Course Name - Operations and Revenue Analytics

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Welcome friends. So, in our previous session, we discussed in detail about two class problems that how to allocate our capacity between two classes and we see that Littlewood's formula is very very helpful in simplifying the entire calculation which is supposed to be done for the capacity allocations. And then we also discussed that in practical cases, we generally have many more classes, just not two classes. Whether it is airlines, whether it is hotels, whether it is any other kind of restricted capacity system, you may have many classes. Sometime the classes may range up to 10 also.

So, therefore, for more practical purpose, multi-class model is required. And we discussed the convention of that multi-class model also in our previous session. Where we discussed that there can be n classes in a particular server and for each classes we discussed the capacity to be b1, b2, b3 up to bn. We discussed the price we are going to charge is from p1 to pn and very interestingly now we have to decide that what should be b1, what should be b2, what should be b3 and so on. So, that our total revenue from this server should be maximized.

Here, one important assumption if you remember we discussed in that class that at a particular time only one class is being booked. If I am booking for third class at this time, so, I only need to see that how much capacity I have to allocate for third class. Once, I close the third class then I will reach to the second class and in the second class again, when I am starting the booking I have to see that how much I have to keep for the second class and remaining will go to the first class. You see in a more retrospective manner that when I am doing booking for fifth class, I am concerned with what I am going to keep for fifth class and the remaining will go for first, second, third, fourth. So, in a way, whenever I am booking for a particular class, I will see that this multi class problem can also be broken as a two class problem.

That the current class and all the higher classes which are available at that particular time. So, that is a very interesting thing which will help us in simplifying our multi class models also. So, in this particular session, we will see that how we are going to apply the concept of two class for multi class also with the help of a decision tree. And then there are certain heuristics which are available both are result of a PhD thesis, expected marginal seed revenue that is the way of improving the calculation purpose of this multi-class problem. And there are two versions of this EMSR, version a and version b. Version a and b, b is more computationally sound, sound means faster, efficient, so more, more and more organizations are using EMSR-b version for knowing the booking limits for a particular class.

And then finally, if we compare a and b, so, we will have some kind of numerical data for comparing EMSR-a and EMSR -b. Let us first start the multi class problem as a decision tree. Let us say I have a situation of three classes. Class 1, where the limit is b1, class 2 b2, class 3 b3. The fare I am charging p1, p2, p3. It is simple to understand that p1 is greater than p2 and p2 is greater than p3.

The convention which we followed. First booking will be done for class 3, then class 2 and then class 1. All of you can easily understand when I am at a situation of class 2. So, I have to decide how much I have to keep for class 2 and how much will go for class 1. Then at this particular stage, it is a very clear without any additional efforts, it is a two class problem.

But, when we are considering that how much I need to allocate at what should be b3? Should I allocate one seat extra to b3? Can I b3 plus 1? This is the question. And here when I am answering this question that should it be b3 or b3 plus 1, it becomes a multi-class problem because now you have to see that how much is going for class 3, how much will go for class 2 and how much will go for class 1.

Multiclass Problem

3 Classes

b_1 , b_2 , b_3

$p_1 > p_2 > p_3$

First booking will be done for Class III → Class II → Class I (Two Class Problem)

What should be b_3? → Can I $b_3 + 1$?

But, just now I told you that we assume that at a particular time, I am going to allocate only for that particular class and all the other classes, upper classes will be opened after I close this particular class. So, when I am doing this class 3, so I have to decide class 3 versus class 2 plus class 1. So, in this way, this is also a two class problem where I have to consider class 1 and class 2 as a single thing. Now, if I allocate one extra sheet in b3, there are two possibilities which may happen. These possibilities may be like demand for b3, demand in class 3 can be more than b3 or demand in class 3 is less than b3, if demand in class 3 is less than b3 that is the original capacity.

So, allocating this extra seat to class 3 is not going to give you any impact, because then this extra seat will go to the upper classes whenever the upper class booking will be open. But, if the demand in class 3 is higher than the original limits of b3. Then this extra sheet will be used by class 3 and if this extra sheet is used by class 3 it is coming extra sheet in class 3, if consumed it is having three possibilities. Either, there is a possible higher demand in class 2. So, either loss of class 2, you have compromised one customer of class 2 or loss of class 1, you have compromised one customer of class 1 or it is possible

Class III Vs Class II + Class I

$d_3 > b_3$ or $d_3 \leq b_3$

Extra seat in Class III if consumed it is — either loss of Class II or loss of Class I or No loss in Class I and Class II that in class 1 and class 2, there is no extra customer coming the demand remains low in class 1 and class 2.

So, no loss in class 1 and class 2. So, if your extra seat is consumed in class 3, these are the three possibilities which may happen and all of these things can be represented very conveniently with the help of a decision tree. Let us see how? I hope we are clear with the scenario and let us see the development of the decision tree. So, that decision tree is I am considering this class 3 situation that starting of class 3 booking.

And when we are starting the class 3 booking, we have to only see how much we are going to allocate for class 3. That is the question in front of us. Now, one answer of this can be very easy that you just keep b3. Keep your b3 constant, do not increase your capacity. That is the first thing.

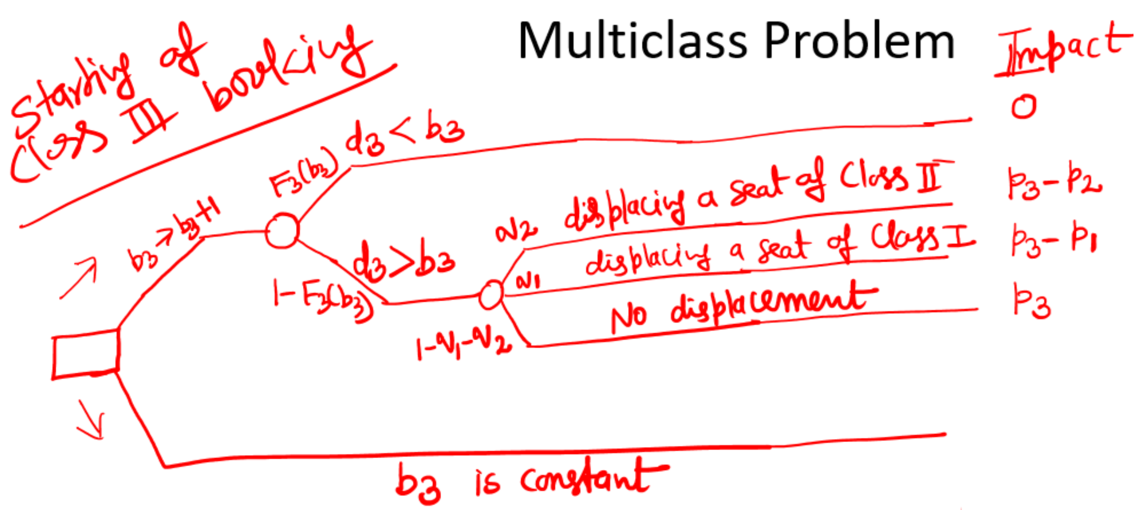
Second is, you are going to increase the capacity of your system and when you are going to increase the capacity of your system, there are two possibilities which may happen. One possibility is your demand of class 3, d3 remains less than the b3 . And if your demand remains less than b3, we will see what is going to be the impact of this. The other possibility is that demand of b3 is, demand d3 is more than b3. And if demand d3 is more than b3, there are three possibilities as we just identified that either there is a loss from class 2 or loss from class 1 or there is no loss if there is no higher demand in class 1 and class 2.

So, if d3 is more than b3, there are three possibilities which may happen. Now, these three possibilities, one is loss of class 2. So, loss of class 2, that is, with let us say class 2 is coming with Q2 probability that you are displacing a seat of class 2. This Q2 is the probability I am assuming. Q1 is the probability I am assuming for displacing a seat of class 1. And neither you are displacing a seat of class 1 nor class 2, this will be coming with a probability of 1 minus Q1 minus Q2.

So, it means no displacement. So, these are the multiple scenarios which are possible. If you are having a demand which is less than b3, so by adding this new customer, let us see what is the impact, what is the final outcome coming? This will give no impact because additional customer is not giving any revenue from class 3 and there will be no displacement because of that. So, this additional allocation is giving no impact.

Now, when you are having one additional customer and that customer is used, so you are getting P3 revenue from that customer. p3 is revenue from customer in class 3. Now, when you are displacing this customer from the cost of class 2. So, class 2 customer is going to give you p2 revenue. So, you are getting p3 minus, p3 is the positive revenue at the loss of 1 p2 revenue.

If you are displacing a seat of class 1, this is p3 minus p1 and if there is no displacement, so it is purely your additional impact that you are going to get p3 seats and then If you are not increasing any seat in your class 3, obviously, there will be no displacement. So, there will be no impact of this. So, these are the five scenarios which will emerge as a result of your allocation of a seat and you see that we have made it two class problem.



This is one side, this is another side. Now, you can see that what are the probabilities of all these things.

So, the probabilities are that here we are increasing b3 to b3 plus 1. So, the probabilities are F3, b3 and this is 1 minus F3 b3. So, these are the probabilities and you can see that this can be calculated the total expected revenue by multiplying the relevant probabilities to the impact. So, this can be solved with the help of Littlewood formula because that Littlewood formula is very much applicable for our all the two class problems and if I try to summarize that how are we going to solve this. So, the simple mathematical equation for solving this that the optimal booking limit is the smallest value of 0 to that particular booking class to C2.

Such that C2 minus b2 is less than 1 minus b2 upon p1. So, this is the knowledge which we are taking with the help of Littlewood formula. And this formula can be applied to all the multi class problem, but it will take lot of computational efforts while using this Littlewood formula. So, there is a easier way for handling such kind of multiclass problems in a sequential manner and that is very easily done with the help of this expected marginal seed revenue systems. So, it is a kind of a heuristic which will help us, which will simplify our process of solving such type of situations, where Littlewood formula is to be applied for multi-class problems.

And for that purpose we will see this simple example. And here, we have multiple classes and we are given with prices which we are going to charge for these classes. Here, we have four class 1, 2, 3, 4 and the price which is applicable one is the highest fare full fare and then 420. 290 and the discount fare is available at 125. The mean demands are given and their standard deviations are also given to us. That mean demand in different classes are like this and their standard deviations are also given to us.

Class 4 is not operating. So, there is no mean demand, no standard deviation available for class 4. So, it means that we have to divide the overall capacity in these 3 classes 1, 2, 3 and therefore, the objective is to find the protection labels Yj for all the classes in order to maximize the total revenue for this particular system. Now, as I already mentioned that we have not provided the mean and standard deviation for the lowest fare class it means that fare class is not operating. And we will see that how EMSR will help us in solving this particular situation that how much capacity we are going to allocate for a particular class or the protection label for a particular class.

Now, as in the beginning of this particular session I said that, there are two versions of EMSR are available EMSR-a and EMSR-b. And their logic is almost similar there is some minor difference which actually gives more efficiency in EMSR-b. So, EMSR-a is the heuristics which is based on the Littlewood's rule and for considering the protection label Yj for a particular class j. Let us say for class 3, we have to calculate the protection labels. So, you have to calculate the protection label Y3 with respect to all the higher classes available at that time. So, since if I calculate for Y4, so I have to calculate for 4 to 3, 4 to 2 and 4 to 1 and then Y4 will be Y43 plus Y42 plus Y41.

$$y_4 = y_{4-3},\ 4-2,\ 4-1$$

$$y_4 = y_{43} + y_{42} + y_{41}$$

Since, class 4 is not operating in this particular case, so we have started this particular solution for class 3. So, Y3 the protection label for class 3 will be the sum of protection label for particular class with respect to a particular class. So, Y31 plus Y32 these are the two classes, these are the protection label for Y3 with respect to 1 with respect to 2 and so the total protection label will be simply the sum of these things. So, if there is a situation where more classes are there. So, you have to do this calculation for all those classes with respect to the present class and then the summation of that will give you the protection label.

So, already we have seen in our Littlewood's formula that how this protection label for a particular class is calculated, if it is a because Y31 or Y32, these are simply a two class situation. You have a class 3 and then class 2. You have a class 3 and then class 1. So, you are calculating the protection label Y32, Y31 as a two class problem. So, directly this formula of Littlewood is applicable in calculating these protection labels and that will help us in getting the total answer.

So, here we can use our excel knowledge and using that excel knowledge we can actually calculate normal inverse function using the data given to us of standard deviation and

$$y_3 = y_{31} + y_{32} = F^{-1}(1 - p_3/p_1) + F^{-1}(1 - p_3/p_2)$$

mean available for a particular class. So, like for example, if we want to calculate Y43, Y43 if we want to calculate. So, this function will be applicable that is normal inverse, normal inverse of 1 minus p4 upon p3 this is the mu of 3 this is the standard deviation of

j	p_i	μ_j	σ_j	y_{4j}	y_{3j}	y_{2j}
1	$500	16.5	5.6	20	15	11
2	$420	44.2	15.0	52	37	
3	$290	35.1	11.2	37		
4	$125	--	--			
Total				109	52	11

3. So, you can refer this data from this particular table that this is your p4, this is p3, this is mu3, this is sigma3.

And we have put all these appropriate values in this particular p4, p3 and we got directly from the excel calculations these formula are inbuilt in the excel.

$$NORM.INV\left(\left(1 - \frac{125}{290}\right), 35.1, 11.2\right) = 37$$

So, you will get the value of Y43 as 37. You can similarly calculate the values of Y42, Y41 in excel and that will give you the values of Y4 equals to Y43 plus Y42 plus Y41 and that is 20, 57, 37 this is Y43, this is Y42, this is Y41. Similarly, you can do, I request all of you to do these calculations on your own. This is Y31, this is Y32, this is Y21. So, now, you have a all the information available with you and before you see our answer on the screen, we request you to do these calculations of Y31, Y32 and Y21 on your own and then you will see that this is actually the sigma of individual columns.

These are the sigma of individual columns. So, it says that if you have a total capacity of your class 4, the 109 seats from the total seats should be reserved for the bookings for class 3 and higher. So, when your customers are coming, starting the booking, 109 seats you are keeping for class 3 and higher. 52 seats you are keeping for class 2 and higher and 11 seats are available exclusively for class 1. It is also interesting, if I ask you that what is the value of b1 in this particular case?

What is the value of b2? What is the value of b3? And what is the value of b4? That is also a kind of additional question to all of you that you should be able to answer. So, this is how our EMSR-a is done where we have to calculate important thing is protection label with respect to each higher class which is available at that particular class label.

The other method is EMSR-b, which is a computationally slightly faster method. Here, we are not going to create so many classes for the higher classes. What we are going to do is aggregate them. We will take the average of mu's, the average of sigma's, and based on that, we create a virtual class. That virtual class is used for, you can say, getting a direct level of calculation. So, all these sigma activities will be minimized, and you will have your direct answer with that updated method of EMSR-b. So, all the higher classes which are available, we will be able to aggregate their requirements, and we will see that.

So, this artificial class or virtual class has the demand equal to the sum of the demands for all the future periods and a fare equal to the average expected fare from future bookings. So, then the application of this Littlewood rule will be much easier for EMSR calculations. So, let us see how we are going to do that for EMSR-b, using the same data of the four-class problem. Here, for class 3, the weighted fare will be based on the class fare multiplied by the mean demand which you are expecting for these classes. So, for class 3, this will be the expected fare; for class 2, this will be the expected fare; and for class 1, this will be the expected fare because it is the highest class.

So, there is no need for aggregation. Similarly, the aggregate demand is the demand for all the upper classes. For class 3, these are the three classes, and that is the total aggregate demand for class 2 and class 1. And for class 1, it is only that class. So, that is the way of calculating the aggregate demand and aggregate price—or rather, it is better to say weighted price—for the upper classes.

$$Class\ 3 = \frac{290 \times 35.1 + 420 \times 44.2 + 500 \times 16.5}{35.1 + 44.2 + 16.5} = 386,$$

$$Class\ 2 = \frac{420 \times 44.2 + 500 \times 16.5}{44.2 + 16.5} = 442\ and\ for\ class\ 1 = 500$$

Similarly, the aggregate demands for

$$Class\ 3 = 35.1 + 44.2 + 16.5 = 95.8$$

$$Class\ 2 = 44.2 + 16.5 = 60.7$$

$$and\ for\ class\ 1 = 16.5$$

So, this is the standard deviation because standard deviation cannot be added. So, we have to calculate the variance and then take the summation of the variance and then the square root of that. So, these are the standard deviation in a additive form for the upper classes.

$$Class\ 3 = \sqrt{5.6^2 + 15^2 + 11.2^2} = 19.53,$$

$$Class\ 2 = \sqrt{5.6^2 + 15^2} = 16\ and\ for\ class\ 1 = 5.6$$

Again the same formula we have to apply that now this is the p4 and this is weighted fare. This is weighted p3 in fact and this is aggregate demand, aggregate mean demand and this is aggregate standard deviation.

So, for using this type of formula directly you are getting the protection level of 105. Similarly, for class 2 against class 3, you get the protection level like 54.

Based on the values obtained in the previous calculations, protection level for class 3 against class 4 can be obtained by

$$= NORM.INV\left(\left(1-\frac{125}{386}\right), 95.8, 19.53\right) = 105$$

Similarly, protection level for class 2 against class 3

$$= NORM.INV\left(\left(1-\frac{290}{442}\right), 60.7, 16\right) = 54$$

And if I see that these are the weighted fare, we already did the calculation and aggregate demands, aggregate standard deviations. Now, when I compare EMSR-a and EMSR-b. Here, you see that the protection label for class 3 using EMSR-b is coming 105.

j	p_j	μ_j	σ_j	Weighted fare	Agg. Demand	Agg. Std. dev.	EMSR - b	EMSR -a
1	\$500	16.5	5.6	500	16.5	5.6	**11**	11
2	\$420	44.2	15.0	442	60.7	16	**54**	52
3	\$290	35.1	11.2	386	95.8	19.53	**105**	109
4	\$125	--	--					

We just saw this calculation 105. Now, there is no need of doing that summations which we did for getting 109 for this particular class, where we are calculating 3 protection labels and then doing the summation. So, in just one single normal inverse function, we got 105 and it was 109 in the case of EMSR-a. Similarly, for class 2 it was 52 and 54 here and for class 1 in both the method it is coming 11 and 11. Now, you can I leave it to you that please calculate the total revenue you are going to make using EMSR-a and EMSR-b. In fact, there are no uniform outcome that EMSR-a is better than EMSR-b or b is better than a. Sometime a gives you better results, sometime b gives you better result.

But, because B gives you faster calculations, you are not going to allocate capacity just for one system the whole day. So, you have to allocate capacity for, let us say, you are an airline and you have hundreds of airplanes, and on a daily basis, you have to take care of the booking limits for different types of fare classes. So, you need more efficient calculations. So, EMSR-b is more efficient in its calculations, and therefore, EMSR-b is generally used to do the allocations in modern service systems, while if you see numerically, the numbers sometimes may favor a, sometimes may favor b also. In our next class, with the help of some data, we will focus on this particular aspect: how a and b can have their own advantages because complete merit is not available in just one system.

With this, we come to the end of this particular session. Thank you very much.