Course Name - Operations and Revenue Analytics

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Lecture - 23

Welcome friends. As we discussed in our earlier videos that whenever we are talking of revenue optimization through dynamic pricing, we discuss that generally in those cases wherever you have limited supply, resources are limited and those resources are perishable also. And we discussed the example that how from airline industry this particular concept is started that with the limited number of seats in your aircraft how can you maximize your revenue. And for that purpose we discussed that you can have the issue of nesting which allows you for better management of your available limited capacity. And we discussed that wherever you have a perfect market, where you are a price taker rather becoming a price selector, you are a price taker because market decides what should be price of a particular commodity.

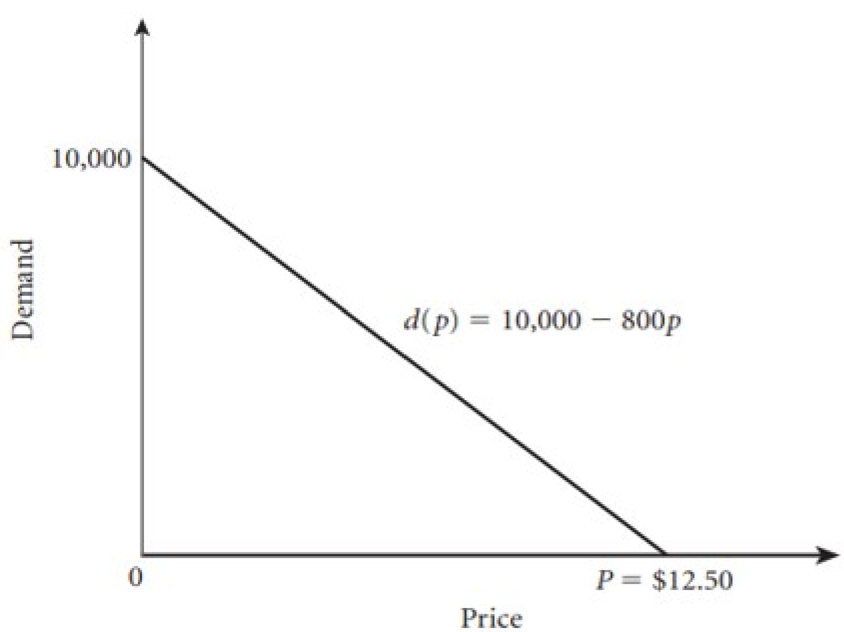
As an individual company you have no influence. on the market conditions, large number of buyers are there, large number of sellers are there, each seller has almost perfect knowledge, there are no entry exit barriers. In that scenario, you have very limited scope to decide the price, but whenever we have limited capacity, constrained capacity, you have a scope of deciding the prices and you can actually see that how by dynamic pricing system you can maximize your revenue. So, in this particular session, we will understand the issues related to this constrained supplies that if supplies are limited you have some kind of limitation like in airline example, how to decide the price which can maximize my revenue? So with this for this purpose, Let me first take you to a case of unrestricted supply. So, that during the discussion we will be able to understand that what do we mean by unrestricted supply and what do we mean by restricted supply.

And therefore, we are considering a very simple example. We are calling it as prototype example and in this prototype example we will be seeing a simple price response function that price response function is represented with this equation d(p) equals to 10,000 minus 800p and we have given this superscript plus.

$$d(p) = (10{,}000 - 800p)^{+}$$

Now, this price response function is basically specifying that how many more of potential customers would buy this particular product. This is a widget, I am talking but you can consider any product for example, automobiles for example, mobile phones for example, laptops etcetera that how many more new customers will be added if we lower the price and how many our current customers will not buy this product if we increase the price. So, this is the price response function that by changing the price how much of your demand is going to increase or decrease that is this problem.

Now, let us see two extreme situations of this particular price response function. Now, at p equals 0, at p equals 0, if the price is 0, the demand is 10,000 units. This is on the Y-axis; you have demand, and this is the Y-axis. This is the X-axis, which represents the price. So, when 10,000 is the demand, it is at p equals 0. If I start increasing the price—let us say \$1, \$2, \$3—and if I reach the price level of 12.50, here the demand is 0. So, from 10,000, which can be the maximum demand, 10,000 is the maximum demand you will get at a price of 0. If you increase the price continuously up to 12.50, the demand will become 0. So, this is the price response function.



In this particular case, we are considering this as a linear function for our initial understanding. This price response function is a linear function. However, in reality, sometimes this may not be a linear function; it may be a quadratic or some other kind of function. But just to understand the concept, it is a linear function at this moment. Now, here, if you see, we have used this sign '+' also.

$$d(p) = (10{,}000 - 800p)^{+}$$

This is just to signify the superscript that demand cannot be negative. So, somebody says that if I reduce the price below 0, can there be any change in the demand?

So, just to signify that you cannot reduce the price to less than 0. So, the demand cannot be beyond that. Similarly, if you increase the price beyond 12.50, it will result in a negative value of d(p). So, d(p) has a minimum value of 0 and a maximum value of 10,000, and that is controlled by this superscript '+', where, as per your price, you are able to supply that much. For example, you keep a particular level of price—for example, your price is 10. So, your demand will be 10,000 minus 8,000, which is 2,000 units, and you have the capacity to supply 2,000 units.

Now, further you increase the price—sorry, decrease the price—and let us say the new price is 8 rupees. So, in that case, your new demand will be 10,000 minus 6,400, that is 3,600, and you are able to supply this increased demand also. That is the meaning of this unrestricted supply: as prices are changing, you are ready to supply that much quantity to the market. Now, in this particular case, when we are doing this unrestricted supply, what should be the optimal price? What should be the optimal price? That is the question because you can supply from 10,000 to 0—all these possible levels of demand are there.

Now, what should be the price that will maximize your revenue? That is the question. Now, for that purpose, let us see how we get this optimal price. Those who have gone through the courses of maxima and minima in their plus-two level know how to determine that level of price. So, if p is the price, and d(p), as we already know, is the corresponding demand—d(p) is 10,000 minus 800p—that is our price response function. Now, the contribution of each unit can be expressed as the margin we are getting: p minus c. p is the price; c is the cost.

$$m(p) = (p - c)d(p)$$

So, p minus cost multiplied by the demand is the margin at a particular price level we are getting. So, if I am setting a particular price—let us say, as we discussed, 10 rupees—what will be the margin at that? This will be the formula through which we will be able to get the margin. Now, what should be the maximum margin?

For that purpose, we will take the derivative of this m(p).We have to do this d(p), sorry, this is not d(p), this is d of m(p). We have to differentiate this margin with respect to price, and that will give us, if you do this differentiation, you get this equation: minus d into p, p is the constant minus c. And by putting this value into this formula of d(p), you get this: 10000 minus 800p bar. Let us solve this. This is that price equals to p bar; cost price is, let us say, something into 8 d(p), that is 800.

And when you solve this, let us take cost equals to 5 rupees, and when I put this cost equals to 5 rupees, we get a particular level of optimal price, that is 8.75, by solving this equation number—let us say this is equation A, this is equation B, and this is equation C. So, by solving equation number C, I got this optimal price level as 8.75, and when I put this optimal price in this price response function, the demand at this 8.75 is 3000 units, and then correspondingly my margin, which I can calculate from equation A. That is p minus c, that means 8.75 minus 5, into 3000, and that will be 11250 dollars or rupees, whatever currency you are considering.

So, if you have unrestricted supply, you should set your price at 8.75, and then you will

Equating the first derivative m'(p) =0

$$d(p^*) = -d'(p^*)(p^* - c)$$

Putting the values in equation, we get

$$(10{,}000 - 800p^*) = 800(p^* - 5)$$

$$optimal\ price\ p^* = \$\mathbf{8.75},$$

$$demand\ at\ optimal\ price\ d(\$8.75) = \mathbf{3000\ \mathit{units}},$$

$$total\ contribution\ m(\$8.75) = \$\mathbf{11250}$$

be having the demand of 3000 units, and that will give you a contribution or margin of 11250 units, 50 rupees. After understanding, because here, it is possible that you can have a situation of 3000, 4000, 5000—all you are able to fulfill, you are able to satisfy.

After understanding this case of unrestricted supply. Now, let us understand the case of constrained supply, where any quantity cannot be fulfilled, where you have a limited quantity. So, the supply of the product is constrained; the treatment of a supply constraint depends on its nature. A hard constraint is one that cannot be violated at any price. Particularly, you can fulfill the demand below that constraint level. If you have a supply of, let us say, 3000 units, so obviously you can fulfill 1000, 2000, 2500, 2999, or maybe up to 3000.

But, you cannot fulfill the demand of 3000 first unit. So, that is a very hard constraint which cannot be violated. Like, Air freight carriers often have the option to lease space on other carriers to carry cargo in excess of their capacity. The freight carrier's physical capacity is thus a soft constraint. What it says that sometime as we discussed in our tactical level of discussions yesterday in other classes, we discussed that there is a system of overbooking.

Now, if you overbook some seats in your aircraft, let us say 10 additional passengers you booked. Now, you are booking these extra customers anticipating that there will be some cancellations, there will be some no shows also. But, on a particular day, all the passengers are ready to board. So, you have only 100 seats, 110 customers are there who are ready to board. Now, what will you do?

Then you see that can you shift some of your passengers to some other carrier. There may be some other flight also may be after 1 hour, 2 hour, 3 hours, 4 hours. So, some of your excess passengers will be shifted to that flight and in that way this becomes a case of soft constraint. And maybe you have to pay some kind of penalty or some kind of you can say charges to your these customers to whom you are shifting from one airline to another airline. Same is the case in the case of cargo also that if you overbooked your cargo and in that case you can take some space, you can leave some space in some other carrier also.

So, all these are wherever such type of possibility exists that by paying something extra, you can use the capacity of some other alternatives or your competitors, then it becomes a case of soft constraint. Now, whenever you are dealing with cases of constrained supply, there are two or three options. One option, which is always there, is to do nothing—like keeping the price at 8.75. As we just discussed, the optimal price for the unconstrained supply is to let customers buy on a first-come, first-served basis until the supply is exhausted. At 8.75, we need 3000 units; the expected demand is 3000 units, but you have a supply of only 2000 units.

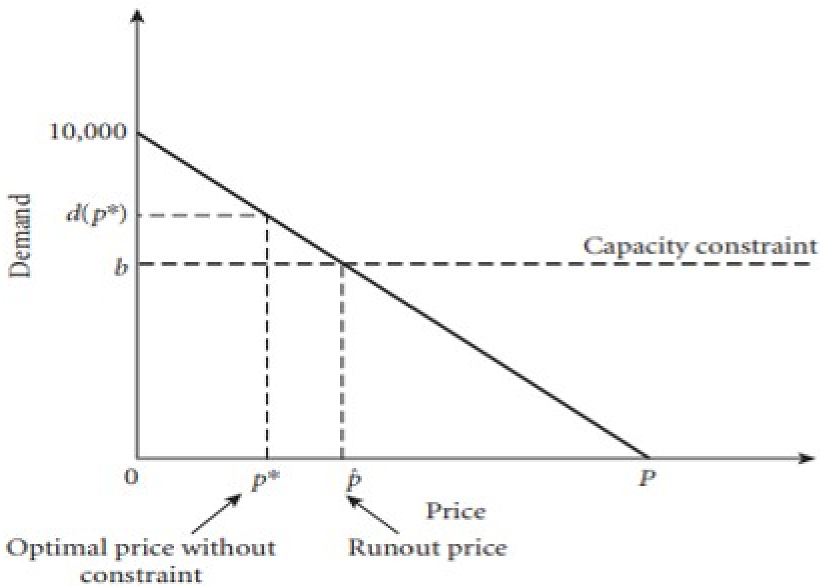
Now, the 'do nothing' approach means you do not take any action; the first 2000 customers will be served from the available 2000 units, and that is all—that is the maximum revenue you will earn if you serve only the first 2000 customers. Because that is your capacity. You are keeping the same price, 8.75. So, certainly, you can now understand that there may be some scope for increasing the price from 8.75—let us say to 9 rupees or 9 rupees 50 paisa—and then, with these 2000 customers, you may have slightly higher revenue. So, that is possible, and I think most of you must be thinking along these lines. But if I say, from the alternatives available to you, you do nothing, and whatever revenue you get is all you will earn. The second possibility is that you are not following a first-come, first-served approach. The second possibility is that you are allocating supplies to some favored customers.

They may not be the first customers, but you understand that they are important customers—they are your loyal customers. So, you want to give the limited capacity to those you can call VIP customers, so that you may have some other kind of indirect benefits from these favored customers. And the third option is that you start increasing the price, like the function is 10,000 minus 800p. Now, the capacity is 2000; the capacity is 2000. So, you can see that we can apply this concept, and this becomes p equals to 10. So, rather than keeping the price at 8 rupees 75 paisa, I will increase the price from Rs. 8.75 to Rs. 10, and if I keep the price at Rs. 10, the corresponding demand will be 2000 units.

The corresponding demand will be 2000, and this will give me higher revenue. You can easily understand that I have increased the price by 1 rupee and 25 paisa. So, that is also a possibility that you can increase the price until you have the demand which meets the supply. So, these are the three things you can do in the case of constrained supply. Let us understand a few more important things.

So, if this supplier of a prototype company is unable to segment his market or give favored treatment to a limited number of customers, it is a particular type of segmentation activity. So, if he is unable to identify who is my favored customer and who is my normal customer. So, that is something where you are not able to segment your market, considering all customers are equal to you. So, in this case, the option available to him is first-come, first-served basis or to raise the price. So, only option 1 and option 3 are applicable; option 2 is not applicable in this case.

Now, let us assume that he decides to raise the price to maximize his short-term contribution margin; then he needs to solve the constrained optimization problem like this. And we have already done that; this is now you can see in the form of this particular diagram.



Where this is our price response function, which, if you remember, is 10,000 minus 800p, and the plus sign indicates that the demand cannot be negative. Now, here we have two prices on the x-axis. The p-star is the optimal unconstrained price.

So, this is p-star optimal unconstrained price, which is 8.75, you may recall. We just solved it, and we got this 8.75. If the supplier does not need to do any further calculation, p-star is also the optimal constrained price. But, we just discussed that if, on the other hand, he finds that D(p-star) < B, then he needs to charge a higher price to maximize his contribution. So, we discussed that he should charge from 8.75 to 10, which is the higher price. This higher price on the x-axis is p-cap. This is p-star, and this is p-cap, and this p-cap is 10 rupees, as we just calculated.

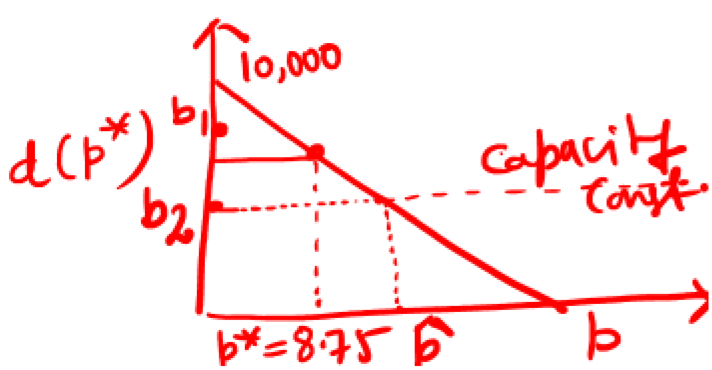
And this p-cap is coming from this particular diagram, as we need to draw it again so that you can understand it better. Let us see: this is our X-Y axis. This is my price-response graph with 10,000 and p. Here, the price without any restriction is 8.75, which I am calling p-star, and this corresponds to the optimal capacity or the optimal demand requirement at this particular optimal price. Now, I know that I have a supply constraint. Now, supply constraints are possible in two ways.

A supply constraint is possible where you have a supply constraint that is more than this optimal demand. Let us say you have a supply constraint, b1. So, if you have a supply constraint b1, where b1 is higher than D(p*), then do nothing. Here, you cannot increase the price. You cannot increase the price.

Why can you not increase the price? Because by increasing the price, demand will further lower, and you will have some excess capacity which remains idle. So, in fact, you cannot increase the price. So, in case B1 is more than your optimal demand level, it is a case—it is a case, practically, of unrestricted supply. If B1 is more than D(p), B1 is a restricted quantity.

For example, your system has a capacity of only 5000. If your system has a capacity of 5000 and 5000 is more than 3000. So, for us, in this case, it is a case of unrestricted supply only. But interesting things come if there is a different level, B2, and B2 is less than this optimal demand level. Now, this B2, if you start touching this price response function, it will touch at this particular point, and this is the capacity constraint.

Now, the point of intersection of this capacity line and the price response line is giving you the optimal price for this constrained capacity, and this I am calling P_cap—this P_cap. And we call this P_cap the run-out price. This is the run-out price, which is higher than the optimal price without the constraint. So, whenever a constrained supply comes, the optimal price is known as the run-out price, which is generally higher than the optimal price we got without the constraint for the same price response function. So, this is again the same diagram on a slightly bigger screen.



The diagram says that this run out price, this is the price on x axis, this is y axis and here you see that the capacity constraint is saying that you have a maximum capacity of 2000. This was you remember is 3000. Now, at optimal unconstrained price was 8.75 and the demand was 3000 units. Now, with unrestricted supply the total contribution which we were getting is 11250 dollars. Now, when you are selling only 2000 units so the profit which you will be getting 2000 into price minus cost, price now we are getting is 10 rupees minus 5. So, it becomes 10,000.

So, because of the constraint, your total contribution has reduced, but it is higher. For example, if you do nothing, our contribution would be 2000 into 3.75. So, by understanding the concept of this run out price, we are able to you can say that it is coming 7500 rupees, so our contribution has increased from 7,500 to 10,000 because of this concept of run out price. So that is how we need to see that what should be the price for the constrained supply which maximizes my revenue and generally, you need to understand this concept of run out price which will help us in improving our profit from 7500 to 10,000. And this is you need to please remember this term run out price that is the price the definition you can say at which demand would exactly equal to the supply constraint.

So, the definition of run out price is this is the price label at which in a constrained supply case our demand will be exactly equal to the supply. So, in all these cases, we have to determine that what should be the run out price, so that you can take the maximum benefit of available capacity and with this I think we are able to understand that generally, the run out price is higher than the optimal price we are setting for unconstrained supply. And the case of constrained supply is only applicable please remember whenever the constraint of supply is less than the demand which is available at the unrestricted case. If a constraint is higher then the demand at the unrestricted stage, then there is no meaning of that constant supply that becomes for the practical purpose as a case of unrestricted supply only. So, with this we come to end of this particular video.

Thank you very much.