**Course Name - Operations and Revenue Analytics**

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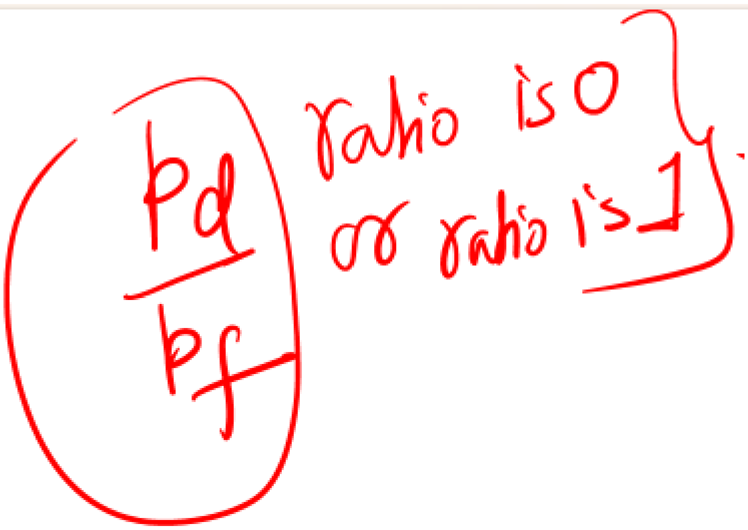
**Week - 06**

**Lecture - 28**

Welcome, friends. In our previous session, we discussed how to start addressing the issue related to capacity allocation. We took a very simple case of two classes: there is a server with a capacity known as capital C, a discounted fare category, and a full fare category. We also discussed how to decide the booking limit for the discounted fare and, based on that, the protection level for the full fare case. We discussed the equation Y equals C minus b for protection level under two classes. Remember that capital C is our total capacity, and b is our booking limit.

We also discussed the use of the decision tree approach for determining b to minimize the issues of spoilage and dilution. Finally, we discussed Littlewood's Rule, which makes it very convenient to decide the booking level using the ratio of discounted fare to full fare. So, basically, pd and pf—discounted fare and full fare—become key factors in deciding the booking limits. We discussed two extreme cases: in one case, pd equals 0, and in another, pd equals pf—or, in other words, the ratio is 0 or the ratio is 1. These are the two extreme cases, and when they occur, the entire server is just a single class.



If pd equals 0, theoretically, the entire system is allocated to full fare seats. And if pd equals pf, the entire system is allocated only to discounted fare seats. When there is only one class, it is meaningless to distinguish between discounted fare and full fare. There is only one fare. In this, we realized that using pd upon pf, Littlewood's Rule makes it very easy for us to proceed.

Now, let us start this particular session with one or two simple examples, and then we will try to extend this knowledge to a multi-class allocation problem also. This is the simple example which is based on our previous session, and this will give some kind of recap also of that particular class. Now, you have a particular situation where you have one airplane which has a capacity of 150, and out of those 150 seats, you have kept a booking limit of 80. So, C is 150, B is 80. So, you can easily understand that the protection level Y is 70 that you are keeping, and as you remember, b for full fare bf, in this case, will be how much?

That is 150. So, here the protection level for the full fare class is 70. By this time, I hope we all are very much aware of what the meaning of protection level is. That means 70 seats are exclusively reserved for full fare. You see, you can book more than 70 seats also for full fare.

If demand comes, let us say, for 90 seats for full fare. So, you can book 90 seats also, 150 seats also for full fare. All the seats in the booking limit, meaning all these seats which are available up to that class, will always be available for the higher class. But, if a 90-seat booking request comes for the lower fare class, that cannot be executed. The meaning is, if a request for 90 seats in the discount class comes, you will reject it; thus, this request will be rejected. But, if a request for 90 seats in the full fare class comes, it is accepted. So, you see that this is the nested booking control system which we discussed about four or five classes ago.

So, though only 70 seats are protected for y class for full fare class, but since as per the dynamic nesting system all the inventory which is available at that particular time will always be available for higher class. So, you are ready to give 90, 100 or any number of seats, if the total seats are available in the aircraft is up to that level. Now, I am changing this aircraft in this example from 150 seats aircraft to a 100 seat aircraft. So, the situation is earlier I was having 150 seat aircraft, now I am going to have because of some maintenance issue the 100 seats aircraft. And pd, pf remains as it is, there is no change in the full fare, there is no change in the discounted fare.

And there is no change in the forecast also that for full fare, discounted fare etc etc. So, the question is what will be the protection labels and the discount levels, booking levels for this smaller aircraft. So, if there is no change in full fare demand and the lower fare demand, there is no change in the forecast also. So, otherwise if you use your general knowledge etc, you will try to proportionately reduce the number that 80 in 150. So, how many in 100 that will be the way you will solve your this question, but since it is a situation of revenue optimization, revenue maximization.

So, we will keep our protection label in this case also equivalent to 70. So, therefore, the booking limit for discounted fare will come to 30 seats. Earlier the booking limit for the discounted class was 80 seats, now the booking limit for the discounted class will be 30 seats, 70 seats you will still keep for your, so 70 was the protection level earlier also. And 70 is the protection level for this new situation when other parameters are not changing, only the available capacity is changing. So, this is the simple example which I thought we should know that how are we going to handle our changing capacity, when there is no change in the fares, there is no change in the forecast also.

I hope we all understood this example clearly. Now, moving further to some other kinds of examples in these two-class categories. The mean full-fare demand for a flight with a 100-seat aircraft is 50, with a standard deviation of 100. So, the mean demand is 50, the standard deviation is 100, and the standard deviation of demand is 100. For any fare ratio, pd upon pf, the corresponding optimal booking limit is the smallest value of b for which this relationship will hold.

$$1 - F_f(C - b) \geq \frac{p_d}{p_f}$$

So, let us see how we are going to execute it. So, in this case, we have created a detailed analysis of this particular situation. So, please remember this situation: the mean demand is 50, the standard deviation is 100, and now we will be changing the ratio to see what is the smallest value of b for which the left-hand side is just ahead of pd upon pf. So, since pd upon pf is fixed, we know what the discounted fare is and what the full fare is. We need to determine b, so we will call it b*.

And this will be when 1 minus Ff (C minus b) is just equal to or more than this ratio of pd upon pf. So, you see that the fare ratio we are considering is 0.4, pd upon pf; this pd upon 0.4 is pd upon pf. And for this pd upon pf equal to 0.4, the optimal booking limit is 25, and you will do this on your own when you are going to do it for 0.6, for which it is going to be 76. So, we are keeping different types of situations, and let us see what the value of b will be and how this table is executed. So, take the first part of the table, and

*The value of $1 - F_f(C - b)$ as a function of b when total capacity is 100 seats and the mean full fare demand is 50 with a standard deviation of 100*

| $b$ | $1 - F_f(C - b)$ | $b$ | $1 - F_f(C - b)$ | $b$ | $1 - F_f(C - b)$ | $b$ | $1 - F_f(C - b)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.309 | 24 | 0.397 | 49 | 0.496 | 75 | 0.599 |
| 1 | 0.312 | 25 | 0.401 | 50 | 0.500 | 76 | 0.603 |
| 5 | 0.326 | 26 | 0.405 | 51 | 0.504 | 80 | 0.618 |
| 10 | 0.345 | 30 | 0.421 | 55 | 0.520 | 85 | 0.637 |
| 15 | 0.363 | 35 | 0.440 | 60 | 0.540 | 90 | 0.655 |
| 20 | 0.382 | 40 | 0.460 | 65 | 0.560 | 95 | 0.674 |
| 23 | 0.394 | 45 | 0.480 | 70 | 0.579 | 100 | 0.691 |

then you can do these calculations on your own.

Now, value of this function 1 minus Ff (C minus b) as a function of b, when the total capacity is 100 seats and the mean full demand is 50 with a standard deviation of 100. Now, in this case, if I am keeping it 0, b is 0, the value of this function 1 minus Ff (C minus b) will be 0.309. This is the value of this function. So, I have started increasing the value of b from 0 to 1 to 5 to 10 to 15 and so on. And when I am increasing the value of these b's, we see that when I am just reaching to this b equals to 25. When I am reaching to this b equals to 25 the value of 1 minus Ff (C minus b) is coming to be 0.401 which is just higher, which is just higher than this 0.4.

So, if my, if my pd upon pf is equals to 0.4. my booking limit should be 25 and protection level correspondingly for higher class should be equals to 75. You further increase b and we you can see that we have done this calculation up to b equals to 100, up to B equals to 100, we have done this calculation and here you see that if booking limit is for 0.6. So, you see that 0.48, 0.496, 0.5. So, for example, if my price ratio is 0.5. For example, pd is, pd is half of your full fare, fare.

So, in that case, booking limit is 50. If it is 0.6, the booking limit will be 76. So, you can play, once you have this table ready with you, then you can play with any level of pd and pf. And by knowing that pd and pf, you can easily calculate the value of your booking control using this particular calculations by changing the value of b, we are getting the corresponding value of our booking control. So, you see that we have done this calculation for b equals to 0 to b equals to 100 and we saw that for a price ratio of 0.390 and less the booking limit will be 0.

So, for a price ratio of this 0.390, 309 sorry this is a slight typo here this is 0.309 for this price ratio, the booking limit will be 0. You will keep all your capacity for your higher fare class and for 0.691, it is better to keep only for all your capacity again for same type of fare. So, this is a very simple and useful way of knowing that how much you are going to keep for your particular booking limit, you just need to calculate for this. So, for this you just need to calculate this particular value of the function and for getting this value of the function you can use excel tables. Where you are using this function for calculating the value of Ff (C minus b) and this will be calculated using the value of mean equals to 50, standard deviation equals to 100 and accordingly all these different values are calculated which are directly used in this particular table.

We can also take another example, if you remember that in one of our earlier classes we discussed a very popular case of Newsvendor problem. In our inventory management, in our forecasting also, we use this Newsvendor problem very often whenever we have a single opportunity of deciding how much to keep with you and this issue of optimal discount booking limit is also you can see very much similar to this Newsvendor problem. Because in that Newsvendor problem and this booking system also you have a single opportunity, if you cannot decide properly that how many newspapers you want to keep in the morning, you will lose that opportunity of earning the profit. And same thing if the flight has taken off, if i am considering airline example then whatever spoilage or dilution has happened, it is happened, you cannot undo that so time factor is very important and therefore the two class problem can be very very analogous to our Newsvendor problem. So, as I just mentioned that airline needs to determine how many seats to protect or order.

Here, you can see the seats which you are protecting for full fare is equivalent to the order this news vendor is doing for procuring the newspapers. So, it is very much similar to this. The airlines unit cost from protecting too many seats is a displaced discount fare. The understocking cost is the revenue from a lost full fare booking minus the discount fare. So, this is the cost of understocking.

discount fare: $U = p_f - p_d$

That is the pf full fare minus pd that is cost of understocking that we know that understocking means the profit which is possible to you. But, you are deprived of the profits and cost of over ordering we know that you have actually procured more inventory, you are unable to sell that inventory and therefore, this extra inventory available to you is a cost to you. So, when it is cost to you it is equal to keeping higher protection level, when you are keeping more for your discounted fare and there is a possibility of getting more customers in the full fare that is actually the cost of understocking. So, in this way, you can simply consider that production label should be set using this simple formula which we have already discussed in our classes.

$$F(y) = \frac{U}{U+O} = \frac{p_f - p_d}{p_f} = 1 - \frac{p_d}{p_f}$$

So, you can refer those videos, where we discussed this concept of Newsvendor problem and using this formula of that Newsvendor problem U upon U plus O, that is cost of understocking and cost of understocking plus cost of overstocking, that is our formula for deciding the protection level.

And this can be translated in terms of our discussions of price for full fare and price for discounted 1 minus pd upon pf. So, in fact, this also is very similar to if you remember this particular thing that is pd upon pf is actually 1 minus F(y). This is coming like that and we remember that F(y), Y is our protection level which is C minus b. This is what we are just coming from this Newsvendor problem. Now, go for a minute to this our previous slides, where also we kept the same idea that 1 minus Ff (C minus b) but, that 1 minus Ff (C minus b) is 1 minus Ff (y)should be greater than equals to pd upon pf. So, we are developing same formula 1 minus Ff (C minus b) that is equals to pd upon pf.

So, whether we are coming from the decision tree approach or whether we are coming from the Newsvendor problem approach, you will reach to the same level of calculations. So, this is another way to understanding the same concept as a Newsvendor problem, because we have done that in our previous classes. Now, after understanding this two class system but, generally, you know in our servers we may have multiple classes and in our earlier examples also, when we understood the concept of cancellations, booking control, nested bookings also in that also all the examples were having multiple class problems. You are having various segments of the customers and out of those segments of the customers, you are deciding a particular fare for a particular class. So, there are situations that you are not satisfied with just two classes, but you are going to have multiple classes in your system.

So, again, the whole idea is that you need to decide the booking limits for each of these classes so that you maximize the total contribution, and in this particular case, when we are going for a multi-class model, these are the assumptions we are going to take. So, the product has n fare classes, each with an associated fare. Fare classes are numbered in descending fare order. So, you see that class 1—this will be the general terminology we are going to follow, and we have already followed it in earlier classes also. 1 is considered to be the highest fare class, 2 is the second highest, 3 is the third highest, and n is the lowest fare class. So, p1, p2, p3 up to pn if n fare classes are there.

So, this is the highest fare, and pn is the lowest fare. So, demand in each class is an independent random variable. This is also important to note: demand in each class is independent. The meaning is that the demand of one class is not affected by the demand of other classes. Every class has independent demand, and therefore, our solution process will become quite easy because of the independence of each class.

And each fare class is denoted by di for a fare class—that is, the demand distribution—and it follows a probability distribution of fi(x) defined on integer x greater than or equal to 0. So, the probability distribution fi(x) is the probability distribution for the ith class. There are a total of n classes. So, i varies from 1 to n, actually. So, that is the basic—you can say—nomenclature and terminology we are going to follow for this multiple-class problem system.

So, this fi(x) is the probability that demand for fare class I will be x, and fi(x) denotes the probability that di is less than or equal to x. Demand books in increasing fare order, like in the two-class system. Also, we see that first the lower fare class is completed, and then you start for the higher class. The same is going to happen here if there are n classes. So, first the nth class will happen, then the n minus 1, then the second, and then the first if n equals 4. That is, the lowest fare will be booked first. There are no no-shows or cancellations.

So, here we are assuming for the time being that there are no cases of no-shows or cancellations, and the objective function is to maximize the expected revenue. So, the time convention which we are going to follow for this multi-class model, if you see this particular figure, is that the booking starts with Xn. That is, the booking starts with this Xn amount for the nth period for the lowest fare. This will continue for the time for the lowest fare class, and then it will start for the n minus 1th period and continue for higher fare classes. So, it is for Xn for this particular fare class, which is the lowest, and it will continue for n periods as we are coming closer to the time of departure of the particular flight. So, you have divided the time zones also. That may be if I am starting, let us say, this is the time of departure: June 2024.

So, if I am starting my booking, let us say, in March 2024, then maybe up to the 10th of April this booking, the 20th of April this booking, then maybe the 30th of April, maybe the 15th of May, maybe the 20th of May, and maybe the 1st of June. That is how the time spending is happening, and the fare classes are changing. This is the lowest fare, and as we are coming closer to the time of departure, we are reaching the highest fare class. And these are the number of bookings which are happening in these particular classes. So, this is the time convention for the multi-class model in our discussion. Now, a few more important things before we actually go for the model building.

The airlines decision at the start of each period J is to how many bookings to allow for that particular class. Now, it is possible that during this particular period bookings are coming for Xn, Xn bookings are coming for this low fare class, but you may have a let us say limit for bn. If Xn is more than bn some of the bookings will be rejected, but if Xn is less than bn or equal to bn all the bookings will be accepted. So, this will keep happening for every class that if the requests which are coming are less than the quota which you have kept for that particular class, you are going to accept the bookings. Otherwise, you will reject some of the bookings and these bookings may go to the subsequent higher fare classes, if customer wishes for the same.

So, this is the booking till the beginning of the period J is the these and unbooked capacity is Cj minus this.

$$\sum_{i=j+1}^{n} x_i \text{ and un-booked capacity } C_j = C - \sum_{i=j+1}^{n} x_i$$

So, total capacity of your system is C that is the total capacity and out of the total capacity for any particular time you have booked if I am let us say I want to know at this particular time the beginning of third period. So, all these Xn, Xn minus 1, Xn minus 2 up to this point, sigma of that is what is represented here. That is the sigma of that, that all these seats are already booked and C minus this sigma is the unbooked capacity available at that particular time. And we will see that more demand is expected in the higher fare class.

Using the power of analytics, these days we can have dynamic booking control also that you initially decided that we are going to have b1, b2, b3 kind of booking, but b1 can also change, b2 can also change, b3 can also change in real time. So, that is the power of analytics that you may not have the static quota allocation for your classes, rather it can be a dynamic allocation for your classes. Airline we assume is known of the demand distribution for the coming periods. Airline needs to find the maximum number of booking for class J to accept in order to maximize their expected revenue. The solution approach in this particular case which we are going to discuss in our subsequent videos that the airline only needs to calculate one booking limit at the start of each period, since, it will receive booking only from one fare class during a period.

This is very important. When you are starting at a particular time, let us say I am starting this third booking period. Now, in the third booking period, how many seats should I allocate for this fare class P3? I do not need to know about P2 or P1 at this time. Because in this third period, I am going to accept bookings only for this particular class.

So, I am more interested, or rather only interested, in knowing the booking limits for the third class, rather than calculating for all available classes at that time. You see, when I am going for the P3 class, the Pn, Pn-1, Pn-2, etc., are already over. So, I am not going to go back in time. I am not going into the future either. So, I am only talking about the present booking class for which I am going to accept bookings in this particular period.

So, that makes our life slightly easier, as rather than doing the entire calculation for all booking classes available at a particular time, you only have to calculate for the specific booking class for which you are accepting bookings in that period. So, these are some of the conventions we are going to follow when developing a model for the multi-class model, and we will cover that model in our next classes. So, with this, we come to the end of this particular class. Thank you very much.