Video Course on

Stochastic Processes

by

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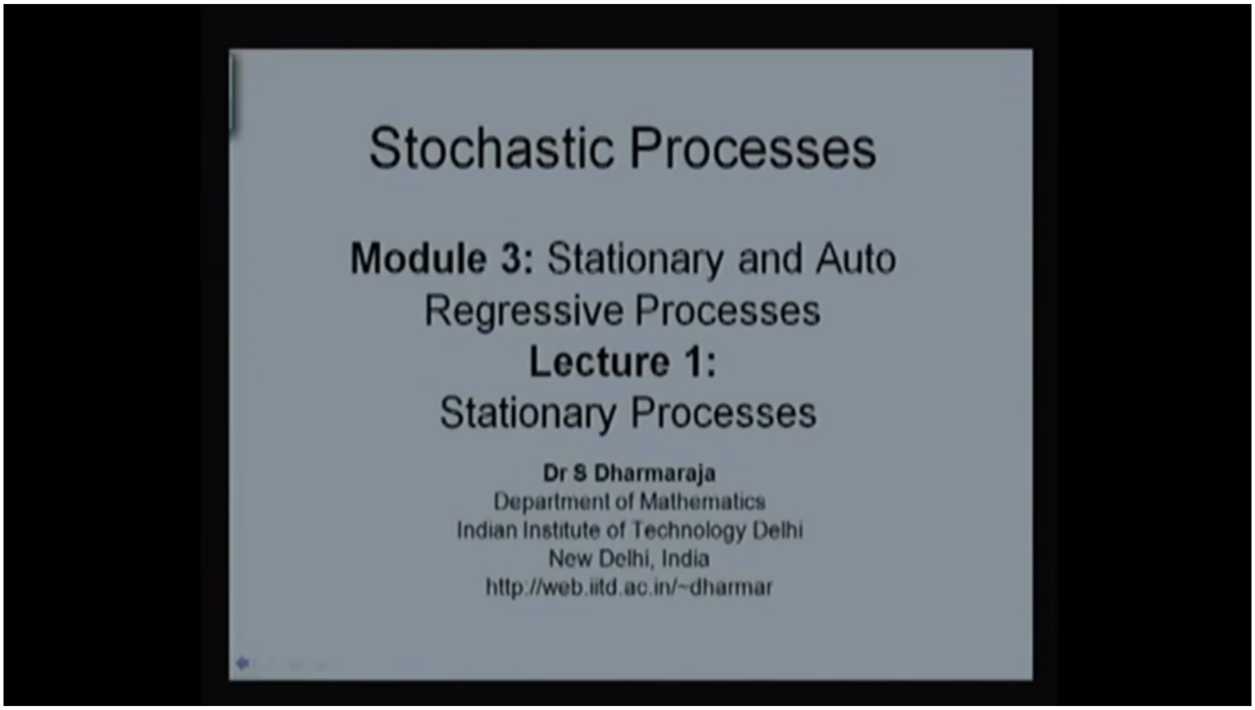
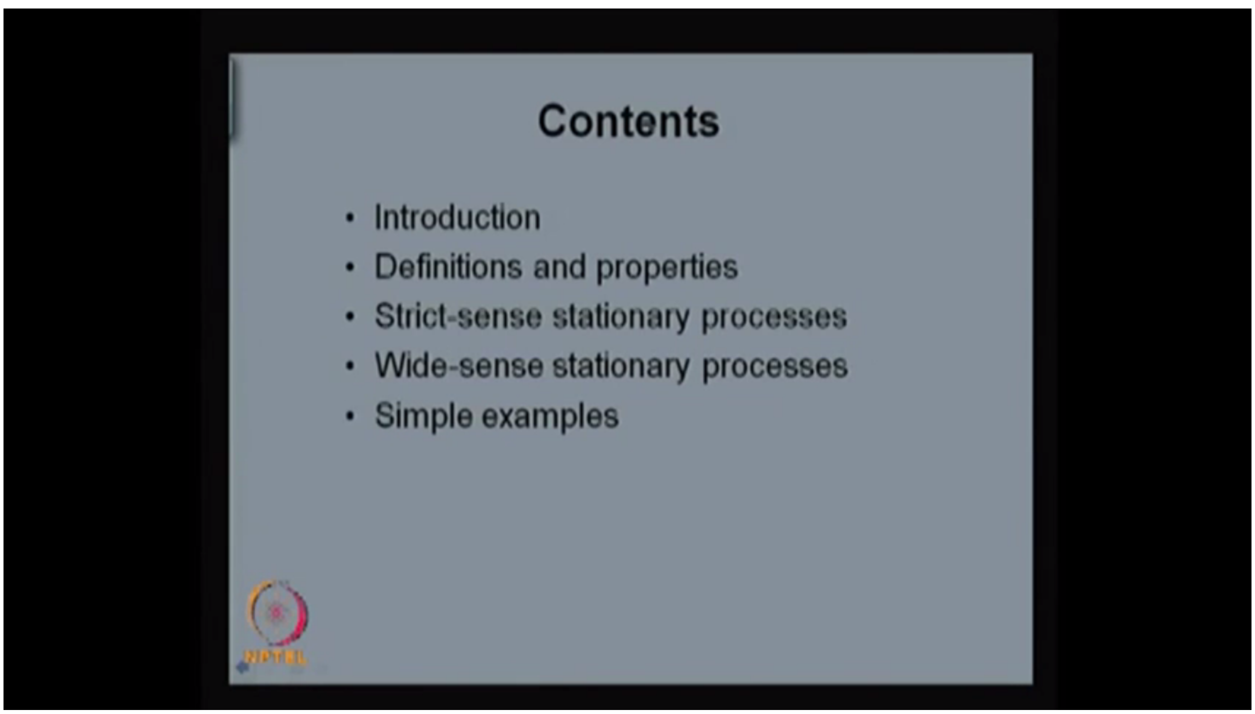
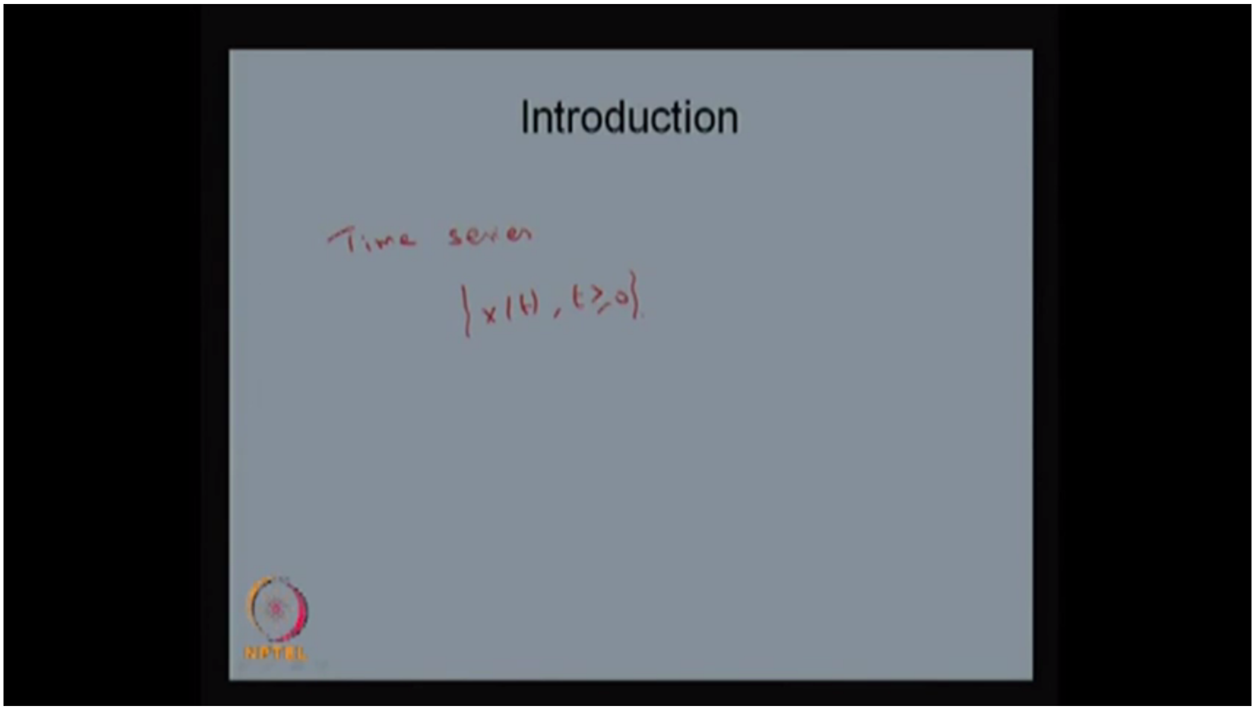
Module # 3
Stationary and Auto Regressive Processes

Lecture # 1
Stationary Processes





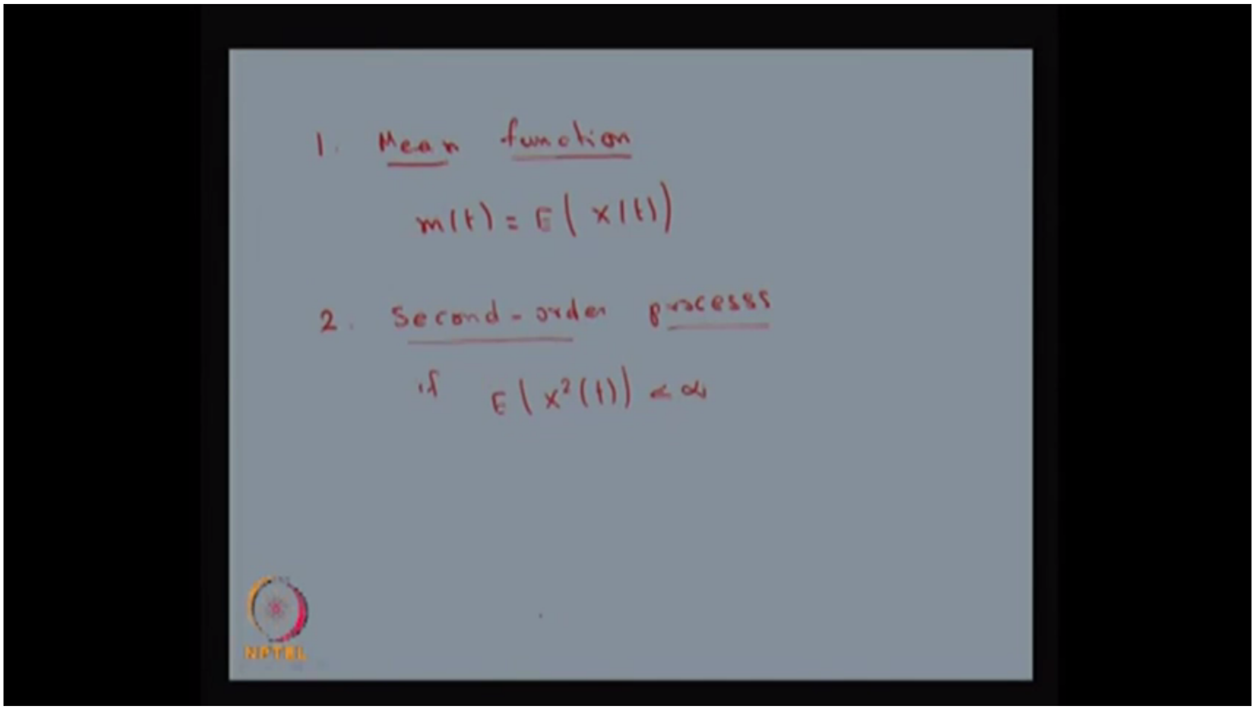
This is the core stochastic process and in this we are starting model 3 that is stationary and auto regressive process. This is a lecture 1 on the stationary process and I will be talking the auto regressive process in the lecture 2. In this talk I am going to cover the introduction of the stationary process and few definitions and the properties of the stationary process. Then there is a two important stationary processes. One is the strict sense stationary process. The second one is the wide sense stationary process. After this I'm going to give few simple examples of stationary process.



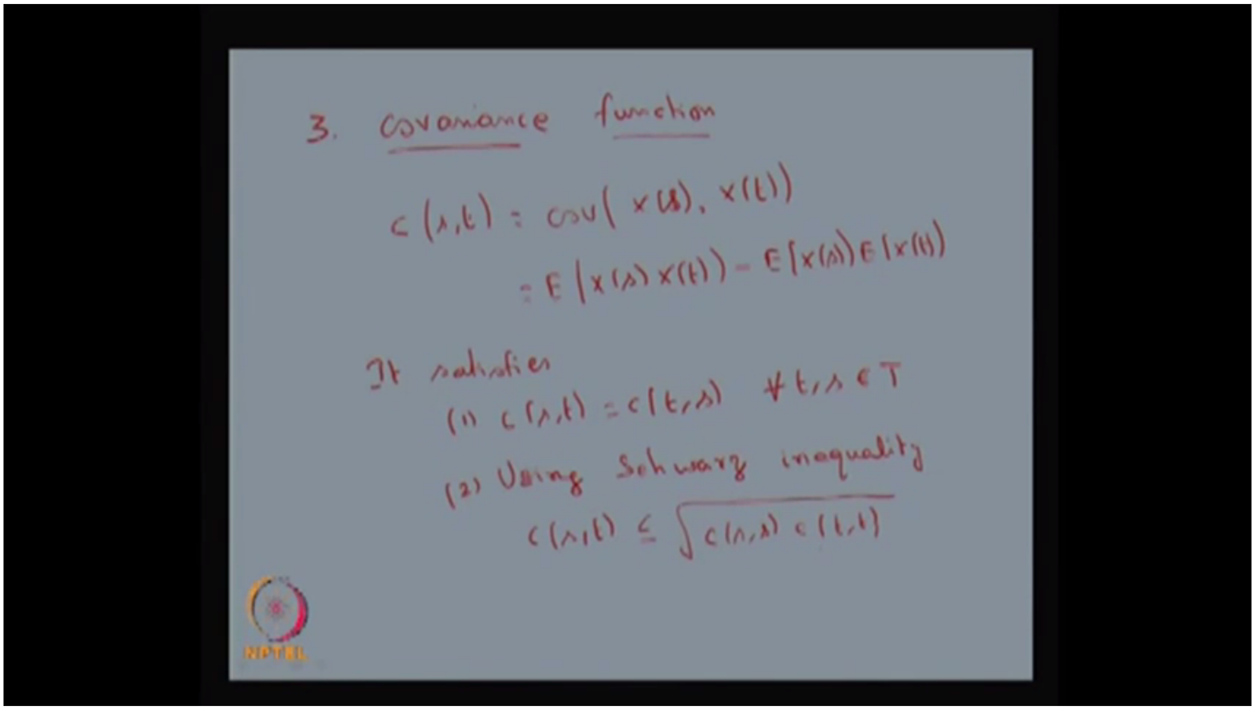
Introduction: A stationary process is a stochastic process whose [Indiscernible] [00:01:23] laws remain unchanged through shifting times or in space. Stationarity is a key concept in the time series analysis as it allows powerful techniques for modeling and forecasting to be developed. What is the meaning of time series? Time series is a set of data ordered in time usually recorded at regular interval of -- regular time interval. In probability theory a time series if you make out a time series is a collection of random variable index by time. Time series is a special case of stochastic processes. One of the main features of time series is the inter dependency of observation over time. This inter-dependency needs to be accounted in the time series data modeling to improve temporal behavior and forecast of future movement.

So basically the stationary is used as a tool in time series analysis when the raw data are often transformed to become stationary. That means if you collect the raw data and that raw data need not be satisfying the times. It may not satisfy a stationary property but using the stationarity property the time series of that raw data is transformed so that you can model as well as you can forecast for the future moment by using the stationarity property. There are different forms of stationarity depending on which of the statistical properties of the time series are restricted. The most widely used to form of stationarity or strict sense stationarity and mix sense stationarity. So basically before we go to the two types of two important types of stationary property that is weak sense stationary property and strict sense stationarity property we will just see few definitions followed by these two important stationarity property.

The first one is the mean function. Mean function is defined as the with the notation M of t that is nothing but expectation of the random variable X of t. So here the stochastic process is the collection of a random variable X of t over the t belonging to T and you are defining the mean function as the function of t that is expectation of random variable X of t. Sometimes this is going to be a function of t. Sometimes it is going to be independent of t. According to the function of t or independent of t we can classify the stochastic process later. So this definition is going to be very important that is mean function.



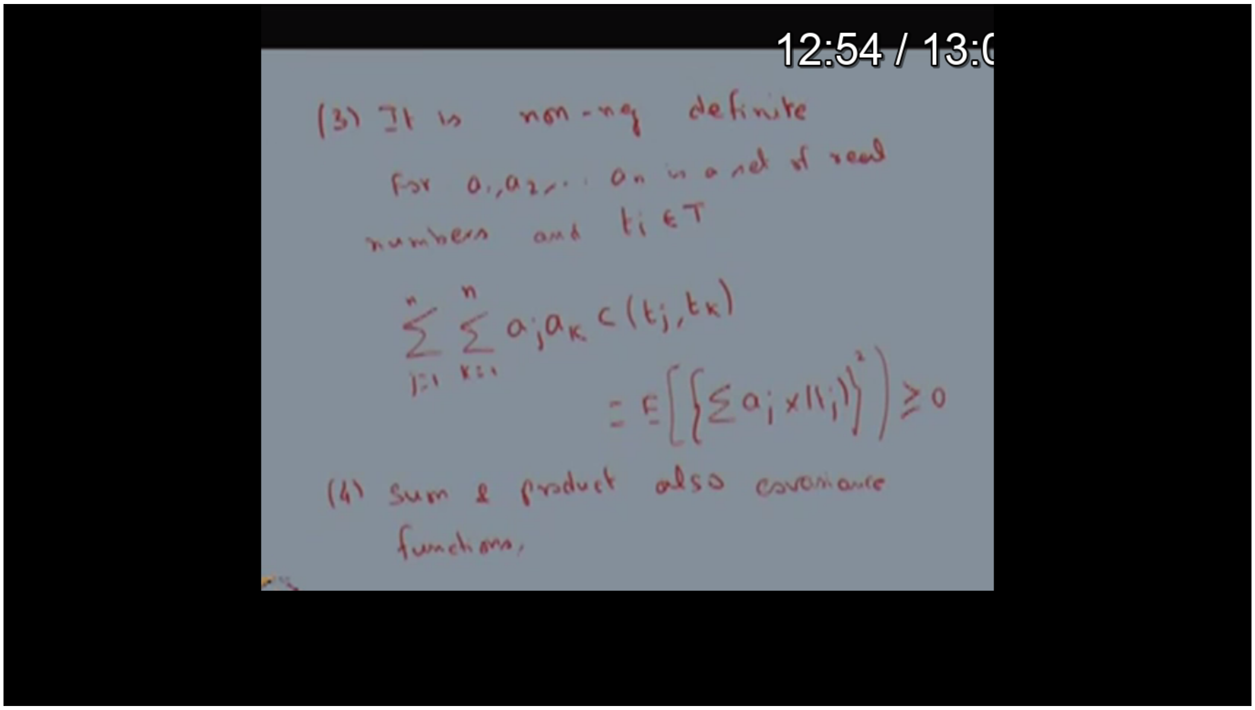
The second one it's a second-order stochastic process. When we see a stochastic process is going to be a second-order stochastic process if it satisfies the condition the second order moment it's going to be finite for all t. If this condition is satisfied that means if a random variables with the finite second order moment then that corresponding stochastic process is called a second-order stochastic process. That means there is a possibility the stochastic process may not satisfy the second order moment may be infinite or it won't exist in that case it is not going to be call it as a second order process. So whenever you collect the random variables from a stochastic process and satisfying the second order moments are going to be finite for all t then we see that stochastic process is going to be a second-order process.



The third definition is a covariance function. How to define the covariance function? Covariance function in notation it is C of s,t that is nothing but covariance of two random variables Xs, Xt. Since it is a collection of random variable so for each t you will have one random variable so that means here you have taken two s and t and you got the corresponding random variable and you are finding the covariance of these two random variables. That is nothing but the expectation of X of s, X of t minus expectation of X of s and the expectation of X of t. Obviously since you are finding the covariance of any two random variable obviously this stochastic process must be a second-order stochastic process so that the second order moments exist and you are able to find out the covariance of this one that means the existence of the second order moment is going to be finite that is assume it to be – that is assumed and therefore you are getting the covariance of these two random variables. So using that you are defining C of s,t that is a covariance. Since it is nothing -- it is expectation of the product minus expectation of the individual one it is going to satisfy the first condition the C of s,t is same as C of t,s for all t,s belonging to T where T is a parameter space. From the parameter space if you take any two t and s then if you find out the covariance function of a s,t is same as t,s.

The second prop property using Schwarz inequality you can have always able to see the upper bound is going to be C of s,s and C of t,t this is going to be exists because the second order moments are finite therefore C of s,s that is nothing but the variance of X of s and this is going to be the variance of X of t and therefore this is nothing but the product of the variance and the square root. So this is going to be a finite quantity. Therefore this has the upper bound of C of s,t has the upper bound the square root of product of variance of X of s and X of t.

The third property it is the covariance matrix non-negative definite also. That means for a1, a2, an that is a set of real numbers and if you take ti's belonging to T and if we find this double summation of j running from 1 to n and the k running from 1 to n aj and ak these are all the real numbers with covariance functions of tj, tk that double summation is nothing but the expectation of summation of aj's x of tj's whole square this expectation quantity is always going to be great or equal to 0 since it is a whole square. So the expectation of whole square quantity is always great or equal to 0 for all the set of all real numbers a1, a2, an and the ti's are belonging to E and this is nothing but the expectation of this quantity that quantity is always going to great or equal to 0 so you can conclude the covariance of function is going to be non-negative.



The fourth property the sum as well as the product of any two covariance functions also covariance functions. The sum and the products also going to be the covariance function. This property needs elaboration. However, we assume these for this course. So this four property is going to be used later whenever you would like to cross check whether the covariance function is going to be satisfied or how to find out the covariance function so these properties will be used.