INDIAN INSTITUTE OF TECHNOLOGY DELHI

NPTEL

National Programme on
Technology Enhanced Learning

Video Course on
Stochastic Processes-1

by

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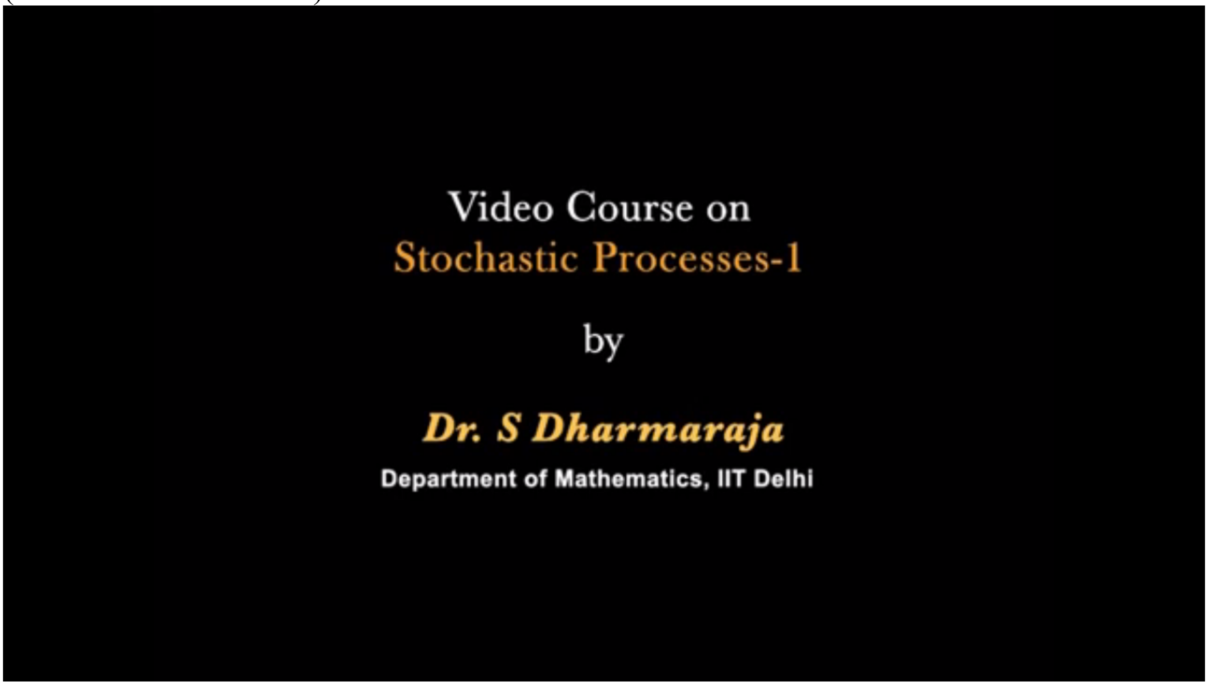

Module 1: Probability Theory Refresher

Lecture # 1
Introduction to Stochastic Processes

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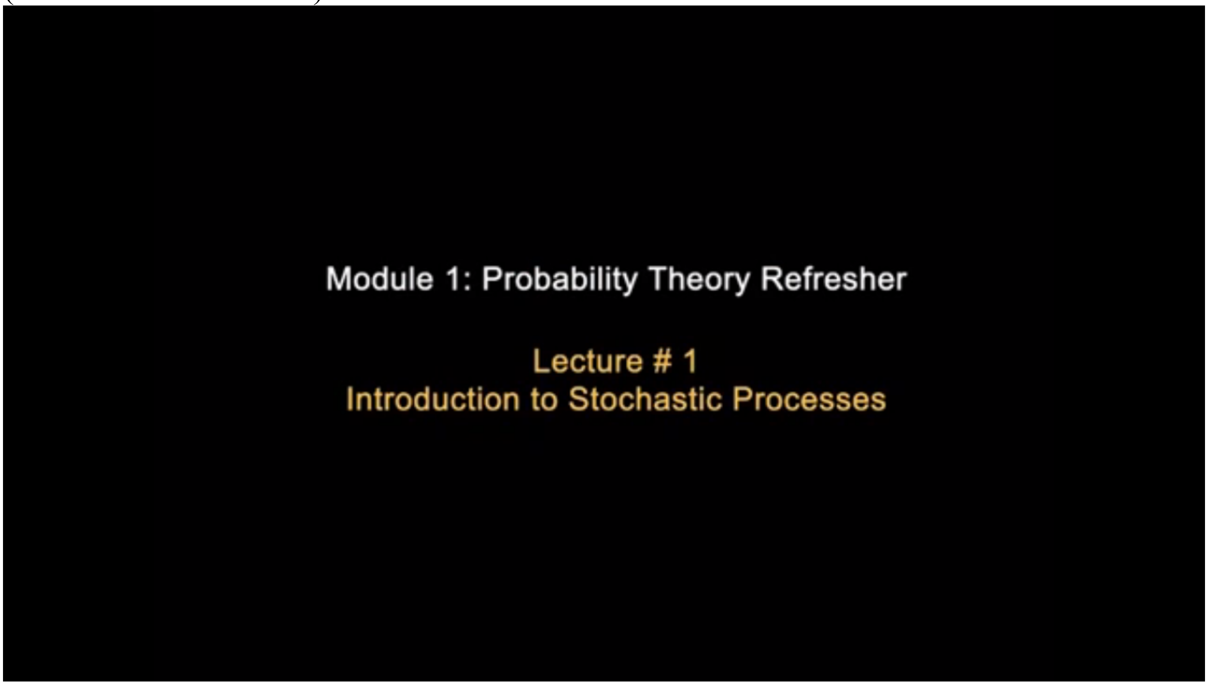

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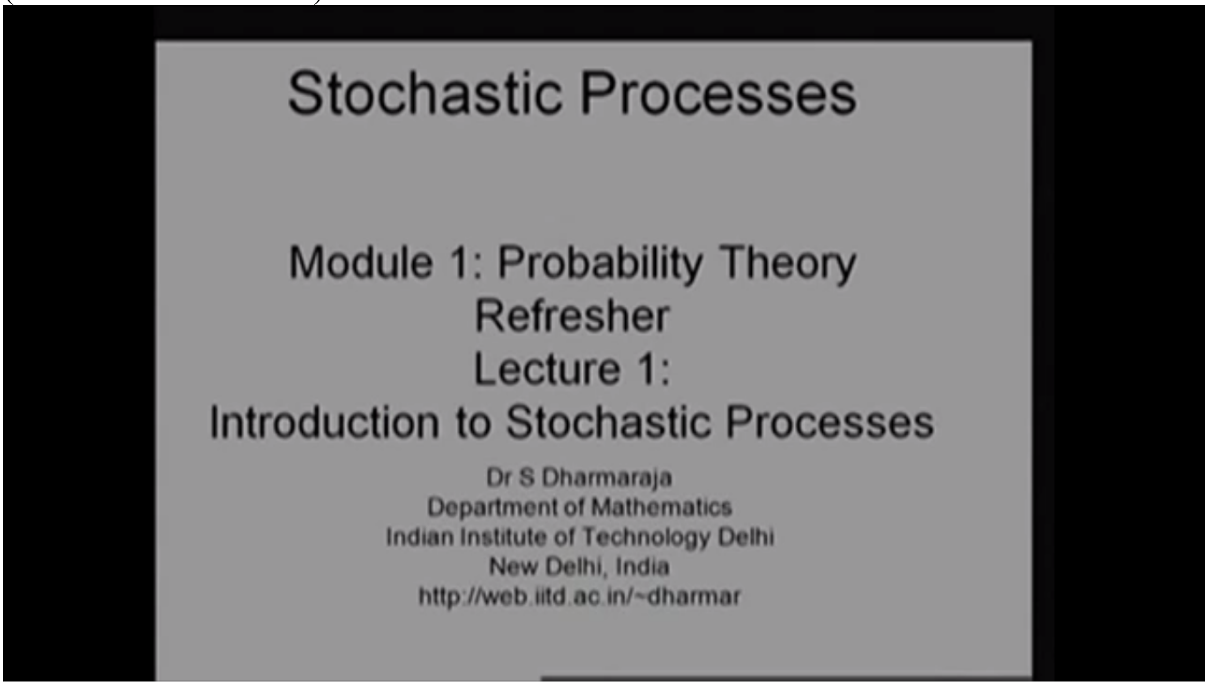

Our lecture is a stochastic processes.

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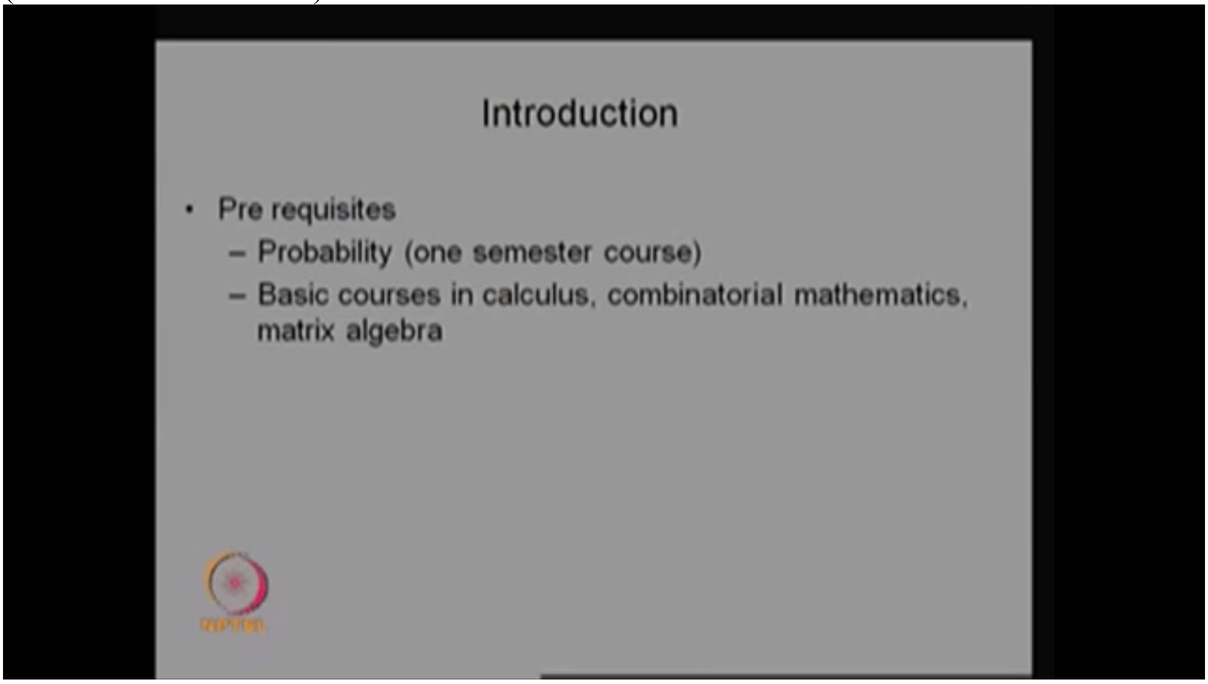

This course needs a prerequisite of a probability as a full one semester course. So most of the universities they have a course of probability theory along with the stochastic processes or random process or probability and statistics. So whatever the courses we have, at least some 30 lectures of probability theory is needed for this stochastic process course as a prerequisite.

Other than probability course, we need a basic course in calculus and some mathematical background over the combinatorial problems and also the matrix algebra.

So these courses would have been covered in the Maths 1 or Mathematics 2 courses. So that is enough for to understand the stochastic process course.

So what we are saying is we need a prerequisite as the probability theory as well as the Maths, Mathematics 1 and Mathematics 2 course is enough to do the or to understand the stochastic process course.

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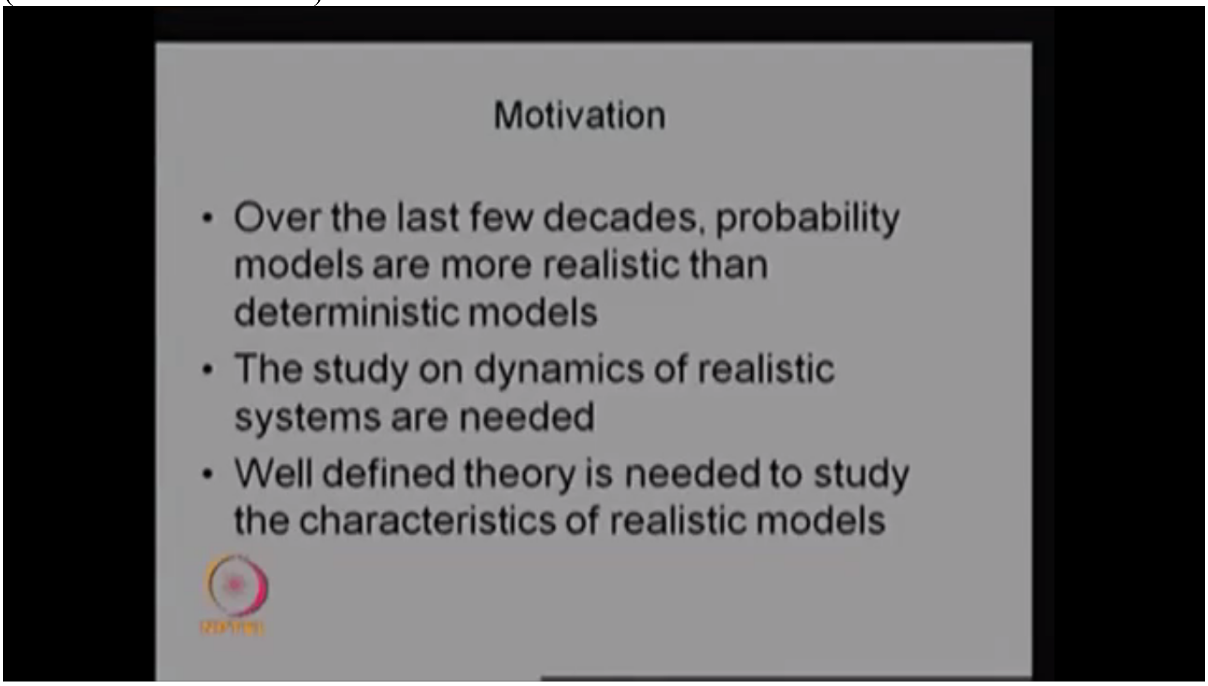

So before we move into the stochastic process, I am going to give what is the motivation behind a stochastic process. When we see the last few decades' problems, more of the probability models are not the deterministic. That means you need more probability theory to understand the stochastic, to understand the system. Then only you can study the dynamics of the model.

If you see the -- if you want to study the dynamics of the system, then you need a more probability theory. So the simple probability theory may not be enough to study the more, more study on the realistic system. The way this realistic system behaves in a very dynamical way, it is not easy to capture everything through the probabilistic or usual probability models. That means that you need more than the probability model -- probability theory to understand the system or to study the system in a well-behaved way.

For that the one of the important thing is a stochastic process. It deals about the collection of a random variable so that you can study the dynamics of the system in a better way. Even though I am giving very light way of saying the collection of random variable, first we should know how the random variable can be defined so that you can study the collection of random variable in a better way.

So for that we are going to spend few examples through that how the more realistic models needs more probability theory other than the usual probability theory so that the stochastic process definition and those things I am going to cover it later part.

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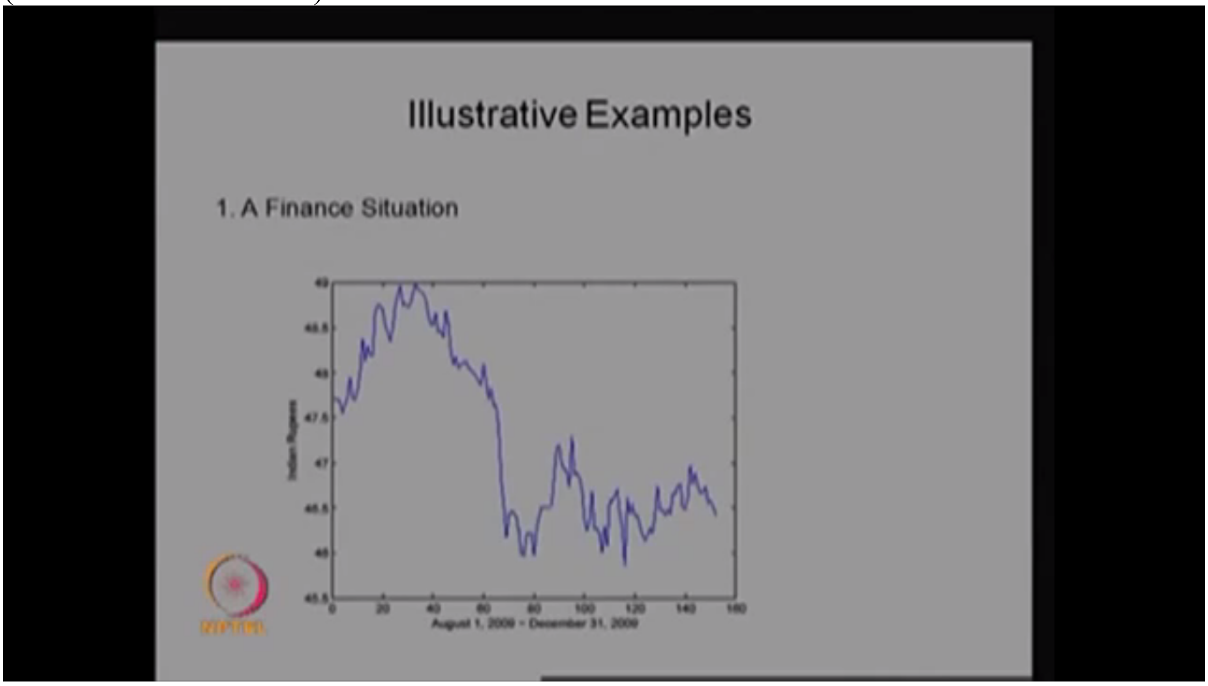

First let us see the first example that comes in the finance situation. This is the actual data which captured over the period of time from August 1, 2009 to December 31st 2009 of what is the current price of the 1 U.S. dollars in Indian rupees.

So if you see the graph, you can make out August 1st 2009 the price of 1 U.S. dollar was 47 rupees 57 or 58 paisa. And if you see the dynamics over the years, over the days from August 1st 2009 to December 31st 2009, it keep on changing and it takes some values higher and after that it goes down and it fluctuates and so on.

So this is the actual data which captured from the -- which we have captured and from that our interest will be what could be the US dollar price after some time. If I know till today what is the price, my interest will be what could be the price after one or two days or after one month or after six months? That means I should know how the dynamics keep moving over the days and what is the hidden probabilistic distribution is capturing over the time so that I can identify what is the distribution behind that.

Therefore, I can study the future prediction. I can study the dynamics of the -- this particular model in a much better way. That means I need what is the background or what is the hidden distribution playing or hidden distribution, which causes the dynamics of the system.

After identifying what is the distribution, my interest could be what could be the some other moment over that time. That means what could be the average value or what could be the second order moment if it exists and so on that can be obtained if I know the actual distribution in the underlying model.

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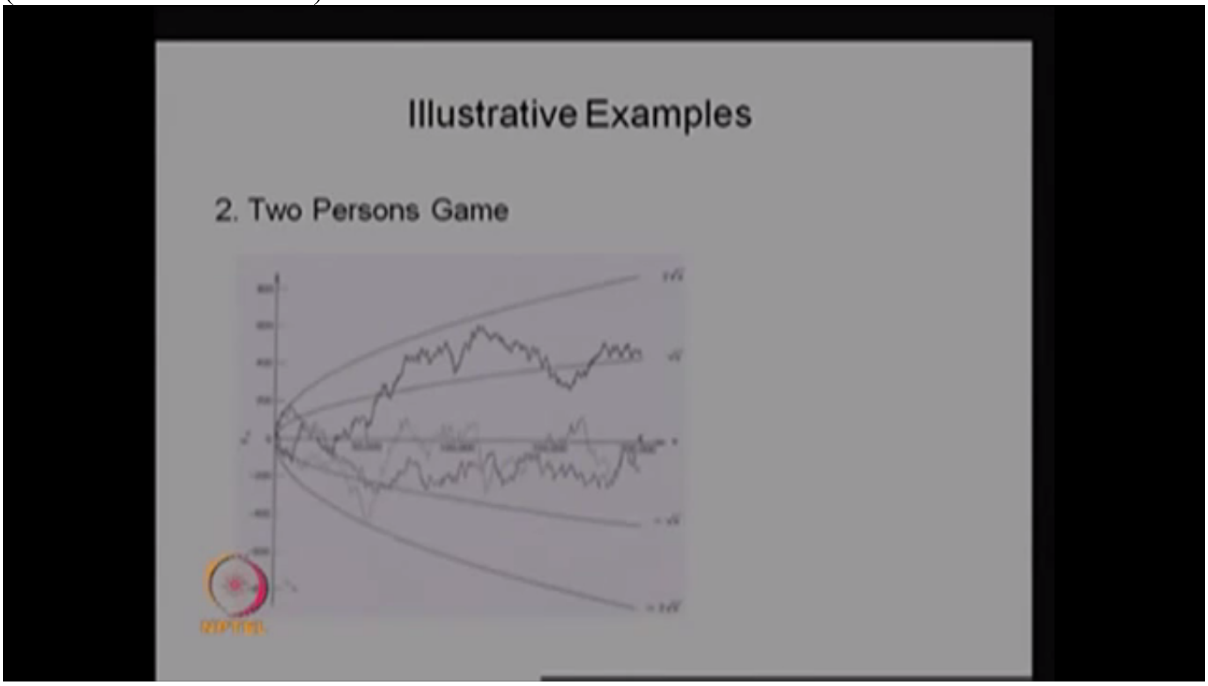

If I see the second example, I am just changing into the another model in which there are two people playing a game, the person A and person B. Whenever the person A wins, he gets the rupees 1. Suppose the person B wins, then he will get the 1 rupee and at the same time, the person A loses 1 rupee the same way and the play is keep going.

Suppose you make the random variable as X_n is the amount of the person A has at the end of n^{th} game. If you make out the random variable X_n for the person A has the amount at the end of the n^{th} game, then at the way the game going on, the value of the X_n will be keep changing and if you make out the another random variable X_n is the sum of X_i where i is running from 1 to n. This gives what is the total amount, the total amount of the person A has.

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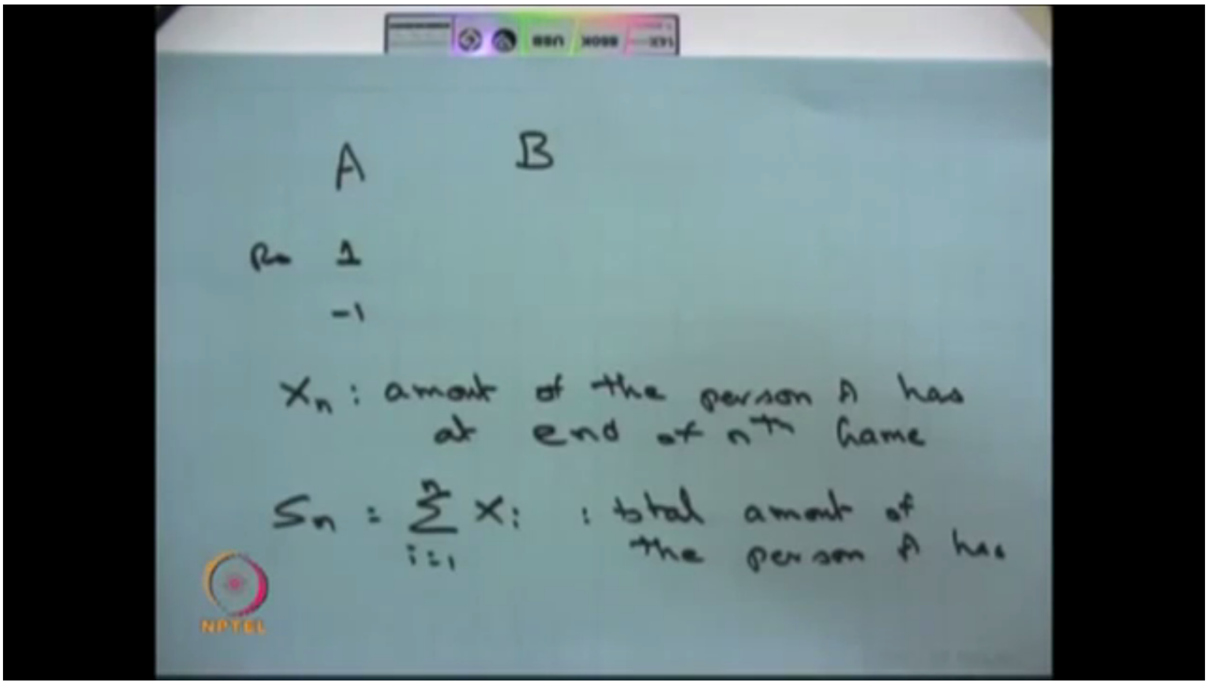

The diagram in which the S_n gives what is the way the dynamics goes and over the n. And if you see the diagram, you can make out the whole dynamics goes -- the -- how the game is going on in the first few games. Accordingly, it changes the positive side or it goes to the negative side and if the n is goes large, then the dynamics of the S_n over the n will be keep changing over that time and you will get the realization of the S_n over the time.

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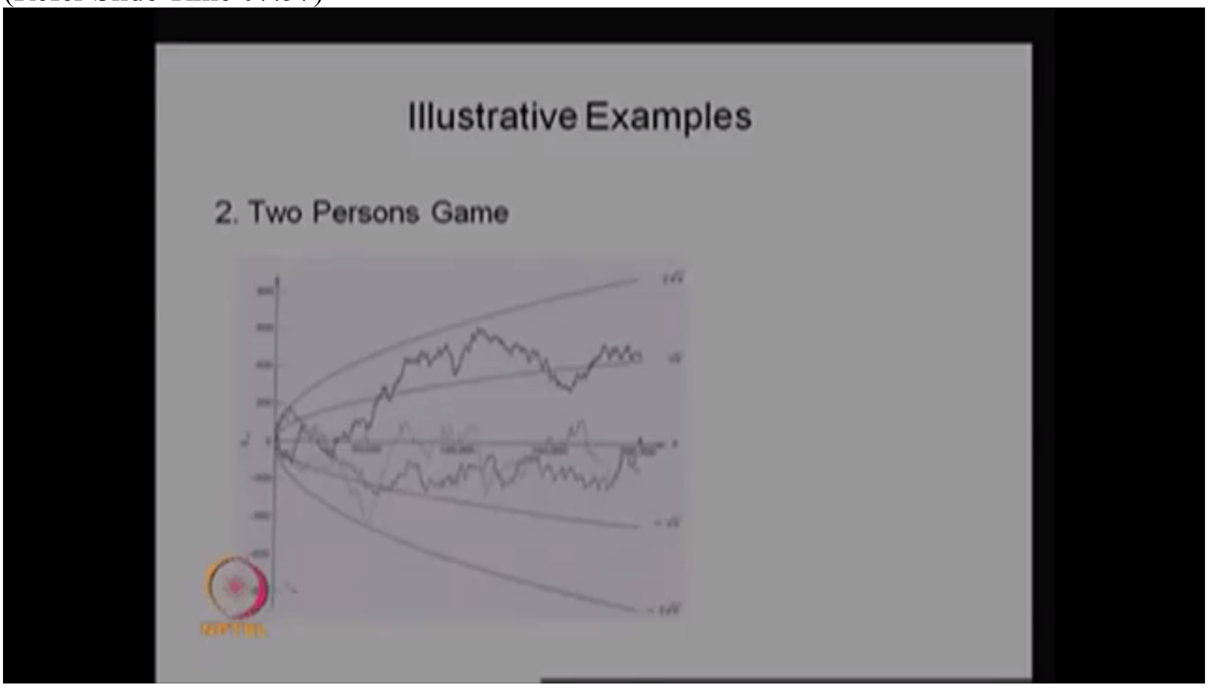

And here I have given three different realization and this diagram is taken out from the book by U.N. Bhat. The title of the book is Elements of Applied a Stochastic Process. So this is one of the motivations behind the stochastic process and from these our interest will be after the -- what is the distribution of S_n at any n? And also as n tends to infinity, what could be the distribution of S_n? That means you need -- you need the distribution of the random variable and also you need what could be the distribution as n tends to infinity or the limiting distribution of S_n? If you know the distribution, then you can get all other movements as for different n as well as the asymptotic behaviour of the random variable S_n.

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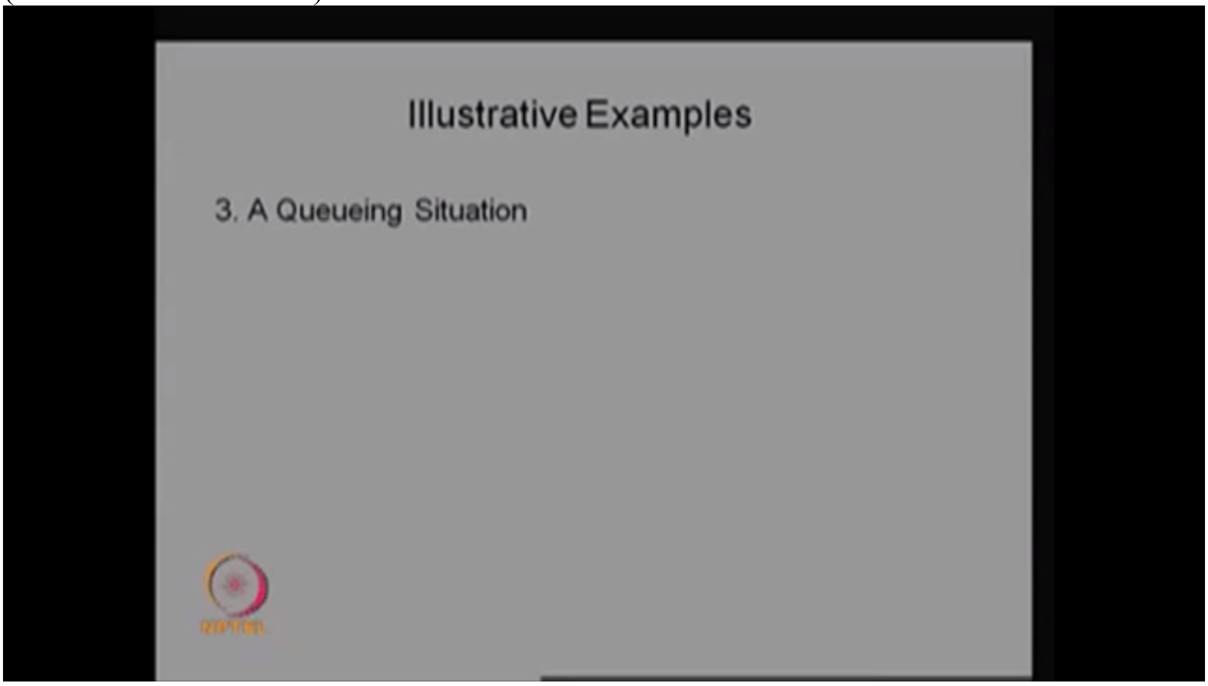

Next I will move into the another example in which it is a queueing situation. The queueing situation here I have taken it as a -- taken a simple example that is a barbershop example in which there is only one barbershop person and who does the -- who does the service for the people whoever entering into the barbershop and there are only a limiting capacity in which there is a maximum 10 people can stay in the barbershop and one person will be under service. Once the service is over and the system will be the customer can leave the system. At any time, maximum 10 people can be in the barbershop and only one person is doing the service for the customers whoever enter into the barbershop.

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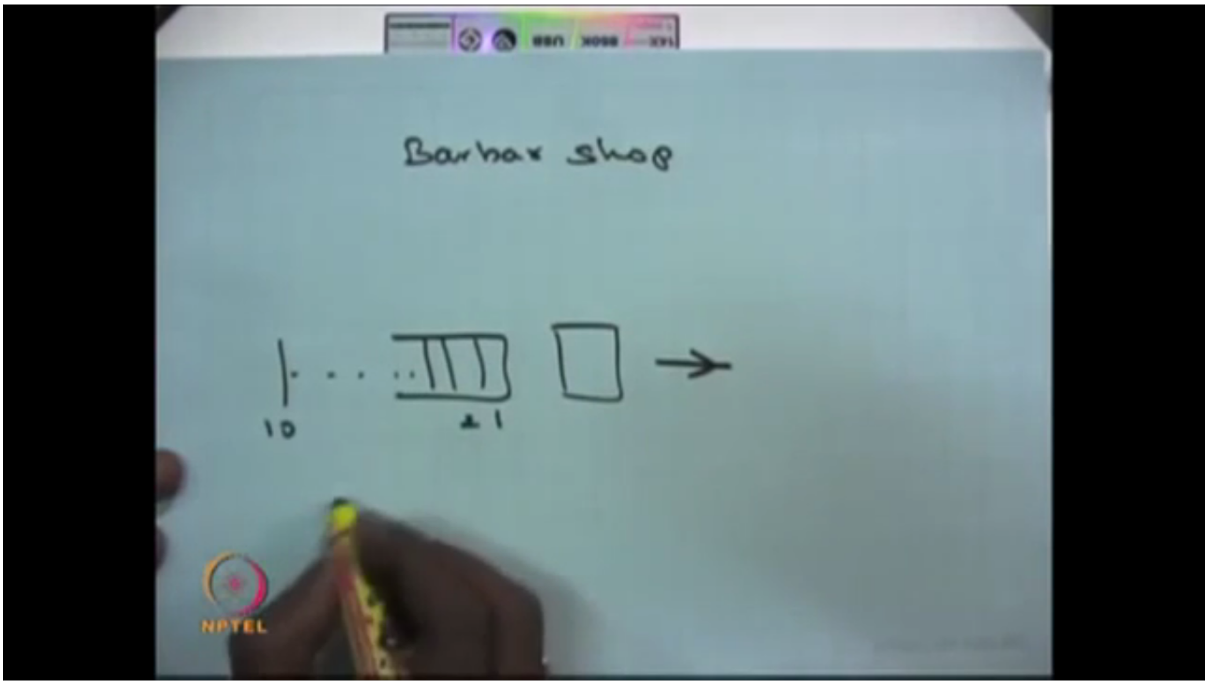

Suppose you take the random variable as X(t) is the number of customers in the barbershop or in the system at time t. The way the dynamics goes, the possible values of X(t) will be starting from 0 to n.

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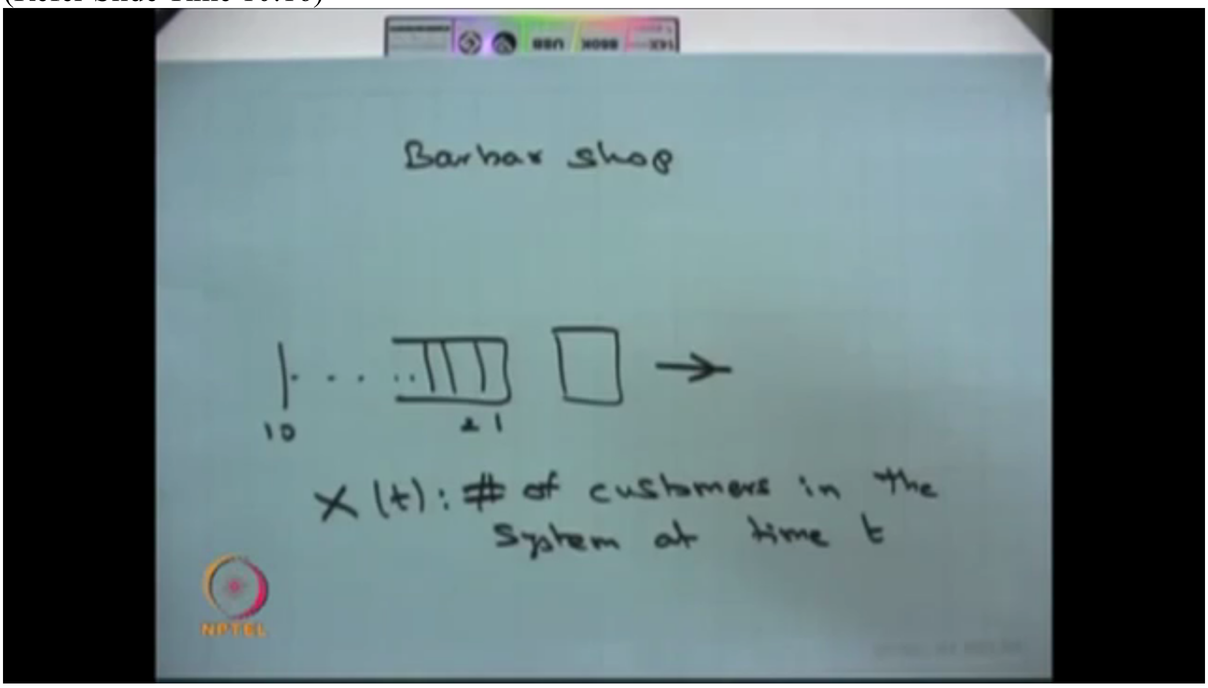

To study this system, you need what is the way the people or the customers are entering into the system and what is the way the service is going on for the customers and what is the discipline in which the customers are getting served also?

Our interest will be suppose we have the capacity of 10, what could be the waiting time whenever the customers are entering into the system? My interest will be one is how to reduce the waiting time on average. In the customers -- this is the customer's point of view.

As the barbershop point of view, how much I can get the more revenue? That means how I can increase the capacity, capacity of the system so that I can make more profit over that time? That means if I know the dynamics of the X(t) over the t for t is varying from zero to infinity, I can -- I can understand the system over that time as well as I can whatever the probabilistic measures or whatever the other measures, average number of customers or average waiting time and so on, I can find out using this type of random variable.

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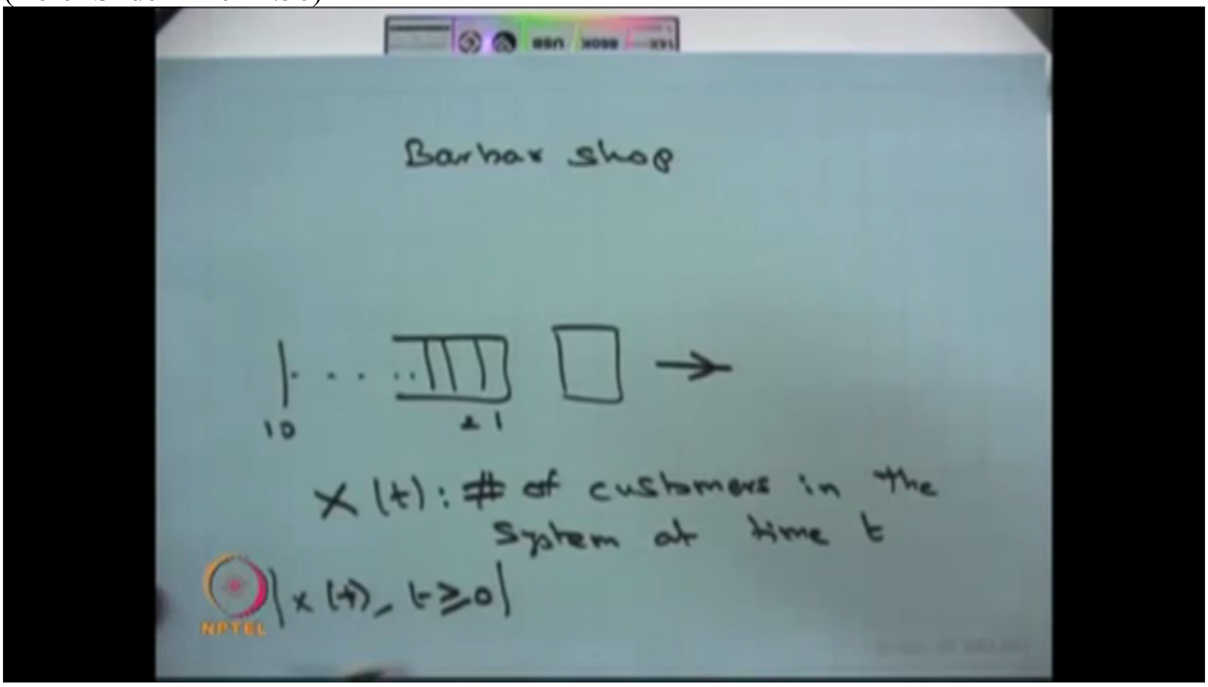

So later we are going to say this is going to be a one of the stochastic process for this example.

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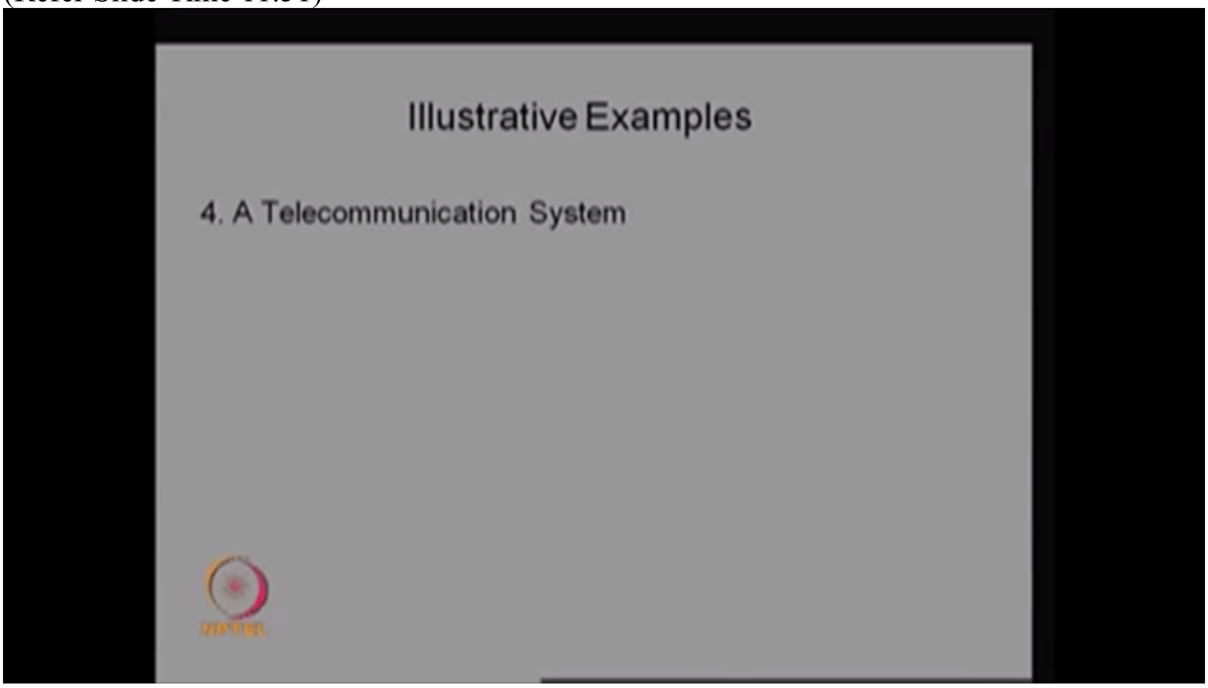

Next I am going to consider the fourth example as the telecommunication system. Suppose you think of a system in which you have n trunks are there. Trunks are nothing but it's a maximum number of calls will be allowed at any time. Whenever a call entering into the system and you have given one trunk to the call and at the end of the call is over, the trunk will be back. So you have a telecommunication system in which n trunks are available at any -- not at any time; n trunks are available.

Suppose I make a random variable X(t) as the number of calls ongoing at time t, here also the dynamics of X(t) is going to be keep changing from 0 to small n over the time and my interest will be how I can -- how I can do the service such a way that the more calls will be entertained as well as how I can find out the optimal n such a way that what is the optimal -- optimal number of trunks such that I can minimize the waiting time or I can maximize the revenue?

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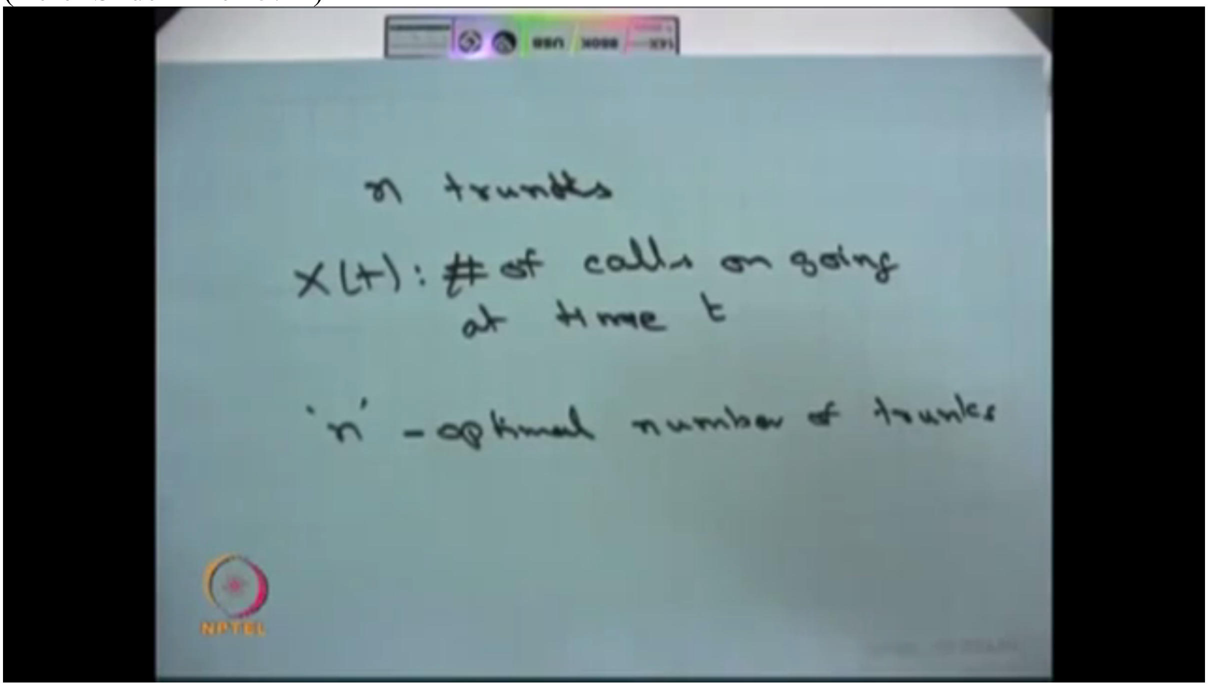

So this is also one of the problems which we come across in the usual -- daily life and so on. So my interest is to introduce the stochastic process so that I can study this type of system in a better way.