Course Name - Operations and Revenue Analytics

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Welcome friends. We were discussing the case of network management that hub and spoke systems are very popular and there are large number of examples, where we need network concept in our booking systems. Because, if I take example of flights we are going from one station to another and from that station to a further new destinations. We are booking hotel rooms for multiple nights, we are taking cars for multiple days. So, everywhere it is not necessary you cannot force customer to just use service for one instance.

There are sequence of services you are expecting and whenever you are expecting sequence of services the concept of network management comes into the picture. We have discussed that concept of network management in our last session also. We discussed that how the concept of linear programming is possible. Greedy algorithm also we discussed. And finally, we discussed that the concept of virtual nesting is going to be the most important interesting which will help us to allocate the capacity using the concepts of EMSR which we have discussed for a single instance cases, where you are going from only one origin to one destination.

But, how it is going to be applicable in network management that we have to discuss. We saw briefly in our last session the concept of indexing, we discussed the concept of bucketing and we discussed the concept of ODF also. In this particular session, we will discuss the concept of opportunity cost. We will discuss the concept of net leg fare and

how using these concepts we are going to make different buckets and then we will see how we are going to use these buckets for our capacity allocation and all these things we will see with the help of examples on excel software. Now, as we discussed in our previous session just for the continuity indexing is the concept which is required for this network management problems.

Now, when you are going from one station to another station, you have to see that this is leg 1, and this is leg 2. If a customer is coming for this complete route, you need to ensure that capacity is available in both legs. There are customers who are expecting services only for this leg or that leg. So, in such a scenario, considering the different types of revenues you may get, whenever you are booking a customer for a particular leg, it may be the total seat from A, B, C. If you are booking for A to C, it may be at the cost of a customer being denied service in A to B, a customer denied service in B to C, or two customers denied services—one in A to B and another in B to C. So, all these possibilities exist, and by denying all these possibilities, you are giving a seat to a customer in A to C. So, whenever you are allocating a seat in a particular sector or for the entire journey, there will be some opportunity cost because you are denying services to some other customers who may come at a later time.

Therefore, we are going to develop a concept known as net leg fare. The revenue you earn by offering a seat to a customer in A to C comes at the cost of denying services to other customers who may come for different types of services using the same resources. So, that is the opportunity cost concept.

So, the net leg fare for ODF i for leg K—for this particular leg K ODF i—there are different types of ODFs. Total fare for ODF i minus the sum of opportunity costs on all resources other than K in ODF i. So, the meaning of this statement—what is the meaning of the sum of opportunity costs on all resources other than K in ODF i—will become clear to you in a few moments.

But you can broadly understand one thing: Net fare equals total fare minus some opportunity cost.

$$\begin{aligned} &Net\ leg\ fare\ on\ ODF\ i\ for\ leg\ k = total\ fare\ for\ ODF\ i \\ &-Sum\ of\ opportunity\ costs\ on\ \boldsymbol{all\ resources\ other\ than\ k\ (if\ any)in\ ODF\ i} \end{aligned}$$

We will see what this total fare and opportunity cost are with the help of examples in subsequent slides. So, if we see using this formula of net leg fare—that is, with the help of an example we have been continuing since our previous session—we see that NLF calculation comes from total fare minus opportunity cost. Now, in this particular case, we have this example where you are traveling from Mumbai to Delhi to Roorkee. Now, if you are traveling from Mumbai to Roorkee in a single flight, the fare we are expecting is 300 dollars. Now, based on our previous experiences—how many times seats between Mumbai to Delhi are occupied, how many times Delhi to Roorkee seats are occupied.

You are saying that you generally may get, you generally may get $180 between Mumbai to Delhi and $105 between Delhi to Roorkee. That is the possible profits. These are the possible profits between Mumbai to Delhi and Delhi to Roorkee respectively. So, therefore, these are opportunity cost. When you are taking a fare of 300 dollars from Mumbai to Roorkee directly, you are having the opportunity cost of 180 between Mumbai to Delhi and 105 for Delhi to Roorkee sector.

Now, here comes the concept of net leg fare. So, you are taking the net leg fare for leg 1 and leg 2. For calculating the net leg fare for leg 1 that is Mumbai-Delhi, you will minus the opportunity cost of Delhi-Roorkee sector from the total fare you are expecting from origin to destination. So, 300 is actually the ODFI that is for the entire route from origin to destination minus the opportunity cost of this particular leg that is going to give you the NFL for this particular first leg that is 195. And similarly, NFL for second case will be 300 minus 180 that is 120.

This concept of NLF is going to be very useful for the purpose of indexing, for the purpose of bucketing and therefore, we will be actually developing one full example so that you can understand that how we are going to use this virtual nesting for our capacity allocation.

			Flight 1		Flight 2	
Product	Class	Total Fare	NLF	Bucket	NLF	Bucket
Mumbai to Delhi	Discounted	400	400	3	NA	NA
Mumbai to Delhi	Full fare	650	650	2	NA	NA
Delhi to Roorkee	Discounted	550	NA	NA	550	3
Delhi to Roorkee	Full fare	750	NA	NA	750	2
Mumbai to Roorkee	Discounted	1300	500	3	1050	1
Mumbai to Roorkee	Full fare	1800	1000	1	1550	1

Now, you consider a scenario where Mumbai, Delhi, Roorkee same example and we are given the opportunity cost of 250 and 800 respectively based on our historical data and now we want to develop three buckets using these three things. We will discuss that how we are deciding the ranges for these particular buckets. For that purpose, the data which we are going to use, you are already familiar with this data. Right now, I will request that do not focus on the entire table.

Just focus on the first three columns of this table. The first three columns on the table. You are flying from Mumbai to Delhi, Delhi to Roorkee, and Mumbai to Roorkee. Now, in every product, you have two types of fares: discounted fare and full fare. So, if you are flying from Mumbai to Delhi, the discounted fare is four hundred dollars, and the full fare is six fifty dollars.

Similarly, Delhi to Roorkee is five fifty and seven fifty respectively, and for the entire journey, it is thirteen hundred and eighteen hundred. You are given the opportunity costs like 250 and 800. So, for this sector, the opportunity cost is 250, and for this sector, the opportunity cost is 800. So, for the calculation of net leg fare, when I am traveling from Mumbai to Delhi, there is no opportunity cost. Similarly, between Mumbai to Delhi on the full fare, there is no opportunity cost.

Flight 2 is not required in this case. So, NLF and all these things are not applicable. Flight 1 is not operating between Delhi to Roorkee. So, there is no question of NLF for these two cases. Similarly, flight 2 is only operating between Delhi to Roorkee. So, there is no case of opportunity cost minus, and that is the total fare is there, net leg fare also. Now, when you are giving priority to Mumbai to Roorkee customers, whether they are discounted or full fare.

You see why this opportunity cost issue is coming in this case. Because if you take a customer who is traveling from Mumbai to Roorkee on a discount. So the discounted customers or full-fare customers may travel from Mumbai to Delhi, let us say 650 and 750. So, you are giving a seat to a complete customer, and the loss which is possible may be 6 plus 7, 1400. So, you could have earned 1400 dollars if you get two customers who are ready to pay full fare, one between Mumbai to Delhi and another between Delhi to Roorkee. So, there is an opportunity cost you are incurring if you are offering a seat to a Mumbai to Roorkee customer.

The same applies to a Mumbai to Roorkee full-fare customer that there may be some kind of opportunity loss you may have because there may not come any customer from Mumbai to Roorkee, and there may be customers who may come between Mumbai to Delhi and Delhi to Roorkee. And if you keep seats blocked for your Mumbai to Roorkee full-fare customer, declining services to Mumbai to Delhi and Delhi to Roorkee individual customers. So, holding the seat for these customers is again coming at an opportunity cost. So, the NLF concept is applicable for these categories only. It will not be applicable to these categories. Now, for the calculation of NLF for flight 1. Flight 1 is offering between leg 1 and flight 2 is leg 2.

Now, the opportunity cost is given to us that for leg 2, the opportunity cost is 800. This is leg 2 and this is leg 1. So, when I have to calculate the opportunity NLF for leg 1, I have to subtract the opportunity cost of leg 2. So, 1300 minus 800 is 500 and 1300, sorry this is 1800 minus 800 is 1000. 800 is the opportunity cost of leg 2. And in this case, 1300 minus 250 that is 1050 and 1800 minus 250 that is 1550, that is the NLF. So, now we have NLFs for all these possible cases.

In this question we have given you the rules for making the buckets. Generally in a practical scenario these rules you have to create on your own. You have to make your own rules and there is no magic in developing these rules. So, what you can do that you can put all these NLFs in ascending or descending orders. For example, in this case let me put 1550, then 1050, then 1000, then 750, then 650, then 550, then 500 and 400. These are the different NLFs.

So, you have 1, 2, 3, 4, 5, 6, 7, 8. So, you can divide in some suitable number of buckets these different NLFs. Maybe you can take 3 buckets, maybe you can take 4 buckets. Since we have done this example with the help of 3 buckets, so one possible bucket is this area where you have more or less similar values of NLFs. However, one NLF is 1000 and another NLF is 1550, but on a larger level it appears to be okay.

Another is this second bucket where you have between 600 to 1000 and third bucket where you have less than 600. So, these are the three you can say categories of NLFs which are helping us in making different buckets. So, these are the buckets and these are the NLFs and on the basis of that if you see here we have written this 400 falls in bucket 3, 650 falls in bucket 2, then 550 falls in bucket 3. 750 falls in bucket 2, 500 falls in bucket 3, 1050 falls in bucket 1 and 1550 all are in bucket 1. So, in this way our bucketing is done and with the help of this bucketing we know that which particular product and a particular class Mumbai to Delhi discounted class, Mumbai to Delhi full fare is coming in which bucket.

So, this completes our issue of bucketing. Now, we will see how we are going to develop the capacity allocation and, with the capacity allocation, the calculation of its total revenue with the help of an Excel diagram. So, as we just discussed, now we are coming to this Excel sheet, and if you want, you can all try simultaneously.

			Flight 1		Flight 2	
Product	**Class**	**Total Fare**	**NLF**	**Bucket**	**NLF**	**Bucket**
Mumbai to Delhi	Discounted	400	400	3	NA	NA
Mumbai to Delhi	Full fare	650	650	2	NA	NA
Delhi to Roorkee	Discounted	550	NA	NA	550	3
Delhi to Roorkee	Full fare	750	NA	NA	750	2
Mumbai to Roorkee	Discounted	1300	500	3	1050	1
Mumbai to Roorkee	Full fare	1800	1000	1	1550	1

•Bucket 1: Net leg fares ≥ $1,000
•Bucket 2: $600 ≤ Net leg fares ≥ $1,000
•Bucket 3: Net leg fares ≤ $600

Net leg fares in decreasing order

1550
1050
1000
750
650
550
500
400

for flight 1	Demand	Std Dev
1000	16.5	5.6
650	44.2	15
500	35.1	11.2
400	40	15

Aggregating demand and standard dev for bucket 3
Demand 75.1
Std Dev 18.72

Calculating weighted average fares

Bucket	WT fare	Muj	sigmaj	Avg wt f	Demand	agg SD		Total revenue	50	60	70	90
1	1000	16.5	5.6	1000	16.5	5.6	14.3422		14342.2	15081.3	15081.3	15081.3
2	650	44.2	15	745.1	60.7	16.0112	56.6629		23177.6	27508.4	27508.4	27508.4
3	446.73768	75.1	18.72	580.1						1490.83	8669.14	14893
									37519.8	44080.5	51258.8	57482.6

for flight 2	Demand	Std Dev
1550	18	8
1050	30	20
750	38	16
550	40	15

Aggregating demand and standard dev for bucket 3
Demand 48
SD 21.541

Bucket	Weighted far	Muj	sigmaj	Avg wt f	Aggregate demand			Total revenue	50	60	70	90
1	1237.5	48	21.54	1238	48	21.54	42.2043		9647.16	9647.16	9647.16	9647.16
2	750	38	16	1022	86	26.8323	83.4328		5846.76	13346.8	20846.8	30921.3
3	550	40	15	872.2								3611.98
									15493.9	22993.9	30493.9	44180.5

Here we have in this column B the different kinds of products: Mumbai to Delhi, Delhi to Roorkee, and Mumbai to Roorkee. For each of them, we have discounted and full fare. In column D, we have written the total fares which are possible, and in columns E, F, G, and H, we have given for leg 1 and leg 2 respectively their net leg fare and the bucket classifications.

Now, we are following this rule of bucket 1 for net leg fare more than 1000, for 600 to 1000 we are keeping bucket 2, and for less than 600 we are keeping bucket 3. And as I told you in the PPT, we have arranged these net leg fares in ascending and descending orders. Here we see that up to this label, this is bucket 3, this is bucket 2, and these are bucket 1. We have respectively copied these numbers in columns F and H. Now, for flight 1, we have this data. That if the fare is 1000—because if you see for flight 1, there

are 4 different types of NLFs which are available. For 1000, which is between Mumbai to Roorkee, the demand data is given to us in terms of mean demand and its standard deviation.

So, this is basically mean demand, and this is the standard deviation. So, mean demand is 16.5 units, and the standard deviation is 5.6. For 650 and 500, the mean demands are 44.2 and 35.1 respectively, with standard deviations of 15 and 11.2 respectively. For the Mumbai to Delhi discounted fare class, the mean demand is 40, and the standard deviation is 15. Now, for example, if I am calculating the aggregated demand—because now for further calculations, we have to apply how much capacity I am allocating for different buckets. So, in a way, you have flight 1 and this is flight 2.

And here you have bucket one, bucket two and bucket three. Similarly for flight two also you have bucket three, bucket two and bucket one. Now you can understand easily that bucket one are having the highest capacity of flight one, bucket two second highest. The total capacity minus protection label you are keeping for bucket 1 that is the capacity available up to bucket 2. And for bucket 3 you will have capacity which is available total capacity minus the protection label up to bucket 2 and same applies for flight 2 also. So, we have to see that what should be the booking limit protection labels for different buckets.

So, you need to keep this visual impression in your mind that where is bucket 1, where is bucket 2 and where is bucket 3. So, you can see in this diagram that we have just made. Now, for this bucket 1, bucket 2, bucket 3 for our flight number 1 in this case the weighted fare that is a term we are introducing because of different fares which are possible under one bucket. Like if you see this diagram there are three types of fares which are applicable in bucket three, two fares are available in bucket two and three fares are available in bucket one. Let us say there are few fares which are available in flight 1 and few fares which are available in flight 2.

So, if I say that 1550 is in flight 2, 1050 is in flight 2, this is in flight 1, 750 is in flight 2, 650 is in flight 1, 550 is in flight 2, 500 is in flight 1 and 400 is also in flight 1. So, 4 fares are in flight 1 and 4 fares in flight 2. So, and in every category every bucket there are

more than 1 fare in a particular case. So, we have to find that with respect to bucket what is the weighted fare for that particular category. Like in bucket 3.

you have only one fare in 1000 category. So, you are going to have weighted fare equal to 1000. You have only one fare in bucket 2 also for second bucket that is 650. So, this is coming as it is. But you have two fares for bucket 1 in flight 1 that these are 500 and 400.

So, you take a weighted average which is there in cell C 23 and if you see that how we have calculated. So, if I go to cell C 23 you will see in the formula bar the formula which we have used for that purpose. So, it this formula says that it is the sum product of B 15 to B 16 B 15 B 16 to C 15 C 16 their sum product divided by total demand this. So, this is giving you the weighted fare for bucket 3 in case of flight 1 and then you are calculating the mean demand.

So, that is also coming in the similar fashion you have to do aggregate demand for all the possible such cases like 16.5 as it is coming here. 44.2 as it is coming here and then you have to aggregate the 35.1 and 40 for getting the aggregated demand for the bucket 3 for flight 1. and then you have to calculate the sigma label the standard deviation 5.6 as it is coming, 15 as it is coming and then you have to calculate the variance of 11.2 and 15 and you can see that how this 18.72, that is this cell number G 17 where we have put the formula for calculation of standard deviation and we have directly copied this 18.72 from here to here. So, in this way our table is complete. Now, after this you have to start that what is the average weighted if you see this particular cell F 20 that is giving you average weighted fare, average weighted fare.

So, for class bucket number 1, Since we have only one fare that is 1000 so the average weighted fare in that case is 1000 and similarly for other two categories you have the weighted fare like 745 and 580. Now we come to particular case of demand that is also given to us that is coming with the help of concept of aggregation. We have discussed in our earlier sessions. I am not going to discuss that you have to follow this EMSR-B approach. You may remember that we have followed EMSR A and EMSR B and in that class you may remember that EMSR B was giving us better results. So, we actually came with this EMSR B formula here and we applied this EMSR B and finally if I go to this

column i and see cell 21 and cell 22 where we have applied that formula and you see that this is the normal inverse where we require three things normal inverse, probability, mean and standard deviation. So, we have taken the probability mean and standard deviation in this case for cell I 22. Now here if you see the value of probability is coming 1 minus C 23 by F 22. Now C 23 is basically 446. or you can say 447 and F 22 is coming my average weighted fare that is 745. Then mu is my mean demand that is 60.7 and then the standard deviation that is 16.01.

So, that gives us the value of protection label we are going to keep for bucket 2 that is coming around 56.66 or you can say round off in 57. Similarly, I will calculate the protection label for my bucket 1 that is coming in the similar fashion you can just check the formula if I go to cell I 21 the formula is available here and this is similar to that the only thing is the values of C and F will change. The value of mu will change and value of sigma will change but the formula will remain in the same fashion. And now I got the production labels for bucket 2 and bucket 1, 56 and 14 if I say in the round off. Now you see that there may be different types of capacities of your aeroplane.

Now, if you have an aeroplane with a capacity of 50, on the very first request If there is a booking request coming which falls in bucket 3, like bucket 3 booking limits, which may be possible like 446, and in fact, let me just clarify to all of you, this is my bucket 3 here, otherwise it is creating some confusion, and this is bucket 1. So, if a booking request is coming in this bucket 446. For example, anyone who is requesting a Mumbai to Roorkee discounted fare class ticket, I will not give that ticket to that person because this is coming in bucket number 3. And there is no capacity available for bucket 3 if the flight size is 50.

But if the flight size is 60, I may give 2-3 bookings to those requests which fall in bucket number 3. And accordingly, we have calculated the revenues for different capacities of 50, 60, 70, 90 types of servers, that in all these cases, you are keeping these number of facilities for your upper classes and all the bookings which are coming in bucket 3 can be accepted depending upon the size of your aeroplane, and as the size increases, you see that your total revenue also increases. A similar calculation you will also do for your

flight 2, which is the second leg and that is because it is not necessary that in both legs you have the same kind of aircraft.

So, if you have one aircraft with, let us say, 80 seats, and in another flight, you have an aircraft which has, let us say, 50 seats. So, on the basis of these two things, based on the protection level, you just calculated the protection level available on the same line for my second flight, which is given to you in this fashion, that is 83 and 42. These are the protection levels available for the second leg. So, unless you have a flight capacity of 90 before that, you are not going to accept any booking which may fall in bucket 3 for flight number 2, and therefore, you need a minimum aircraft capacity of 90 if you are going to accept some bookings in bucket 3 case also.

So with this, you are able to see that depending upon the size of your aircraft, how many seats you are going to give for different type of products and for different types of fare classes. So, this example gives us the practical implication or practical way of handling the network management problem using the concept of virtual nesting and where finally we have applied the AMSR rule for getting the protection labels for different product classes. So, now after completing this excel diagram excel calculation we see that what are the limitations of virtual nesting. So, now virtual nesting has some disadvantages also that is the indexing process can create potential difficulties in data collection and forecasting because in the indexing process we are using the concept of opportunity cost. Now, how are you calculating the opportunity cost that may have some variations and because of those variations the calculation of NLF may vary also.

So, therefore, that is one difficulty in using the concept of virtual nesting. Now, if data is collected at the virtual class level then re-indexing can alter the parameters of demand in a virtual class by changing which products are mapped into each virtual class. Because here we are creating the virtual classes and you have mapped your different products into different buckets 1, 2, 3. Now, it is quite possible that a bucket number 3 where you have one product which is a direct product also Mumbai to Roorkee and in the same bucket you have one product which is Mumbai to Delhi and another Delhi to Roorkee.

So, there may be some combinations of products which may come into a particular bucket which may find it difficult to execute also that we have just seen in the case of excel diagram also. So, these are some of the potential difficulties in virtual nesting. So, sometime because of the convenience we may go with greedy algorithm itself because that is very easy to operate. Otherwise, in case of deterministic demands we may go with the linear programming also. And if we want to have more involved calculations, we will go with the virtual nesting system.

So, with this, we come to the end of the discussions on how network management is handled with the help of virtual nesting systems. With this, we come to the end of this video. Thank you very much.