard deviation. So, this probability CSL will come from this calculation, and mu and sigma will be available to you.

So, using this information, we can actually calculate the cycle service level, as you have seen on your slides, that in this case, mean demand is 250, and the standard deviation of the demand is 90. The cost price is given to you as 120. The ceiling price is given to you as 280, and the salvage value is given to you as S. So, the cost of overstocking becomes C minus S: 120 minus 80 equals 40. C_U is P minus C like this, and CSL is the cost of understocking upon the cost of understocking plus the cost of overstocking: 160 upon 200, that is 0.80.

And now you have to apply this F inverse CSL mu sigma, and that, I said already, is the normal inverse function in our Excel. So, this is written as the normal inverse function. 0.8, 250, 90, and you are getting 326 as your optimal stocking quantity for a CSL of 80 percent.

Now, for example, this CSL is coming from our this particular calculation of Cu upon C naught.

A manager must decide how much winter apparel they need to keep in stock for the winter. The forecasted mean demand is 250, and standard deviation is 90. Each unit costs 120 Rs and sells at 280 Rs. The unsold units are disposed of at the rate of 80 Rs.

$$C_o = c - s = 120 - 80 = 40$$
$$C_u = p - c = 280 - 120 = 160$$
$$CSL^* = \frac{C_u}{C_u + C_o} = \frac{160}{160+40} = 0.80$$
$$O^* = F^{-1}(CSL^*, \mu, \sigma)$$
$$= NORM.INV(0.8{,}250{,}90) = 326\ units$$

We may think of, let us say, I ignore this. Let us ignore, and now I am a more customer-oriented organization, and I feel that a CSL of 80 percent is not going to give you many satisfied customers. I want to increase my CSL. Now, I want to keep CSL from 80 percent to 90 percent, and I request now that please calculate what is the optimal stocking quantity, which is the normal inverse of 0.90, 250, 90. Let us see what this optimal stocking quantity is. You can calculate this.

It will be higher than T 326. Let me just do this calculation on my own also and the values are like. So, here the values are 365 point something let us consider it only 365.

So, the stocking quantity increases if I want to have higher CSL. So, more CSL requires more quantity, but the question is whether this higher CSL is justified or not just because of having this kind of emotional understanding

that I will be able to serve more number of customers I am going for higher CSL and higher CSL is resulting into higher stocking quantity. So, before I end this particular session there is a homework for you that now we have two stocking quantities one stocking quantity is 326 which is coming for CSL of 80 percent. Another stocking quantity is 365 which is coming for CSL of 90 percent. I request please calculate expected profit in both these scenarios that what will be your expected profit if your order quantity is 326. and what is your expected profit if order quantity is 365 and then you will be able to see that it is actually not profitable.

It is actually not a good decision to increase the CSL and having more number of quantities because it is actually decreasing your expected profit and that is what we say is prospective analytics that you have scenarios that you can increase your CSLs and you can become more customer oriented organization. But ultimately you realize you will realize that it is actually decreasing your expected profit you are getting the best expected profit when you are having a CSL of 80 percent. Yes, it is quite possible that if because of some issue you are able to increase the salvage value, then all of a sudden your entire expected profit calculation may change. If because of some issue you are able to increase the price of your product then also your Cu will change.

The point is, without changing Co and Cu, it will not be desirable to increase your CSL, and therefore, you will be able to practically understand this point when you calculate the expected profit for these two levels. So, with this homework, we come to the end of this particular session. Thank you very much.