Course Name - Operations and Revenue Analytics

Professor Name - Prof. Rajat Agrawal

Department Name - Department of Management Studies

Institute Name - IIT, Roorkee

Week - 06

Lecture - 27

Welcome, friends. In our previous sessions, we discussed how we are going to use three very important concepts for revenue maximization, and we also discussed that there can be reversible and irreversible ways of handling cancellations. We discussed whether we can reopen a closed class or if we should only go with the existing classes in case of cancellations. And we discussed in detail how you will have, at the end of certain events, available capacities in different classes using reversible and irreversible methods. We completed that exercise up to some of the events, and then I asked all of you to complete the remaining table on your own. I hope you would have completed that remaining table by using the discussion points for the reversible case as well as the irreversible case.

Moving further, whenever we are in such a situation, remember that we divided the total capacity into four different classes. In the beginning itself, we said that we are not going to take any bookings in class 4; we had only class 1, class 2, and class 3 as the operating classes, while class 4 was closed. So, an obvious question will come to your mind: how have we divided the available capacity into class 1, class 2, class 3, class 4, etc.? It is very clear to all of us that our objective is the maximization of total contribution coming from the booking system. So, capacity allocation—how many seats we are allocating for a particular class, how many we are allocating to a different class, etc.—is a very important issue. Before jumping directly into multi-class capacity allocation, let us take a very specific and simple case, which is known as two-class capacity allocation. Because that is the simplest way of understanding how capacities are allocated.

And once we understand the two-class model, we can extend this knowledge with some modifications for multi-class capacity allocation. And in that, there are certain heuristics that will also help us in making our calculations much faster. So, in this particular session, we are going to talk about the two-class model. We will see how we can handle that two-class model using the decision tree approach. We will also talk about a very popular approach for solving such two-class models, which is known as Littlewood's approach.

So, in most cases, this Littlewood approach gives you maximized expected revenue from your booking systems. So, let us see the peculiarity of this two-class model. As the name indicates, we are dividing the total capacity into two classes, A and B. For the sake of simplification and the meaning of these two classes, let us say class A is a discount class and class B is a full-fare class. So, the question is: if my system has a total capacity of, let us say, 100, how many of these 100 will go to the discount class and how many will go to the full-fare class? That is the question. And let us see what the solution will be for the two-class model, which is being developed on the basis of the decision tree approach.

We are considering a low-fare class, which I am calling the discount class, and then this is the high-fare class, which I am calling the full-fare class. For the discount class and full-fare class, we are using the nomenclature p_d for discounted fare and 'p_f' for the full fare. So, 'p_d' and 'p_f' are the nomenclature we will be using in this entire session to represent our discounted class and full-fare class. Now, discount customers each pay a fare p_d. Obviously, p_d is greater than 0, and full-fare customers pay a higher fare 'p_f,' which is higher than p_d. So, you can say that in simple terms. So, you can remember that p_d is greater than 0 and p_f is greater than p_d. You can easily understand that if p_f equals p_d, if p_f equals p_d, then there is no meaning of two classes; the entire system has only one class.

So, p_f has to be greater than p_d. And generally—one more important thing, which we will discuss in detail—in these kinds of systems, you first exhaust all your discounted fare class capacity. Once your discounted fare class capacity is completed—is exhausted—then only full-fare capacity is used. It is generally considered in these calculations that it is impossible for you to still have discounted fare class availability and also be booking in the full-fare class. So, full fare will only come when discounted fares are over. So, these are the three important assumptions: we have a limited number of seats, all discounted fare bookings occur before full-fare bookings start, and the incremental cost of carrying a passenger is 0.

We will be talking in detail, and we have already discussed the meaning of incremental cost in our earlier videos. So, these are the three very important assumptions you should remember. Now, in this two-class model, the important question we are trying to address or optimize is how many discount bookings to allow or how many seats to reserve for full-fare customers to maximize revenue by deciding the discounted booking limits or vice versa. We have discussed the case of protection levels in our earlier classes. So, the protection level for the full-fare class is the total capacity minus the booking limit for the discounted class.

For example, if you have a total capacity C equal to 100 and you set the booking limit for the discounted class at 35. So, the protection level for the high-fare class, the full-fare class, will be 100 minus 35, which equals 65. I request you to give me the answer: what will be the booking limit for the full-fare class in this case? Just think, with this example and data, what will be the booking limit for the full-fare class? We have discussed in our earlier classes what the booking limit for the full-fare class will be.

Let me give you the answer: the booking limit for the full-fare class will be full. It will be equal to the total capacity available for this system, which is 100. So, the booking limit for the discount class I have given is 35, and the booking limit for the full-fare class will be 100 in this particular case. So, this is just a refresher of what we have learned in our previous classes. Now, in this particular case, how will the model be developed?

And before that there are two very important terms which we will be telling you. These terms are one is spoilage and another is dilution. You must have some idea by the name itself, what is the meaning of spoilage and what is the meaning of dilution? Now, spoilage may occur because you may allow more seats, you may allow more seats because you want more revenue. So, you can keep more seats for your full fare customers.

Now, unfortunately, more full fare customers are not available. You kept a very high protection level for your full fare customers. Enough full fare customers are not available. And when your flight took off, all those seats which remained unbooked will be lost that is the lost inventory, lost capacity. So, that is the spoilage.

So, you are not able to use your inventory because you kept very high number of protection level for your full fare case. On the other side, now you become conservative and in that conservative approach more full fare customers will not be available. So, you are keeping a larger booking limit for your discounted fare class. Now, in this case it is quite possible that there were some customers available who are ready to pay full fare also. But, you kept more seats for lower fare class and in that way you are able to fulfill all your available inventory.

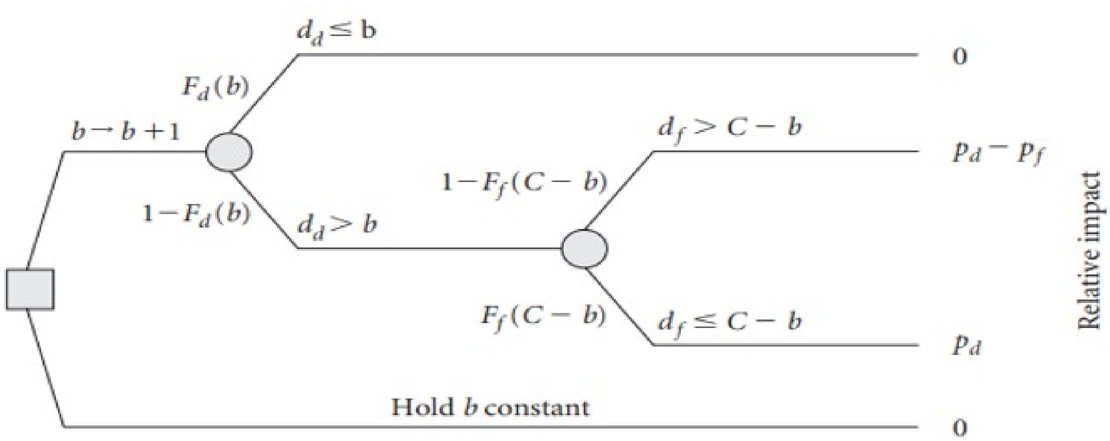
But you have diluted your revenue because the additional revenue was possible because more customers were willing to pay full fare, but they paid a lower fare, and therefore the revenue maximization is diluted. You were expecting to collect, let us say, 1000 rupees from that particular flight, but you actually collected only 800 or 700 rupees. So, you have diluted your total revenue by 200 or 300 rupees because of keeping more seats under the lower fare class. So, we actually want to avoid spoilage as well as dilution. I do not want spoilage; I do not want dilution.

So, no spoilage, no dilution. Because whether it is spoilage or dilution, both spoilage and dilution give you suboptimal revenue. Because in spoilage, revenue is lost due to vacant seats; in dilution, revenue is lost because of lower collection, and you could have higher collection because of your excessive limits for the low fare class. So, please remember these two important terms whenever we are talking about capacity allocation: the tradeoff between spoilage and dilution. We do not want either of them. So here, if you remember, in one of our earlier sessions a few weeks ago, we discussed a very popular Newsboy problem, and in that Newsboy problem, we discussed the meaning of expected profit, expected cost, expected loss, etc.

So, we will borrow some of our discussion in today's class from that Newsboy problem case. Here, to start our model, we are considering two or three types of probabilities. F_d, where the subscript 'd' is always related to discounted fare, and 'f' is always related to full fare. Now, $F_d(x)$ is the probability that discounted demand is less than or equal to x, and $F_f(x)$ is the probability that full fare demand is less than x. So, whatever this x is, it will actually be some numeric value, and we are keeping these probabilities for both full fare and discounted classes.

$$F_d(x) = \ probability\ for\ discounted\ demand\ being\ less\ than\ or\ equal\ to\ x$$
$$F_f(x) \ = \ probability\ full\ fare\ demand\ being\ less\ than\ x$$

Now, we are going to consider a popular way of handling some kind of probabilistic situations and this is Decision tree approach which we are going to apply in this particular case.



In this decision tree, the situation is how much to allocate for discounted class and how much to allocate the full fare class. So, considering the case when the booking limit has been increased to b plus 1, you are keeping b for discount class and now I am keeping it b plus 1 from b to b plus 1. If discount demand d_d is less than or equal to b there will be no effect on expected revenue therefore, net change in revenue is 0 shown in the top most branch in the figure of this decision tree.

So, you see that here we are keeping b to b plus 1, this particular part you can see in this available database and b otherwise is keeping constant.

So, two decisions are possible in the starting I keep the booking limit for discounted class as b only or I increase it to b plus 1. Now when I am increasing to b plus 1 then there are two situations possible like demand of the discounted fare is b, Fd(x). You remember we discussed that Fd(x) is the probability that demand is x or less. So, now, I am replacing this x with b here. So, it means that demand for discounted fare is less than or equals to b. So, if demand is less than or equals to b by increasing the capacity to b plus 1, I am not able to use this additional capacity because the demand is only b. So, if I allocate more numbers in that category, there will be no benefit of that.

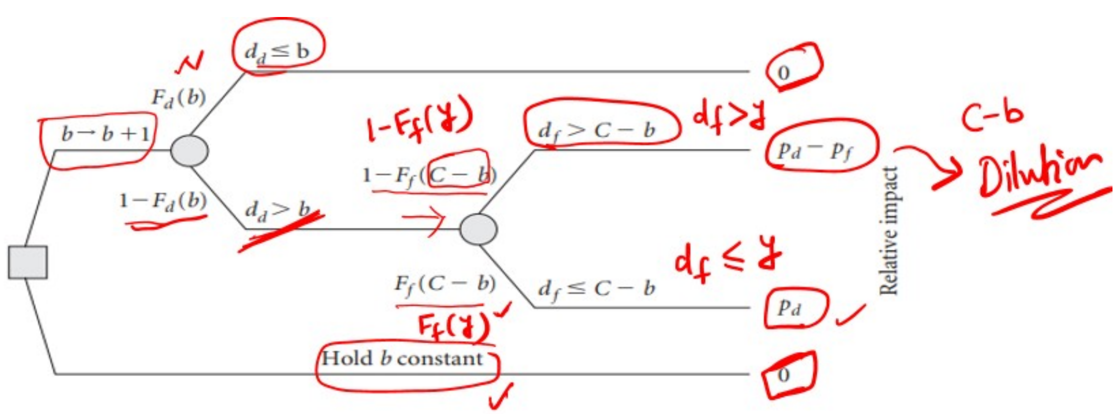
So, the change in revenue relative impact is 0. There is no impact that is here. If Fd(b) is there, so the other probability will be 1 minus Fd(b). That 1 minus Fdb means that demand for discounted fare is more than b. Demand for discounted fare is less than b if probability is Fd(b). If the other possibility at this node is probability being 1 minus Fd(b) and if probability is 1 minus Fd(b) the demand of discounted fare class will be more than b. Now, if demand for discounted fare class is more than b, there are two possibilities there are two possibilities that is further branched here. Now, 1 minus Ff(c) minus b and Ff(c) minus b. Now, C minus b is what?

Can you recall what this C minus b is? C minus b is basically the protection level for the high fare class. You recall that we discussed this particular equation. The protection level for the full fare class is the capacity minus the booking limit for the discounted class. So, Y equals C minus b. I am using this Y equals C minus b to replace this C minus b with that protection level, and this is.

So, there are two possibilities: if demand is greater than b for the discounted class, either you are able to use the protection level for your full fare class—that is, the demand for full fare is less than or equal to Y, the protection level. Or, demand is more—the demand for full fare is more than the protection level. That is the upper part of this node. So, demand for full fare is more than the protection level—this is easy to understand—and demand for full fare is less than or equal to the protection level. Now, please understand this point: when I increase the seats in my lower class—the discounted class—and the demand is more than that in the discounted class. Now, I have to see what the impact of this higher demand in the lower class is, because of my available demand in the upper classes.

Now, in the upper classes, there are two possibilities. One possibility is that you are actually using it when the Ff demand is less than the protection level, and when the demand for full fare is less than the protection level. You are going to get the impact on your revenue by p_d —that is, the additional seat in the lower class will give you one extra revenue from that seat, which is p_d. But, if demand in the upper class is more—if demand in the upper class is more than the protection level—then you are actually getting into this impact: p_d minus p_f. p_d is the discounted revenue, and p_f is the full revenue.

So, in fact, you are eating the revenue of the full fare. You could have earned more revenue in the full fare, and now you are not getting that full fare revenue, and you can



understand this is a situation. This is a situation we just discussed 2-3 minutes back. This is a situation of dilution.

Because you have reduced the capacity, you have reduced the protection level of your full fare class and allocated one more seat to the lower fare class, and there is a chance of more demand in the full fare class.

So, the possible extra revenue you could have earned in your higher class is sacrificed, and that is the situation of dilution. And obviously, these are the three branches we discussed, and the fourth branch is that we are keeping b constant; b is constant. So, if there is no change in the capacity allocation, there is no relative impact. So, out of two, there are two situations where you have no impact, and there is one situation where you are leading to dilution of your revenue—that is how you can understand this entire decision tree model. Let us see further now: when you have the discounted demand more than the capacity b, and if the full fare demand is more than this protection level C minus b, then the loss due to dilution is d_f minus d_d.

But if the demand for full fare is less than this protection level, then the revenue will increase by p_d, as that seat would have been vacant otherwise. So, you have actually minimized the scope of going into the spoiler situation by utilizing that capacity, which would have gone unfulfilled in the higher fare class; you have taken the benefit of that. So, depending on whether you have more demand in the higher fare class or lesser demand in the higher fare class, you can decide whether to increase the capacity in the lower fare or not. But it is not so easy to know. We will use Littlewood's rule for that purpose.

The expected change in revenue from changing the book limit from b to b plus 1 is the probability-weighted sum of the possible outcomes. So, the possible outcomes are 1, 2, 3, and 4. These are the 4 important outcomes we just discussed, and out of these 4, these 3 occur when we increase b to b plus 1. The fourth outcome is the result when we are not changing the capacity at all. Now, using this entire knowledge, our expected revenue increase is the weighted sum of all these three things.

So, all these three possibilities arise because of some probability: this probability leads to this, then this multiplied by this leads to this, and this multiplied by this probability leads to this. So, this has to be a weighted sum of these three possible outcomes, which can be written in a much simpler form as the weighted sum of 3 possibilities, 3 outcomes that are going to happen. If the right-hand side of this equation is greater than 0, then it is profitable to increase the booking limit to b plus 1. So, the point is that, as we discussed a few minutes back, this is 0. So, 0 multiplied by Fd into b will always be 0.

Now, p_d minus p_f—you see that this has to be negative. We will always have this negative because, as you may remember from the beginning of this session, we said that p_f is greater than p_d. So, since p_f is greater than p_d, p_d minus p_f will be negative, and p_d is going to be a positive number. So, out of these three outcomes, one is 0, which will have no impact; one is negative, and one is positive.

$$E[h(b)] = [1 - F_d(b)]\{p_d - [1 - F_f(C - b)]p_f\} \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Therefore, it is quite possible that the right-hand expression in this equation may sometimes be negative and sometimes positive, but whether it is positive or negative depends on two things: one is the relative difference between p_d and p_f, which will tell you how negative the second outcome is.

And second is it will also be depending upon the relative probabilities that what is these probabilities because this 1 minus Fd into b that is common to both these factors. So, what is the full fare protection level probability and 1 minus of that. So on that relative term this expression will depend 1 minus Fd into b and this is the term which is common and then based on this term you will decide that whether this expression is positive or negative. But, if it is positive then you should increase your capacity from b to b plus 1. And in this expression you can see that whether it is going to be positive or negative, it is basically dependent this is the expression which is actually governing the positive or negative aspect.

And therefore, the C minus b which we have already discussed as a protection label and 1 minus is the probability of the full fare booking being more than Y. And for solving this particular equation there comes a very interesting rule that is Littlewood rule.

If the right-hand side of the equation (1) is greater than zero than it is profitable to increase the booking limit to b+1. The key term in the equation is $\{p_d - [1 - F_f(C - b)]p_f\}$
where C$-b = y$ protection level
Therefore, $[1 - F_f(C - b)]$ is the probability of the Full fare booking being more than y

Because, 1 minus Ff(C) minus b one scientist around 1970s, he gave this system that 1 minus Ff(C) minus b to decide what should be the booking limit for your lower fare class should be equal to p_d upon p_f. And this ratio of discounted fare and full fare is the deciding factor basically for deciding the limits for your discounted fare. So, 1 minus Ff(C-b) which is 1 minus Ff(y) protection level equals to p_d upon p_f. So, p_d upon p_f is basically deciding the booking limit.

$$y^* = \min[F^{-1}(1 - p_d/p_f)]$$

Take one or two very extreme cases. For example, take one case where p_d is equals to 0. Now, p_d is 0 means discounted fare is 0. You are not charging anything to discounted fare. So, what does it indicate?

It indicates that the full fare protection level, the full fare protection level is equal to 1. That means there is no need, there is no need to keep any capacity for the discounted class because the discounted class is not adding any additional revenue. So, therefore, you can allocate the entire class, the entire capacity for your full fare customers. And you can consider another situation: this will show you that if p_d equals p_f, now you are increasing your discounted fare. The initial discounted fare is 0, and now you start increasing your discounted fare. So, there will come a situation where p_d equals p_f, and if p_d equals p_f, then it will signify that you should not have any protection level, you should not have any protection level for your full fare class.

That means the full fare and discounted fare are all the same. So, the entire capacity can be allocated to the discounted fare class itself. So, these are the two extreme cases. In one case, and in fact, both these cases result in you not dividing your capacity into classes. You are going to have only one class in your system, whether you call it the lower class or the higher class, but there will be only one class if you have either p_d equals 0 or p_d equals p_f. So, that becomes a very simple way of determining how much capacity you are going to allocate.

So, this Littlewood rule, which actually comes from this entire decision tree model, becomes very simple to execute with the help of this Littlewood rule. And we will continue with more examples of this Littlewood rule in our next class. With this, we come to the end of this particular session. Thank you very much.