Course Name - Operations and Revenue Analytics

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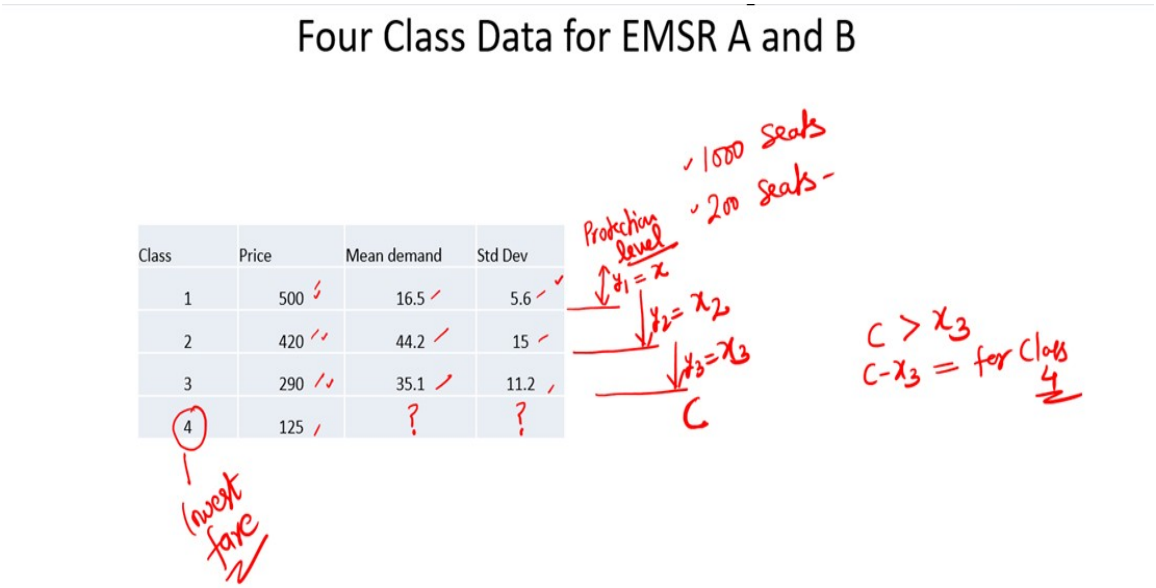
Lecture - 30

Welcome, friends. In our last session, we discussed the use of the Decision tree approach for solving multiclass problems. And in the multiclass problem, we discussed how we actually made it into a two-class problem. We are always seeing it as a two-class problem, and therefore, Littlewood's rule is very easily applicable to get us a solution for a multiclass problem. Moving further, we also discussed that these multiclass problems, with the help of Littlewood's rule, can be very easily solved using one particular approach that we know as EMSR. We discussed two versions of EMSR: EMSR A and EMSR B. Using these two versions, we saw that version B is computationally more effective.

But the economic advantage—whether A is better or B is better—we do not have a clear consensus on that. However, we also discussed that because B is more efficient, most of the time we are using EMSR B for deciding the class allocations. In this particular session, with the help of some numerical data, we will examine this particular statement: whether A is better or B is better. And at the same time, we will also do a complete calculation using Excel in this particular class so that if you get a chance and you have to do this allocation, you can also use Excel software for this kind of division of classes. So, as we already have this data from our previous class, where we have four classes and the price or the fare which is applicable is 500 rupees, 420 rupees, 290, and 125.

We have the data available with respect to the mean demand of each class, and their standard deviation is also available. We generally do not have the data available for the mean demand of the class which is the last class in the given categories. And we will see

the meaning of this thing: why there is no relevance in giving the mean demand and standard deviation for the last class. Because if you have a system which has, let us say, 1000 category seats or another system where you have only 200 seats. Since our class 4 is the lowest fare class, only the leftover seats will be given to this particular class



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The maximum revenue will come from class 1 because the price is highest here, second highest is 420, third highest is 290.

So, we generally are more interested that what should be the protection level and therefore, in all our previous sessions also rather saying that what is the booking limit for class 1, 2, 3, we are talking in the term of what is the protection level for a particular class. So, the meaning of the protection level is that these many seats are available for this particular class, Y1 is the protection level let us say Y1 is equals to X. So, X seats are 100 percent reserved for class 1 this is Y2 which is let us say X2. So, that means X2 seats

are available for class 1 and class 2 together, this is Y3 which is X3. So, total X3 seats are available for class 1, class 2, class 3.

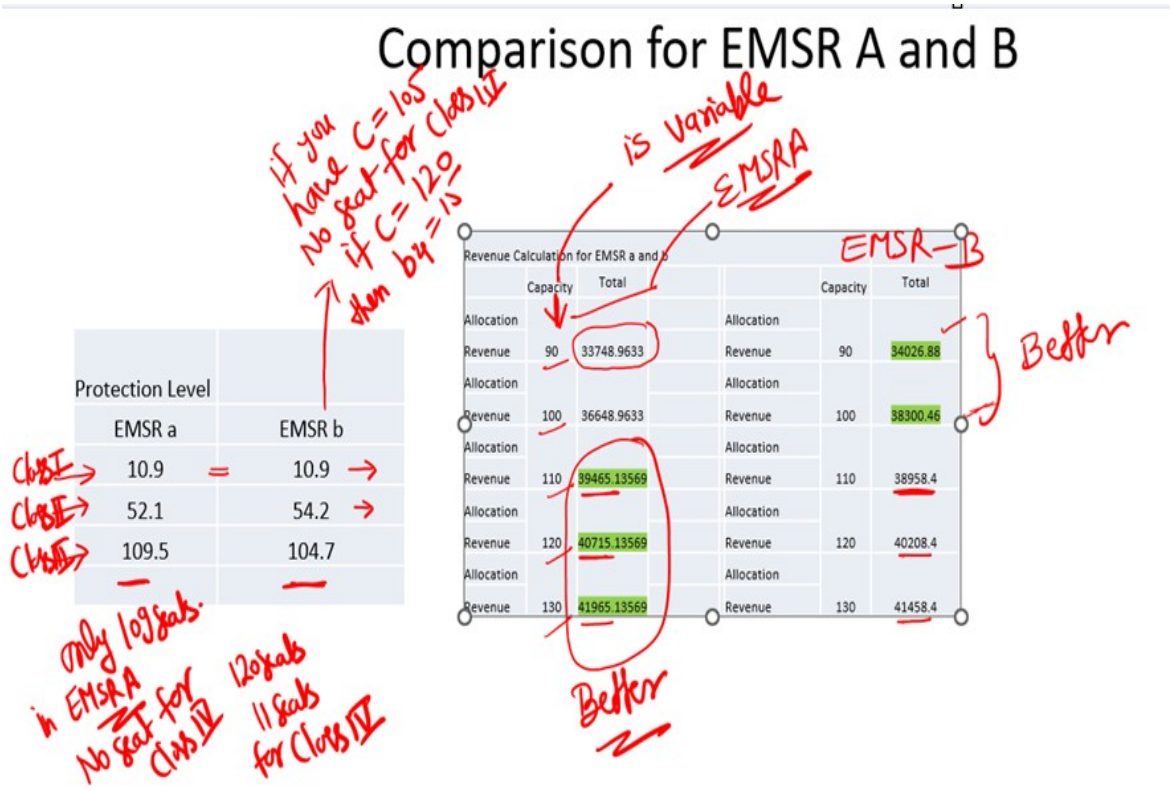
Now, if you have a capacity which is let us say capital C, this is the capacity of your system and if the capacity of the system C is more than X3 for example. So, C minus X3 seats will be for class 4. So, that is how this logic is there that we do not give the mean demand and standard deviation for our last class. Now, as we have done in our previous session using EMSR A and EMSR B, we have calculated the protection level for A and B and that is available here directly you can refer our previous class. We will also show you that how we have calculated these things for our using the excel softwares.

So, if I use A the protection level for class 1, class 2 and class 3. These are the protection level for using A and for B there is some changes, particularly for class A, class 1, the levels are same. Levels are slightly different for 2 and 3. Now, for example, if your system has only 109 seats, so in EMSR A, you will not allocate any seat to class 4, you will not allocate any seat, no seat for class 4. But for example, if you have a system where you have 120 seats, in that case, you will allocate 11 seats for class 4.

So, similarly, if I refer EMSR B. If you have a system capital C equals to 105 for example, no seat for class 4, but if you have, if capital C is let us say 120 then b4 or you will have 15 seats for class 4. So, in this way, the EMSR A, B and there is no need of having the protection level for your class 4 because, that is variable and that depends upon how much capacity you are going to have with you. Then, if you see the right side of this screen, here we have multiple criteria and the multiple criteria is with respect to the capacity of your system. Capacity is variable and we have done this calculation for different levels of capacity 90, 100, 110, 120 and 130.

Now, when we have 90 seats the total revenue is close to 33,748, you forget points into the decimals let us concentrate only on the integer part that is 33.748. While for the same 90 seat server you have revenue equals to 34,026 for your EMSR B. So, this calculation is coming for EMSR B and this is A. And therefore, here it appears that EMSR B is a better way of getting the revenue. Same in the case of 100 capacity also.

Here, my revenue is 36,648, while the revenue is 38,300 in case of B. It is again better. But, when our system's capacity is increasing, it is 110, 120 or 130. Here, you see that revenue in case of A is higher than B. It was 38,958 which becomes 39,465. From 40,208 it has become to 40,715 in case of EMSR A. Similarly, from 41,458 it has become 41,965 in case of A. So, for capacities equal to 90 and 100 B is better and for capacities equal to 110, 120 and 130 A is better.



So, as we discussed in our previous session and in this session also. It is difficult to say whether A is better or B is better. But, you can calculate all these things are possible in a

very automated manner. So, therefore, the logic that computational things are expensive or you will take more time for computing A versus B that logic is not very appreciable logic. And therefore, I request that we can do both the calculations always, A and B. If you are getting more revenue using B, then you allocate your capacity using EMSR B.

If you are getting more revenue, more expected marginal revenue is coming from A, then you should use A, whichever methodology is giving better revenue. In fact, this is the data which we discussed in our previous class and we will show you the complete calculation of these revenue tables that how did we arrive on these revenue values. But before that, before we actually shift to the excel tables, we will also like to refer one paper which is published comparing EMSR A and EMSR B. This was the paper written by Talluri and Van Rysen in 2004 and they in fact went one step ahead. They also used a simulation software and they calculated the optimal protection level using the simulation results and then they also calculated EMSR-A and EMSR-B related protection levels.

So, they also have four classes 1, 2, 3, 4 and the prices are 1050, 950, 699 and 520. The in fact, they have given the mean demand and the standard deviation of demand also for class 4 in this example. However, as I just mentioned you it may not be necessary also. So, mean demand standard deviation for all the other classes are available in this column number 3 and column number 4. Using that they got protection levels for classes like this.

So, you can see that for class 1, the protection levels for A and B calculations are almost similar. While there are some minor changes in the protection level for class 2, and class 3 also has some minor changes for A and B. You can compare these numbers of protection levels with the optimal numbers. So, these numbers of A and B are very, very close. Almost all these 9.7 is the optimal protection level, while A and B are given 9.8 as the protection level. 54 is the optimal protection level for class 2, while it is 50.4 and 53.2. 98.2 is the optimal protection level for class 3, while it is 91.6 and 96.8.

		DEMAND STATISTICS		PROTECTION LEVELS		
Class (j)	Fare (p_j)	Mean (μ_j)	Std. Dev. (δ_j)	Optimal	EMSR-a	EMSR-b
1	$1,050	17.3	5.8	9.7	9.8	9.8
2	$950	45.1	15.0	54.0	50.4	53.2
3	$699	39.6	13.2	98.2	91.6	96.8
4	$520	34.0	11.3			

SOURCE: From Talluri and van Ryzin (2004).

So, you can say that these are more or less very, very close to A and B. Now, let us see the total revenue calculations for these different optimal A and B levels. So, they also ran different capacity levels; they ran it for 90, 100, 110, 120, and 130, the same as we have done for our data. Using the optimal protection levels of 9.7, 54, and 98.2, the optimal revenue collection is given in column 2. This column 2 gives you the optimal revenue collection, like 74,000 and so on.

When you use the protection level given by EMSR A, these calculations show that if the capacity is 90 units, the revenue collected is 73,950, which is around 99.3% of the optimal revenue. This is column 3; this is column 4. So, the calculation of column 4 is with respect to columns 2 and 3. Similarly, for capacities of 100, 110, 120, and 130, we got different levels of revenue, and most of the time, you will say that it is almost optimal. 99.84, that is when the seat capacity is 130.

So, your revenue in the optimal case is 95,054 and if it is EMSR A if you use that protection level it is 94,899. And now come to column 5 and column 6 and in column 5 and column 6 we are using EMSR B method. Now, here you see that revenue is almost equal to column number 2. 74,000 is the revenue which is almost equal to optimal revenue, optimal revenue is just 3 rupee extra to the EMSR B. So, here you see that most of the time it is 100, 100, 100, 100.

Only in this one particular scenario, it is 99.7. So, this is again almost optimal or rather it is not appropriate to say almost it is actually optimal. So, here this data shows that EMSR B is better than EMSR A and EMSR B is giving you almost optimal solution. But, even if you see EMSR A the difference is less than 0.5 percent and as the capacity is increasing,

Capacity	Optimal Revenue	EMSR-A Revenue	EMSR-A % Optimal	EMSR-B Revenue	EMSR-B % Optimal
90	$74,003	$73,950	99.3	$74,000	100
100	$79,429	$79,164	99.67	$79,426	100
110	$84,884	$84,554	99.61	$84,862	99.7
120	$89,879	$89,668	99.77	$89,875	100
130	$95,054	$94,899	99.84	$95,054	100

SOURCE: From Talluri and van Ryzin (2004).

you see that initially it was 99.3 percent of the optimal and when the capacity has increased from 90 units to 130 units, it is now 99.84 percent of the optimal level. So, you can say one thing that as the capacity of your servers increase, airline, hotels, restaurant, if the capacity increases, the difference between optimal EMSR A and EMSR B will start diminishing when the capacity is less.

And in that time, the different approaches may have some significant, some significant differences in the overall calculations or overall revenue which you are getting. So, as you are going up, you will see that there is hardly any difference between A, B and optimal cases. So, this is how we have developed our theoretical understanding. Now, let us go to the excel part. So, that we can understand that the calculations or these numbers which we have discussed with respect to EMSR calculations or with respect to total revenue calculation how these numbers have been obtained.

So, you all can use your excel system and using the excel system you can punch this data and when you punch the data you will see that this data is there you can see on your

screen that we have four classes. We have put the prices of each of these classes, their mean demand that is in column E, then the standard deviation that is in column F and then we have calculated the protection levels the Y4J it represents that protection levels with respect to class 4. So, like, if you see the entry of column G row 5 that is G5 it is Y41 and the formula which we have used for Y41 that is shown in your formula bar. So, let us if you see norm in so you have to give these three values, one is the probability, second is the mean value and third is the standard deviation. So, the probability in this case is 1 minus the price of the base level divided by the price of the class for which we are calculating it.

So, this is D8 by D5, D8 is the base that is 125 and D5 for respect of which we are calculating that is 500 price. So, it is basically, if you understand this is 1 minus P4 by P1, this is 1 minus P4 by P1 that becomes the probability and then you have mean that is E5 and standard deviation that is F5. So, this is the formula which is inserted in this G5 cell similarly, you have the formula for G4 G6 cell, where you have now 1 minus P4 by P2. And then rest of the things remain as it is, standard mean and standard deviation and then you have calculated Y41, Y4 if I insert one column. So, this becomes Y41, this is Y42 and this is Y43.

And then, to calculate my protection level here, Y4, that is the sum of Y41 plus Y42 plus Y43. Similarly, we can do for y3j. Here, we are calculating Y31 and Y32. As it is we have inserted this normal inverse function formula in j5 and j6 and you get y3 in the cell number j9. Then for calculating Y2, we are left with only one cell which is higher than this.

So, we will have only one thing to calculate, which is Y21, and in this way, we get protection levels. So, this is the protection level for class 1. So, you have 109, 52, and 10. In fact, for all practical purposes, it is not possible to have capacity in a particular class in fractions. So, you have to convert this into an integer value.

You can consider it as 11, 52, and 109, or you can consider it as 11, 53, and 110; both are okay. If you follow the scientific way of rounding off, you should take 11, 52, and 105. Similarly, you have to calculate the average weighted fare, and this average weighted

fare, if you see, will only be applicable to the lower part of this table for my class 2 and class 3. So, class 2 will have the sum product divided by the sum of the fares. So, you have three categories for this: 420 and 500, two fares which are more than 290.

So, their weighted fare price will be 441. So, the formula you can see in the formula bar. Similarly, you will have aggregate demand, which is the demand of that particular cell and the upper classes. So, it is 16.5 plus 44.2, which comes out to be 60.7. And similarly, for class 3, the total demand of 1, 2, and 3 will be the aggregate demand.

The aggregate standard deviation is calculated by determining the variance, adding them, and then taking the root of that. Based on that, these protection levels are calculated again using the normal inverse functions. So, you can say that the protection level for class 1 was 10.93, and it is coming as 10.93 here again. Then, for class 2, the protection level is Y2, and for class 3, the protection level is Y3. There is a slight difference between A and class 2 and class 3.

As we have already explained in our PPT, for class 2, the protection level is 52 using A, while it is 54 using B. For class 3, the protection level is 109 for A, while it is 105 using B. If you perform this calculation, we have already done the revenue calculation for you to save time. Here, if you see the upper part, we have directly produced the result of our protection levels for class 1, 2, and 3. Now, using this protection level information, let us consider a situation where we have only 90 seats available. If 90 seats are available, then obviously, I have a protection level of 10.9 for my class 1 and 52.1 for class 2.

So, here it will be because, out of 50.1, 10.9 seats are protected for class 1. So, only the remaining seats are protected for class 2. So, 41.2 seats are available for class 2. For class 3, class 2, and class 1, the total protected seats are 109.5. So, out of 109.5, 52.1 seats are reserved for class 1 and 2.

So, the remaining seats will be used for your class number 3. So, here in class number 3, since we have only 90 seats available in this particular case. So, maximum out of 100 point 109, let us say 52 seats are going to 1 and 2. So, 57 seats are actually possible for class 3, but since our total capacity itself is 90. So, we cannot use the total protection

level for class 3; rather, we will be able to use only 37.9 seats for class 3, and we know the pricing of 500, 420, 290, and 125.

So, we will do the product of fare and allocations, that is, D16 will be multiplied by D17. Similarly, E16 will be multiplied by E17, and F16 will be multiplied by EF17, and their results are in row number 18, like that which is there in front of you. And then finally, this total is the sum of D18, E18, and G18, F18 up to G18. So, but there is no entry in G18 at the moment because there is no allocation in this case. So, we got this 33,748, and you can match this value with our PPT where we have already discussed these numbers.

Now, the same calculation we have repeated for the right-hand side of this sheet, where we see that if we have a 90-seat capacity, out of that, 10.9 will be protected for class 1, 43.3 will be protected for class 2 only, and only 35.8 seats will be available for class 3. And using the product of fare and allocation, we have got the values in row 18 also, specifically in cell M18, N18, and O18, and their total is 34,026. So, you can compare these numbers: 33,748 is less than 34,026. And when you go to the lower part of this allocation table, if I have a capacity of 110, 120, or 130, then the EMSR B is showing the better results. So, I have shown you this particular table, the calculations, and I request you to do these calculations on your own; the data is there in front of you.

You have seen the application of the formula, you have seen the logic also, and using all these things, you should be able to do the revenue calculations for capacities of 100, 110, 120, 130 for both these tables. And then you will understand that sometimes A is better, sometimes B is better, and therefore, this discussion remains slightly inconclusive. I mentioned that because B is working with the help of a virtual class, which is the weighted sum of all the upper classes available at a particular class level. So, people feel that B is efficient, faster, and economically viable. Also, we saw that the results of B are closer to the optimal value than the results of EMSR A. So, with this, we have understood the Excel calculations and a proper comparison between EMSR A and EMSR B. With this, we come to the end of this particular session. Thank you very much.