Course Name - Operations and Revenue Analytics

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Lecture - 07

Welcome, friends. In our last session, we started discussions about different types of forecasting situations, which we are going to help with predictive analytics. We discussed how very simple types of exponential smoothing methods are used in the industry for getting our forecasts. We discussed the case of simple exponential smoothing, where we saw that we are using a smoothing constant alpha for smoothing the fluctuations of our level. But we have seen in the characteristics that our data may have a trend also, data may have seasonality also, and trend and seasonality may also have fluctuations.

And therefore, in this particular session class, we are going to discuss how we are going to smooth our trend fluctuations and seasonality fluctuations. And therefore, we have already covered simple exponential smoothing methods, and now we are going to discuss trend-corrected exponential smoothing methods, which are also known as Holt's method, and then trend and seasonality corrected methods, which are also known as Winter's method. So, we will see the advanced version of analytics with more complications in our historic data. Now, as you remember, we explained to you that this is how we have a simple case, where no trend and no seasonality is present. But actually, the real data is moving like this in a zigzag manner.

Similarly, the trend is like this: you are having an increasing trend, and increasing also, you remember, it is additive, and it can be this way also, which is ratio or multiplicative. Similarly, there can be seasonality like this. Now, in the seasonality also, there may be fluctuations. How can you see that? Like here, the fluctuations are like this.

Similarly, seasonality can be like this. So, it is not a smooth curve which is happening with a smooth increase and decrease at a particular interval; rather, it is again a zigzag curve which has a lot of variations from one period to another. The overall impact can be seen as demand touching a peak at a particular period and demand touching a low level in another period. But, if I see from period to period, it is not a smooth movement; there are a lot of fluctuations. So, the correction of these fluctuations is the Holt model, and the correction of these plus these is our Winter model.

That is what we are going to discuss in this particular class, as we have already discussed in our previous session that we have a case of simple exponential smoothing method where we are calculating the new base St using alpha dt plus 1 minus alpha St minus 1.

$$\mathbf{S_t = \alpha D_t + (1-\alpha)S_{t-1}}$$

And here, this St is basically my forecast for the next period, that is Ft 1 is St. This is what my already discussed phenomenon is: whatever is the base of the current period becomes the forecast for the next period. Now, moving further, you have trend-corrected exponential smoothing. Trend-corrected means you have two important characteristics of your data—level and trend—and now you have to smooth level also using alpha, and trend also using beta.

So, in this model, we are going to use two smoothing constants, alpha and beta, for smoothing the fluctuations of level and trend, respectively. Now, here in this type of case, let us first understand how our forecast will be determined. We will actually move in this way, and in the current period, I am here. So, I will calculate the value of St and the value of Tt for the current period, and this St plus Tt is the forecast for the next period. Here, for simplification, I am writing SSL because L is level and T stands for trend.

So, the current level and current trend, the sum of them is my forecast for the next period. So, to make it more clear, the forecast for the month of July is basically the base of June plus the trend of June. That is the formula you are getting, and then you can repeat the same formula for all the subsequent periods.

$$\mathbf{F_{t+1} = L_t + T_t \text{ and } F_{t+n} = L_t + nT_t}$$

So, as we are going ahead now, when I am in July, when I reach July, I will calculate the base for July and the trend for July, and that will give me the forecast for August. But sometimes, when I am in June only, I want to know the forecast for August also. I am not in July, but I want to forecast for August now.

How can I do that? So, using this information, I will consider that the trend will remain constant. I will consider, I will assume that the trend will remain constant, and that will give me the forecast for August also as L June, and I will multiply because August is two periods ahead of June, July, and then August. So, I will multiply 2 into the trend of June, that will give me the forecast for August, and using this knowledge. Now, I can forecast for September, August also, L June plus 3 into T of June.

However, however, I would like to caution you that F August, F September are not very accurate forecasts. As I am close to July, I have a better forecast for July. Now, comes the role of analytics, predictive analytics. As you are coming closer to August, you will update your L June, you will update your T June, L June will become L July, T June will become T July, and then you will have a better forecast for the month of August. As you further go, you will become closer to September, and then again you will update your base.

You will update your trend and you will have a better forecast for the month of September also. So, in this way, if I develop initial estimations for August, September, October, all these are the initial estimations, and as I am going closer to these periods, my initial estimations can be continuously improved. So, you may start a rough planning with these initial estimations, and as you are coming closer to these periods because you have more data, new data, updated data, it will help you in improving your forecast as you are moving closer. So, here we need to regularly update L. Regularly update L, that is your base, and you have to regularly update T, that is your trend. So, you have to regularly update your base and trend, and that will give you your new forecast. Now, for calculating the updated base and updated trend, you will use this basic equation which we have discussed when we were discussing the simple exponential smoothing method: alpha Dt plus 1 minus alpha St minus 1:

$$\mathbf{S_t = \alpha D_t + (1\text{-}\alpha)S_{t\text{-}1}}$$

and I can make a slight modification in this equation.

You can say that your parameter, let us say St, is alpha Dt plus 1 minus alpha St minus 1. Now, you can generalize this equation by saying that it is alpha A plus 1 minus alpha B:

$$\mathbf{S_t = \alpha A + (1\text{-}\alpha)B}$$

and you can use it for calculating St, Tt, and when we will be reaching Winter's model, we can also use seasonality calculation using this equation. So, this becomes a generic equation for updating my base, trend, etc. Right now, we are talking of Holt's model. So, I am only limiting myself to base and trend.

And let me again caution you all that in this class, I am interchangeably using S and L. Both are meaning base value only. So, now, our updated base Lt1 is alpha D plus 1 alpha Dt plus 1 plus 1 minus alpha Lt plus Tt.

$$\mathbf{L_{t+1} = \alpha D_t{+}1 + (1\text{-}\alpha)(L_t + T_t)}$$

Now, some of you may ask why it is Lt plus Tt. So, you see that in this formula, this B is basically when I am calculating Lt, it is alpha Dt plus 1 minus alpha; this is the previous forecast. St minus 1 was the previous forecast.

So, that is the previous forecast Lt plus Tt for the current period, and similarly, the updated trend T is beta multiplied by the current trend plus 1 minus beta multiplied by the trend for the existing period.

$$\mathbf{T_{t+1} = \beta(L_{t+1} - L_t) + (1\text{-}\ \beta)T_t}$$

So, you will get Lt plus 1 and Tt plus 1 using these two equations. These are important, and as we already know, the values of alpha and beta always vary between 0 and 1, where the common values mostly preferred are between 0.1 and 0.3 for both alpha and beta. Theoretical values range between 0 and 1. So, this is our trend-corrected exponential smoothing model, and now let us see one illustration, one example where we have data for some past 6 periods, which is given to us in this table.

And you can note down the data from this narration, which shows how it is varying from 34 lakhs to 61 lakhs from 2019 to 2024. Here, the data has level and trend also, and the alpha and beta values are given to us as 0.1 and 0.2. Now, in this case, we have to see what the forecast for 2025 is. Now, the forecast for 2025, let us say that these are period 1, 2, 3, 4, 5, 6, and 25 will be 7. So, we want F7, and F7 will be, as we have already discussed, equal to L6 plus T6. F7 is L6 plus T6.

Now, to start the solution, first we need to know L naught and T naught. What is the initialization of this particular process? So, as we discussed in our last class, we can initialize by taking the average of all the L values. So, you can say that, and that is also possible, that we can make a kind of regression equation between these initial estimations, let us say, in this way. You can develop a regression equation between the periods and the corresponding demand in those periods, and with that, the intercept on the y-axis is L naught, and the slope of this regression line is your T naught.

So, from our Excel calculations directly, we can get the values of L naught and T naught, where L naught is coming to 26 lakhs. 4842, while T naught is coming to 5,48,247, and then this is a typo here for period number 2019. Now, the forecast will be simply F1, that is L naught plus T naught. So, this is our, and you can develop in an iterative manner that demand for this period was 34,17,774, as you see here. The forecast came to be 31,53,809. So, there is a forecasting error also, that is very obvious.

Now, you need to update L02, L1, T02, T1, and that will give you F2 = L1 plus T2. Once you have L1 T1, then you will calculate L2 T2, and in this way, in a sequential manner, you will reach the calculation of L6 and T6. We can develop a very good Excel template for showing you how these calculations can be done in a very iterative, simple manner. So, now using the values of alpha and beta given in this problem, you will calculate the value of L1 using this formula: alpha D1 plus 1 minus alpha F1, and that gives you the value of L1.

$$\mathbf{L_1 = \alpha D_1 + (1-\alpha)(L_0 + T_0)}$$

And similarly, you can calculate the value of T1, which is coming to be 5,53,541.

$$T_1 = \beta(L_1 - L_0) + (1- \beta)T_0$$

So, therefore, your F2 is L1 plus T1, which is coming to be 37,33,099. And in this manner, if we are continuing, now you can understand F1 is L0 T0, F2 is L1 T1, F3 is L2 T2, F4 is L3 T3, and going up to F7 is L6 T6. And with this, finally, our forecast for 2025 is 64,39,353. So, in this way, in this iterative manner, we have done the calculations where we saw how the calculations are moving in a simple movement. Now, the important thing here is whether we have considered the correct values of alpha beta. The values of alpha beta, whether these values are correct.

So, for every period when you are calculating these things, you also have the data of D1, D2 up to D6 because F7 has not yet happened. So, up to 6 periods of data are available to you, and forecast data is also available. Now, you can calculate E1 to E6, and when you are calculating E1 to E6, you can also see the average error MAD, and then by calculating the MAD, you will see what the movement of alpha beta is. Because you can do this experiment with different combinations of alpha beta, and those different combinations of alpha beta will help you determine

whether we are using the right values or the right set of values of alpha beta. So, this is one possible case of our predictive analytics where we are able to predict and determine whether we are predicting correctly or not. Moving further, there is another case where we have a trend and seasonality, and as we now know, in this case, there will be three situations: one is the level, which is already fluctuating; the trend is also there, and seasonal variations are also present. So, in this case, in fact, the situation becomes like this: you have

just to give me some kind of a base, let me have now the actual data like this. So, now you can see that though it is the increasing direction of the data, but when you see these peaks, when you see these peaks here, there is an x amount of difference; here, there is no difference in these two peaks, and here it may be an x square difference. So, even the seasonal variation is changing. And this seasonal variation is changing; there is a change in the trend also, and there is a change in the level also.

So, you need three types of smoothing constants: alpha, beta, and gamma. These cases are known as Winters' model, and as we discussed in our last class, the seasonal factor can also have two types of characteristics: either additive or multiplicative. You can simply say that demand is increasing from one season to another season by, let us say, 100 units, or you can say that demand is increasing from one season to another season at the rate of 15 percent or 20 percent. So, these two are the different types of cases. So, depending upon the type of cases, you may remember that we had two types of situations where the forecast can be level plus trend multiplied by the seasonal factor, or the forecast is level plus trend plus the seasonal factor.

So, it depends upon what type of data is there. And let me also tell you that just by seeing the data, for example, there is a set of data available on the slide, and this data is available for about 12 different periods. For now, when you see this demand data, it is almost impossible for anybody to tell whether this data has additive seasonality or multiplicative seasonality. Here again, the role of predictive analytics comes into play to understand the characteristics of the data.

That is, what type of characteristics are there, and accordingly, we will handle this data with an appropriate model, an appropriate decision model, so that you are getting more accurate predictions for your future forecast. So, with this, we would like to stop here, and we will take this discussion to our next session, where we will see how these Winter models are discussed and, with the help of a practical case, we will like to show you, using the Excel tables, how the calculations proceed in a systematic manner. With this, we come to the end of this particular session. Thank you very much.