Course Name - Operations and Revenue Analytics

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Welcome friends. So, in our last session we were discussing about optimal product availability and we discuss the cases in line with our news vendor problem. The characteristic of news vendor problem was that you have only one cycle for procurement and you need to optimize, not optimize, maximize your expected profit that how much we can get maximum and accordingly we need to decide our stocking quantity. But, in reality It is not so. In reality, it is a very rare thing that you have only single opportunity to procure.

You may remember our saw teeth patterns, where we have cycle after cycle and when you have cycle after cycle, you are continuously procuring and when you are continuously procuring even then, if you remember our discussions of CSL in one cycle it is possible that demand is more than what you have. And in another cycle, it is possible that demand is less than what you have. So, now when demand is more and you have a continuous procurement how to handle your optimal product availability in that scenario that is a kind of a improved version of our news vendor problem and that is what we are going to discuss in this particular class. Now, in this case there are two possible situations which may happen and actually practically speaking there will be a in between case of these two extreme situations. In these two extreme situations, that there may be some demand which is more than Q and D minus Q whatever it is let us say delta.

This delta is lost completely and in another case also D is greater than Q, this D minus Q is again let us say some quantity and this quantity is backlogged. Backlogged means ok I am not able to fulfill the demand of this shortage in this particular cycle. So, all these customers will come to me again when my stock gets replenished. But, practically speaking there will be something in between of these two. Some of the customers will come again and some of the customers will disappear.

So, some customers on this scale you have degree of lost customers you can say and this degree of lost customer may vary that 100 percent customers are lost and 0 lost. And this makes a huge impact on our discussions of continuously stocked items. As I say Practically speaking, you will have a case somewhere between these two extremes. All customers are backlogged. no customer is lost. That is also a very hypothetical situation.

However, it is possible that is constrained by some external environment. There is no substitute product. There is no other let us say seller available for those products. So, all these customers will actually wait by the time you get new supply. But, in case and we all know in the perfect market condition there may be competitors, there may be close substitutes and many customers may go to those close substitutes.

So, in that case there will be some position in between of these two extremes. Now, in this particular class we are going to discuss these two extremes. Now, when all the demand is lost that is our this case and when the complete demand is backlogged that is this case. Now, for developing this particular model of our continuously stocked items, continuously stocked means let me just give you the brief this is our very popular saw teeth model. And in this popular saw teeth model, we know that this is Q every time you are replenishing then fixed cost associated with each other that is like ordering cost, you may remember that this is our reorder point every time you reach to this particular label.

You place a new order then the rate at which inventory is consumed that is D units per period. This per period may be per day, per week, per month etc then standard deviation of demand per unit time. The demand which I am representing the consumption line which I am representing as a straight line, actually it is not a straight line it is having the normal distribution. And the normal distribution has a standard deviation of sigma the safety stock this is safety stock And then cycle service label that cycle service label we all know that probability that demand is less than equals to Q. This is the unit cost of the material cost.

The holding cost generally as we know it is given as fraction of product cost per unit time and then capital H will become that is the holding cost for one unit over a period of time is small h multiplied by the material cost. So, this is the terminology we are going to follow in this particular class.

- Q: Replenishment lot size
- S: Fixed cost associated with each order
- ROP: Reorder point
- D: Average demand per unit of time
- σ: Standard deviation of demand per unit time
- ss: Safety inventory (recall that ss = ROP - DL)
- CSL: Cycle service level
- C: Unit cost
- h: Holding cost (fraction of product cost per unit time)
- H: Cost of holding one unit for one unit of time. H = hC

Now, one case where demand during particular stock out is backlogged. All these customers are coming again whenever I am replenishing my inventories. And in this case, the important thing is no demand is lost, because you are not serving these customers in the current cycle.

So, there may be some penalty, some penalty is possible and that some penalty is basically cost of understocking. There will be some penalty, you may remember from our previous sessions, we discussed two types of cost, cost of overstocking and cost of understocking. Since, we are talking of a continuously replenishing system, so whatever inventory is left over in this particular period, I will use that inventory in the next period. So, there is almost nil and cost of understocking also becomes almost nil if no penalty. But, generally in practical cases there is some penalty associated with the cost of understocking, may be a customer will not like in this period when the original demand is there customer is expecting 5 units.

But when it comes to the next cycle, he may require only 3 units. So now, 2 units he is not buying. So in that way, that demand is lost, and that is considered to be a cost of understocking. So, the customer overall is not going anywhere; the customer is not disappearing. The customer is coming again to you. But the customer may not require the full level of demand, that may be the problem because of which the cost of understocking may come into the picture.

And here, let us see how we are going to develop our model. Now, you need to take a call that, okay, I am stocking this much inventory. Should I stock one more unit? This is my safety stock. So, should I take a call for stocking this safety stock to one additional unit? Now, when you are stocking one additional unit, it will have some extra cost also—extra cost of holding the item—and that we have to see the benefit due to the additional unit of safety stock equals the cost of understocking. Whatever cost of understocking you are assuming, that is not going to be there. You are actually realizing the profit, so you will not incur this cost of understocking.

And the probability of demand during the replenishment cycle is such that more than safety stock units of safety inventory are consumed—that is 1 minus CSL. CSL, as we already know—I told you—CSL is the probability demand is equal to or less than Q. Now, the role of this additional unit will only come when your probability is like demand is going above the CSL. So, that is probability equals to 1 minus CSL, that is the probability that demand will be more than the inventory you have in your safety stocks. Now, because of the additional inventory you are going to have, you will incur this cost per replenishment cycle of additional safety inventory of one unit. If you see Q by D into H, what is this Q by D here?

This Q by D is the duration of the cycle. This is the duration of the cycle that is Q by D, and since H calculation is under root H into c—sorry, small h into c—and this small h is per unit time. So, therefore, for the cycle of inventory Q by D, which gives the time of the replenishment cycle multiplied by capital H. That is the increased cost because of additional inventory—one additional inventory—and benefit per replenishment cycle will be this into cost of understocking. So, now when we summarize this whole discussion using additional cost and benefit from stocking one additional unit. Our CSL calculation gives us this: 1 minus HQ upon D into C_U.

$$CSL^* = 1 - \frac{HQ}{DC_u}$$

Now, this is a very interesting formula because of this illustration, you will be able to understand. Now, we have a situation where our lot size Q is 400, reorder point is 300, and average demand per week is given like this. Standard deviation of this demand is 20 units. Unit cost is 3 dollars. Holding cost is at the rate of 20 percent. So, cost of holding 1 unit for 1 year is 0.6 dollars.

Lead time is given as 2 weeks.

Given Data –

Lot size, Q = 400 gallons
Reorder point, ROP = 300 gallons
Average demand per week, D = 100
Standard deviation of demand per week, σ_D = 20
Unit cost, C = \$3
Holding cost as a fraction of product cost per year, h = 0.2
Cost of holding one unit for one year, $H = hC$ = \$0.6
Lead time, L = 2 weeks

Now, if all unfilled demand is backlogged and carried over to the next cycle, what is the cost of stocking out? So, what is the cost of stocking out? That means you have to calculate Cu. Now, in this case, for doing this calculation, we have this demand over lead time: 200. Standard deviation of demand over lead time is sigma D into under root L, which comes to be 20—sigma D is 20—and under root this is a slight typo: this is under root 2.

So, this becomes 28.3, which is the standard deviation of demand over lead time. Now, CSL is calculated as we discussed in our previous session—a function of ROP, DL, and sigma L, which is a normal distribution of 300. This is ROP, as we discussed in this data. DL is calculated here, this is sigma L, and this is the cumulative probability. So, the CSL we are maintaining is 99.98—a very high CSL. When we calculate our cost of understocking using this value, our cost of understocking, derived from the formula we just developed, is this. This is HQ upon DC_U 1 minus CSL, and then HQ equals to (1 minus CSL) DC_U. Therefore, C_U equals HQ upon D into 1 minus CSL. We have used this expression for the calculation of this C_U. Using this calculation, our C_U comes to 230.77 dollars per gallon, which is almost an impossible type of C_U.

- Mean demand over lead time, $D_L = DL$ = 200 gallons
- Standard deviation of demand over lead time, $\sigma_L = \sigma_D \sqrt{L}$ = 2012 = 28.3.
- $CSL = F\,(ROP, D_L, \sigma_L) = NORMDIST$ (300, 200, 28.3, 1) = 0.9998
- We can thus deduce that the imputed cost of stocking out

$$C_U = \frac{HQ}{(1-CSL)D} = \frac{0.6 * 400}{(1-0.9998) * 100 * 52} = \$230.77 \text{ per gallon}$$

The cost of this item is 3 dollars. The selling price is not given. Even if you consider the selling price, let us say, equals 5 dollars. So, the profit per unit is, let us say, somewhere close to 2 dollars. So, if the profit is 2 dollars and all demands are backlogged, it is practically impossible to have such a high value for the cost of understocking.

This means maintaining that high level of CSL is under question. Because when you keep such a high CSL, the corresponding cost of understocking comes to that. So, practically speaking, without putting so much stress, if the profit is 2 dollars per unit and since demand is backlogged, you can easily understand that if the profit does not come in this period, it will come in the next period. And let us say some penalty may be there, and that penalty will be, let us say, 50 percent. So, the cost of understocking, if I assume it to be 50 percent of the possible profit, will be just 1 dollar. If I consider that 1 dollar as my cost of this.

So, let us consider Cu equals to 1 dollar. And when I consider CU equals to 1 dollar, let us see what is the value of CSL coming. This is 1 minus HQ upon DCu and this will be 1 minus 0.6 into Q that was our 400. And D is since our weekly demand is 100 gallons so 52 weeks into 100 into the cost of understocking is 1 we are assuming and this comes out to be 1 minus and if I summarize this calculation so 0.6 into 4 by 52. So, if I do this calculation this comes out to be close to 0.95.

Let us consider $C_u = \$1$

$$CSL = 1 - \frac{HQ}{DC_u} = 1 - \frac{.6 \times 400}{52 \times 100 \times 1} = 1 - \frac{.6 \times 4}{52} \approx .95$$

The earlier values 0.9998. So, you can compare that maintaining a 99.98 percent CSL versus maintaining a CSL of 95 is much much desirable practically possible. So, therefore, as backlogging is possible we can go with lower CSL. So, this is one important take away whenever we have backlogging options, but now another case is possible where we are losing the demand. The case where demand is lost.

Now, in that lost demand case the CSL calculations are done in this way 1 minus HQ upon HQ plus DC_u and this is again shown with the help of a practical demonstration that same data what we are using in the previous example here. The C_u is given as 2 and in this case if I go with the data now the CSL is 98 percent.

Given data:

Lot size, Q = 400 gallons
Average demand per year, D = 100 * 52 = 5,200
Cost of holding one unit for one year, H = \$0.6
Cost of understocking, Cu = \$2

The optimal cycle service level is given as

$$CSL^* = 1 - \frac{HQ}{HQ+DCU} = 1 - \frac{0.6 * 400}{0.6 * 400 + 2 * 5200} = 0.98$$

The store manager at Walmart should target a cycle service level of 98 percent.

So, it becomes very very reasonable understanding that if backlogged CSL is 95 and if lost CSL is 98, because the whole calculation of Cu cost of understocking is making the difference. Because of backlogging Cu is low and here Cu is high. Generally, when you have backlogging it is difficult that there is no direct measure that how to calculate the Cu.

So, therefore, you have to take some kind of proxies, as I consider the proxy that the profit is 2 units, 2 dollars, but because of the efforts of the customers, because of many other factors, there may be some loss of demand, and that may be considered as Cu, a lower Cu value. Now, going further, you can also consider that after this entire discussion. What are the strategies available for us to handle the CSL and optimal product availability? Ultimately, the purpose is improving decision-making, and for that purpose, now comes the role of analytics. We have to improve our forecast, reduce our lead time, improve forecast, and reduce lead time. So, reducing lead time is one aspect.

Improving forecast means reducing the standard deviation of demand data. Like in this particular case, we had the standard deviation given to us as 20 units per week.

Now, if we can reduce this standard deviation, it will help us handle the situation of CSL in a much better way. And for that purpose—how reducing the standard deviation of demand data will help us, how reducing the lead time will help us—let us understand with the help of some data. Consider a situation where, let us say, the cost is 100 and the price is 250. The selling price, let us say, we consider as this. Consider some salvage value also, that is 80. Now, using this information, we can also calculate our cost of overstocking and cost of understocking.

Cost of overstocking is your cost price minus salvage value, that is 100 minus 80, 20. Cost of understocking will be 250 minus 100, that is 150. Now, CSL calculations are possible. That is, the probability demand is less than or equal to Q. The optimal order quantity should be greater than 150 upon 170, which comes out to be 0.88. Now, in this case, I am giving you data. I will do some partial calculations, and I expect you to complete these calculations. Let us consider a situation where the mean demand is 350, and initially, let us say the standard deviation is 150. I request you to consider some more standard deviations by, let us say, an increment of 30.

So, another standard deviation may be 120, another may be 90, another may be 60, another may be 30, and finally 0, and this means we are improving the forecast. We are reducing the standard deviation of our demand data by getting better and better data points. Now, we know that using this information, our optimal level of stocking will be a function of normal distribution. You can use your Excel for this purpose, and this will be CSL mu sigma. So, let us say to start my calculations, if I have sigma equals to 150. You can put that in this formula, and you will get the value of optimal stocking quantity to be, using this in Excel, around 526.

And as you may remember, yesterday I asked you in my last session to calculate the corresponding expected profits also. So, you can calculate the corresponding expected profit also. This expected profit, as I did the calculation, I am already giving you around 47,469. Then, if I reduce this to 120, the optimal stocking quantity will also reduce; it will become 491. Now, you can understand that your cost involved in the stocking will be less. And this will help you in improving your expected profit; it will become 48,476.

You further reduce your sigma to 90, 60, 30, 0, and when you get to 0 level of sigma, demand is fixed mu. Then, optimal stocking quantity should also come equal to this mu level because there is no uncertainty. So, why should you stock extra? So, your optimal stocking quantity will also come to be 350, and when the optimal stocking quantity comes to be 350, the corresponding expected profit will come to be 52,500. So, you can see that by changing the level of standard deviation, by improving the forecast, you are able to continuously improve your expected profit. Your optimal stocking quantities will reduce, and you will have a very good level of profitability. But, this is the extreme case. I do not say that, but as you are reducing this, your profits are also increasing.

So, you can say standard deviation is inversely proportional to my expected profit and similar relations are possible for reducing the lead time also. Lead time reduction is inversely proportional to expected profit. Because, if your read time is less you have to face, you can easily understand the same calculation that this mu and sigma L that mu and sigma will also go down automatically. You will require lesser optimal stocks and lesser of optimal stocks will require will result into higher expected profit. So, in this case we have seen that the practical cases that demand is backlogged or demand is lost, but as I said in the beginning there will be a hybrid case of some demand is lost and some demand is still there.

So, therefore, whether it is lost or whether it is there you have to consider some amount of let us say cost of understocking and using that cost of understocking, you will be able to determine what should be the value of your CSL and accordingly you can play with different levels of CSL to see that whether a particular cost of understocking is desirable or not desirable. Like, in this case we saw with this example that cost of understocking came to be somewhere around 230 dollars per gallon. Which is extremely impossible kind of a case? So, therefore, maintaining such high. So, your inventory policy as well as uncertainty in the data, leading to CSL level and that CSL gives you cost of understocking and then if you have some other proxies of cost of understocking that will help you in getting the CSL and accordingly you can decide your optimal stocking quantity.

So, with this we come to end of this video. Thank you very much.