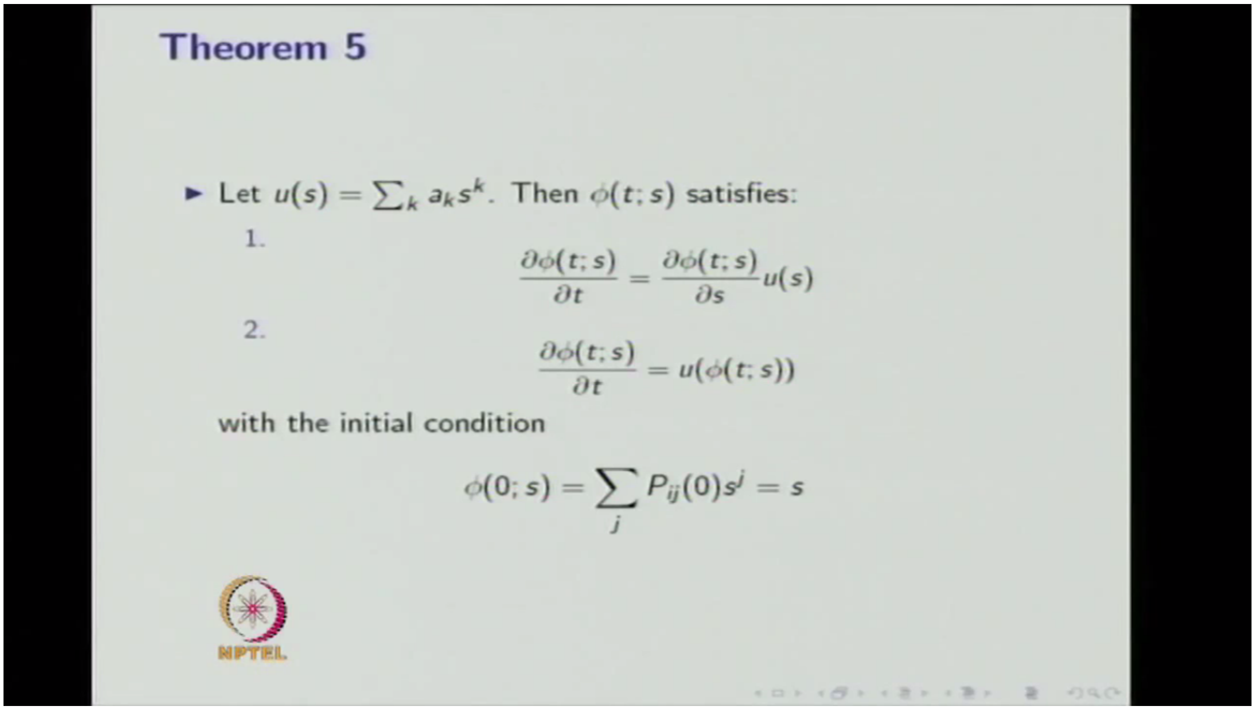
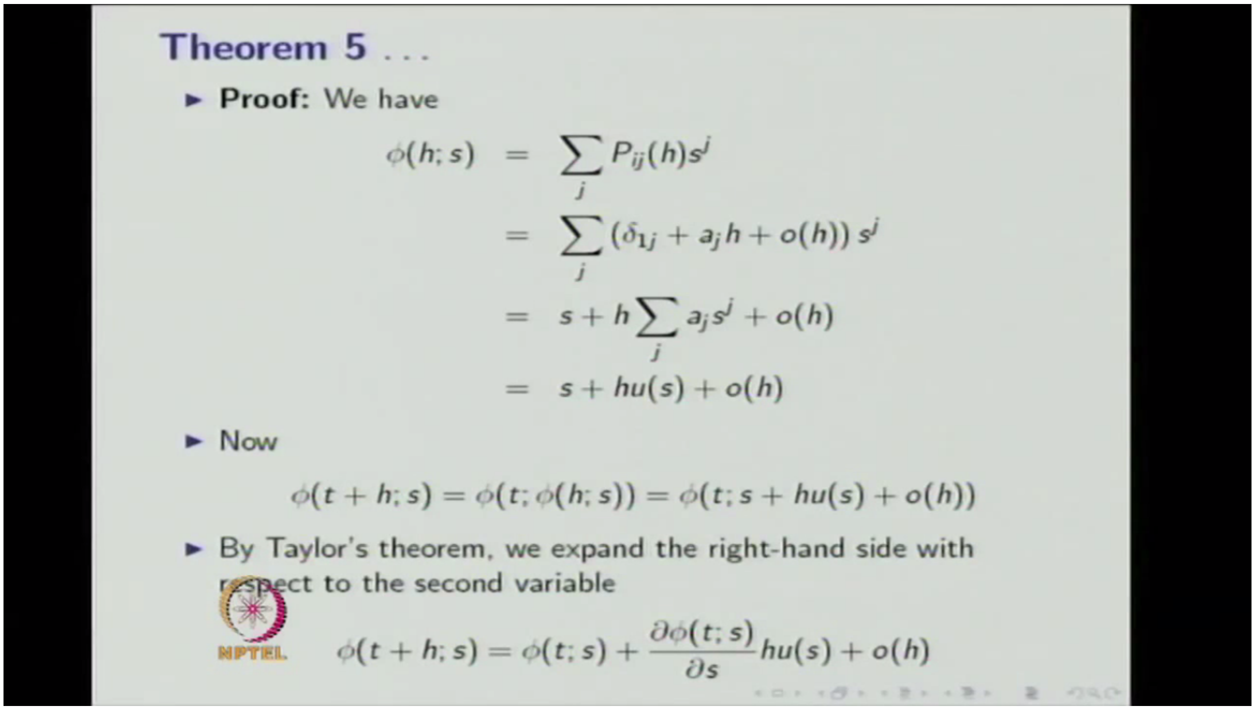
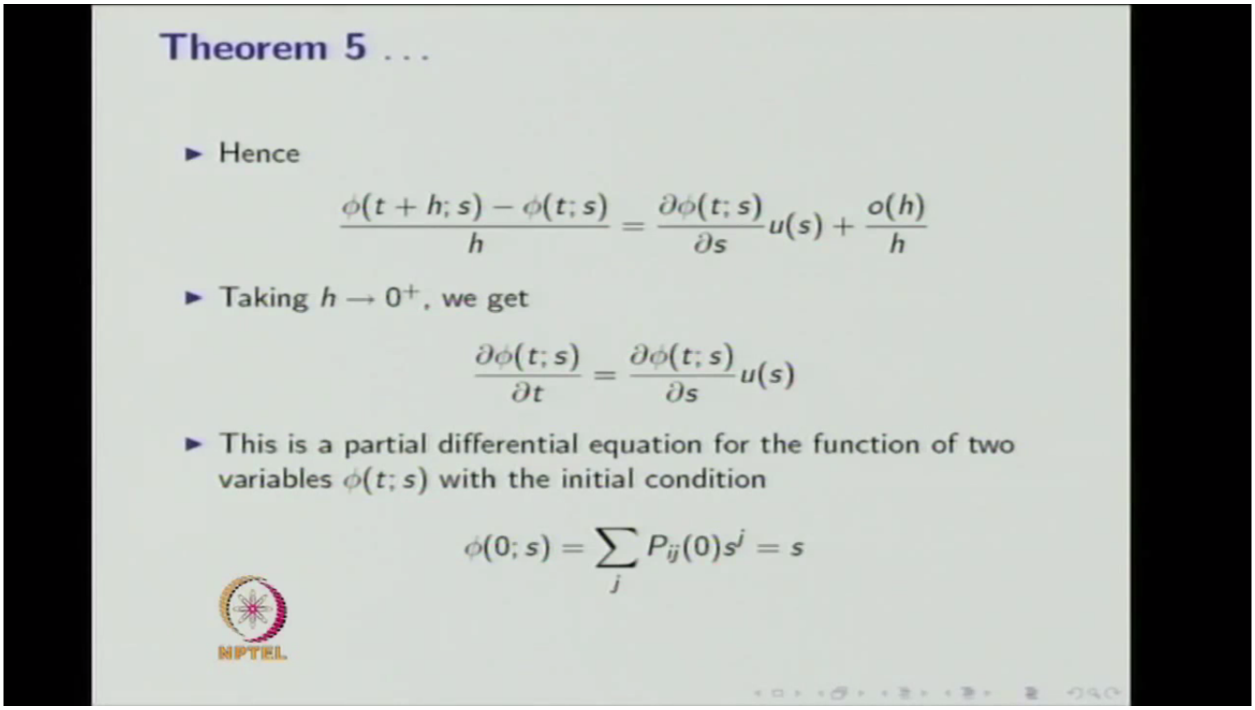


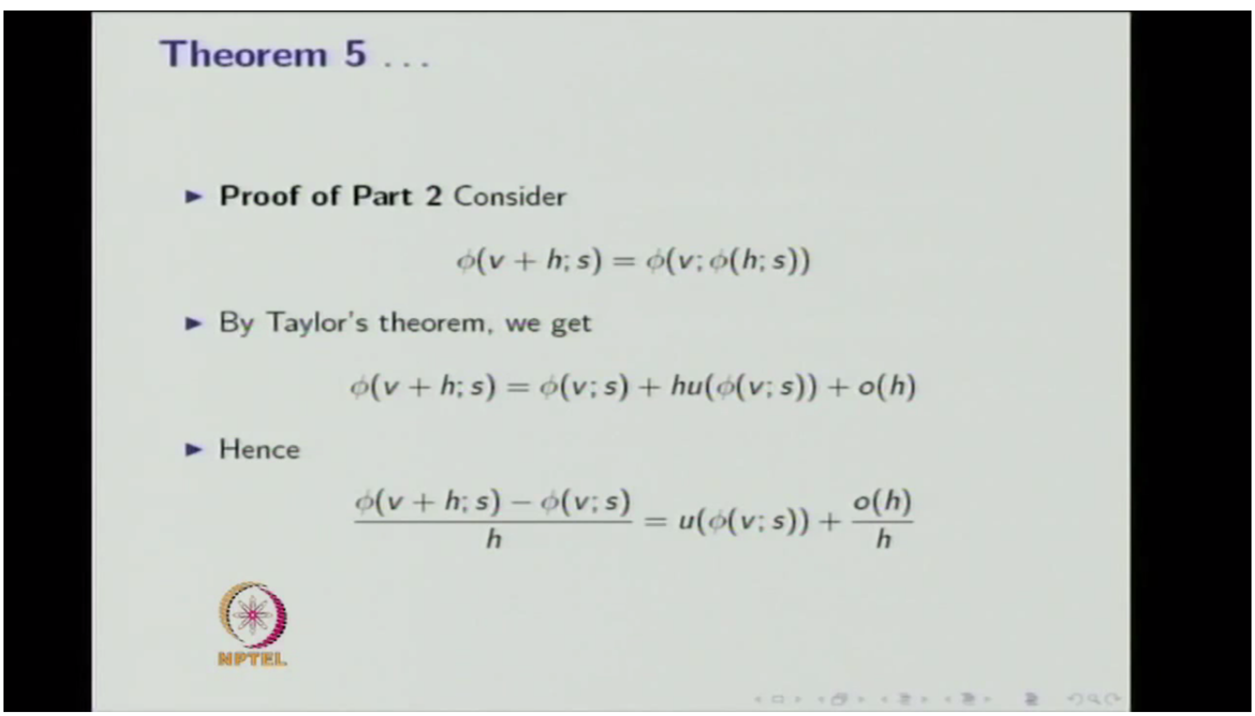
Now we will move into the theorem 5 which discuss the differential equation corresponding to the probability generating function for Pij of t. Let that u of s is equal to summation ak s power k summation over k. Then the probability generating function for Pij of t satisfies partial derivative of Psi of t, s with respect to t is equal to partial derivative of the Psi of t, s with respect to s multiplied us and partial derivative of Psi of t,s with respect to t is same as u of Psi of t, s with the initial condition Psi of 0,s is same as a summation over j Pij of 0 s power j but that is nothing but s. So the theorem 5 gives the partial differential equation and the ordinary differential equation satisfied by the partial by probability generating function of a Pij of t.



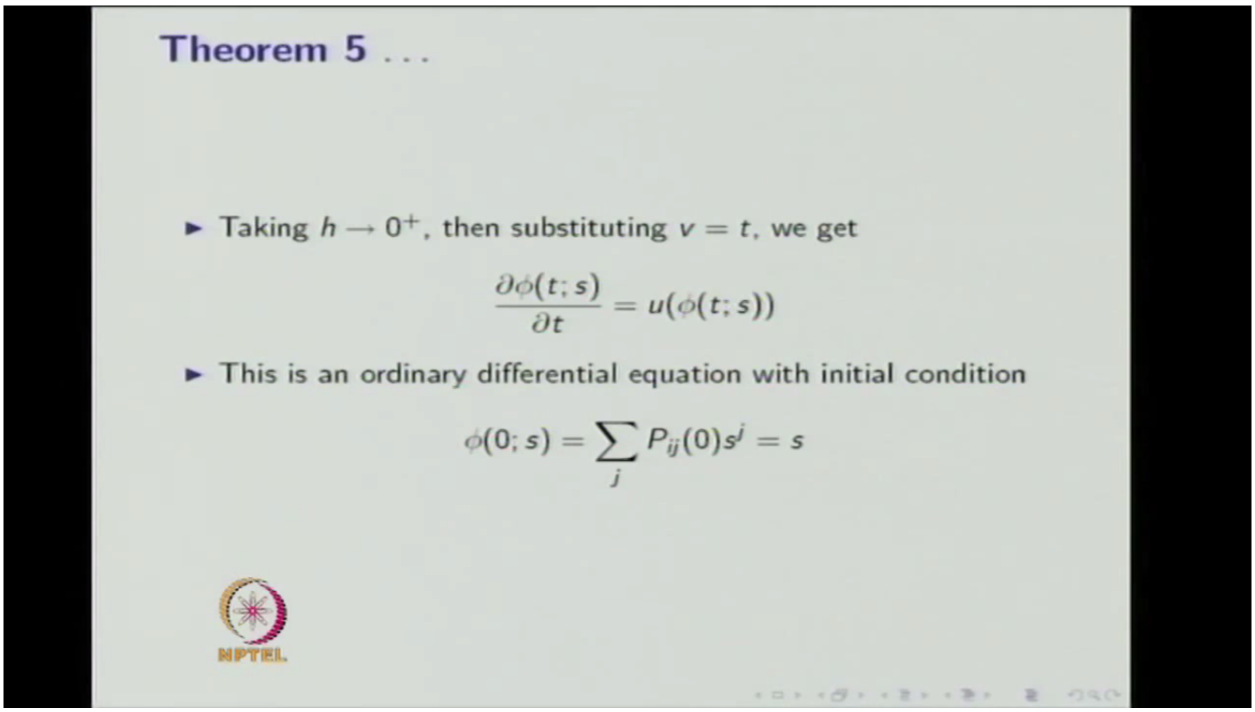
Let us see the proof. We start with Psi of h of s, Psi of h,s that is nothing but the summation over j P of i,j of s, P of i,j of h s power j; substitute the Pij of h and simplify you will get the first term will be s. The second term will be h times summation over j aj, sj the second term will be order of h. You know that u of s is same as summation over j, a suffix j of s power j therefore the probability generating function for Pij of h, s that is nothing but s plus h u of s plus order of h. We know that by the theorem four, psi of t plus h,s will be Psi of t, psi of h of s, h,s. So substitute psi of h, s with s plus h of us plus order of h therefore this will be psi of t, s plus h of us plus order of h. By Taylor's theorem we expand the right hand side with respect to the second variable. Therefore, the right hand side will be psi of t,s. The second term will be partial derivative of psi with respect to s times h of us h times us plus order of h. All the other term vanishes throughout divide by h and take psi of t, s in the left side.



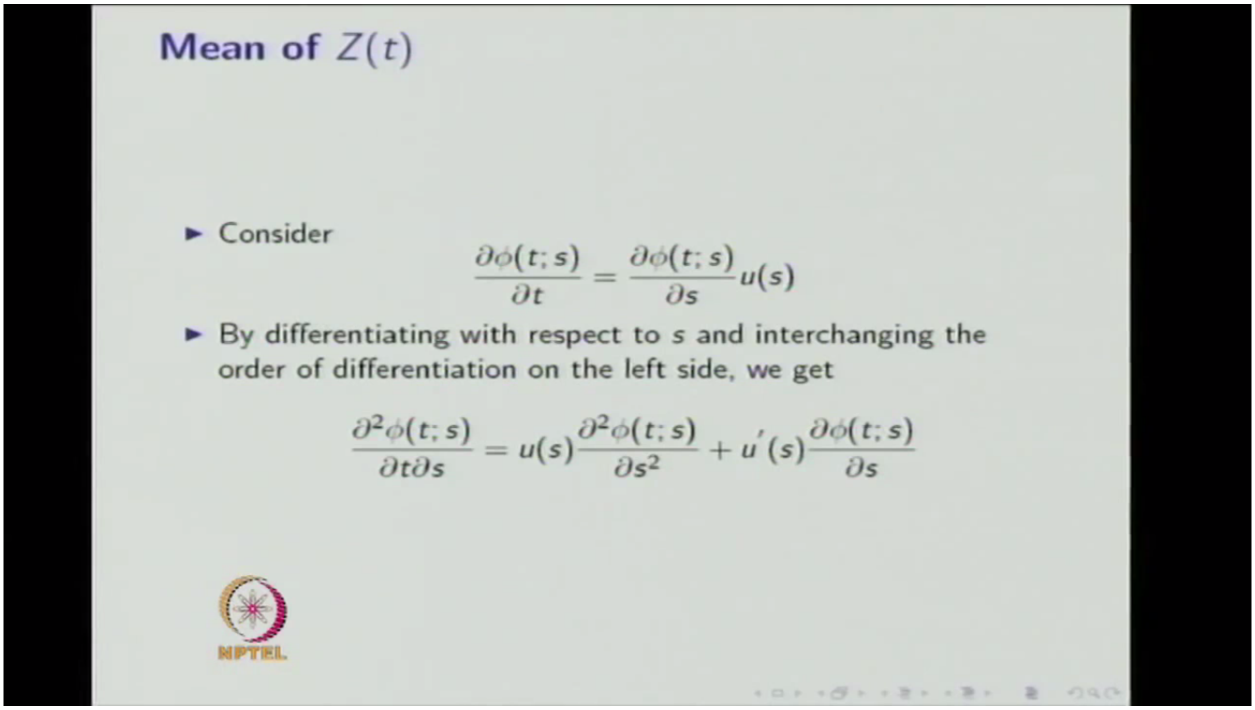
Therefore, the left-hand side becomes psi of t plus h,s minus psi of t, s divided by h whereas in the right hand side will be partial derivative of psi with respect to s times u of s order of h divided by h. Taking h tends to 0 positive we get to the partial differential equation dou psi divided by dou t is equal to dou psi by dou s time us. This is a partial differential equation for the function of two variables psi t,s with the initial condition psi of 0,s is s.



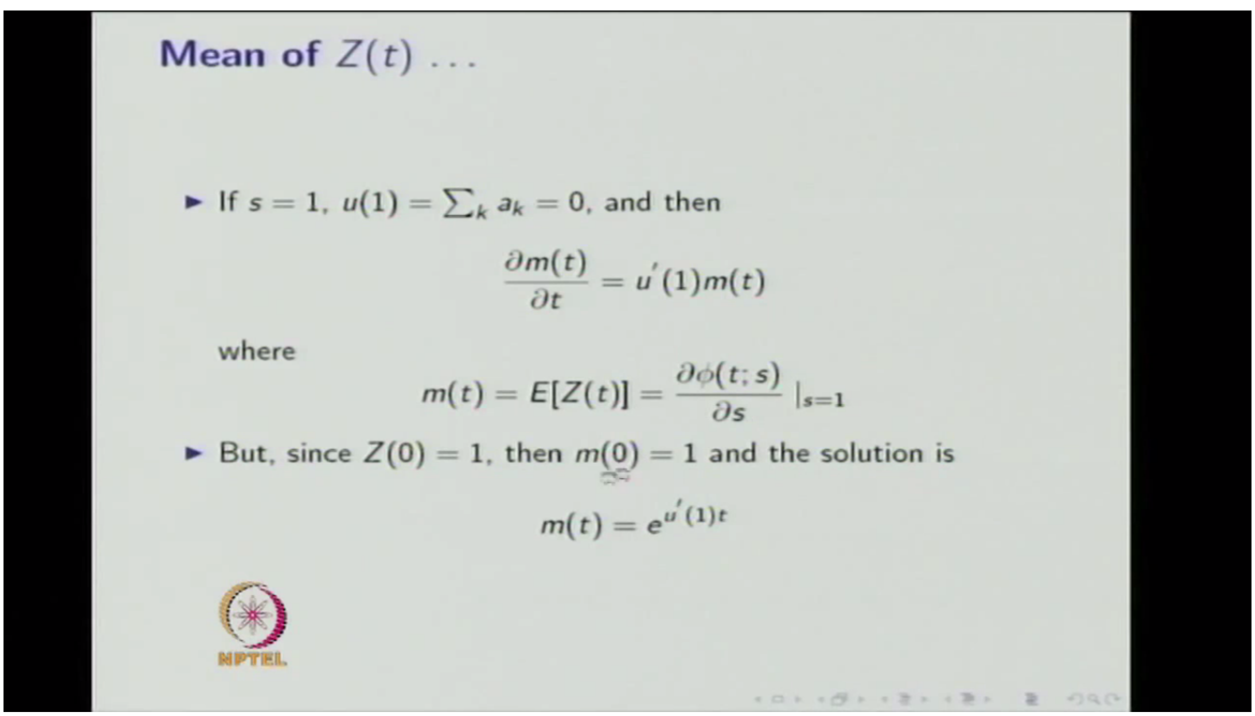
So we have proved the first part of theorem five. Similarly one can prove the second part of theorem five. The proof of a part two. You start with the probability generating function psi of v plus h,s is same as psi of v, psi of h,s. By Taylor's theorem the right hand side becomes psi of v, s plus h of u psi of v,s plus order of h. Then psi of v,s in the left hand side divide throughout by h we will get this equation. Now limit h tends to 0 plus and then substitute v is equal to t in this equation limit meta h tends to 0 plus and substitute the v is equal to h, v is equal to t we get partial derivative of psi with respect to t is equal to u of psi of t,s. This is ordinary differential equation with the initial condition psi of 0, s equal to s. So in the theorem 5 we conclude the probability generating function satisfies the partial differential equation and the initial and ordinary differential equation with the initial conditions psi of 0,s equal to s.



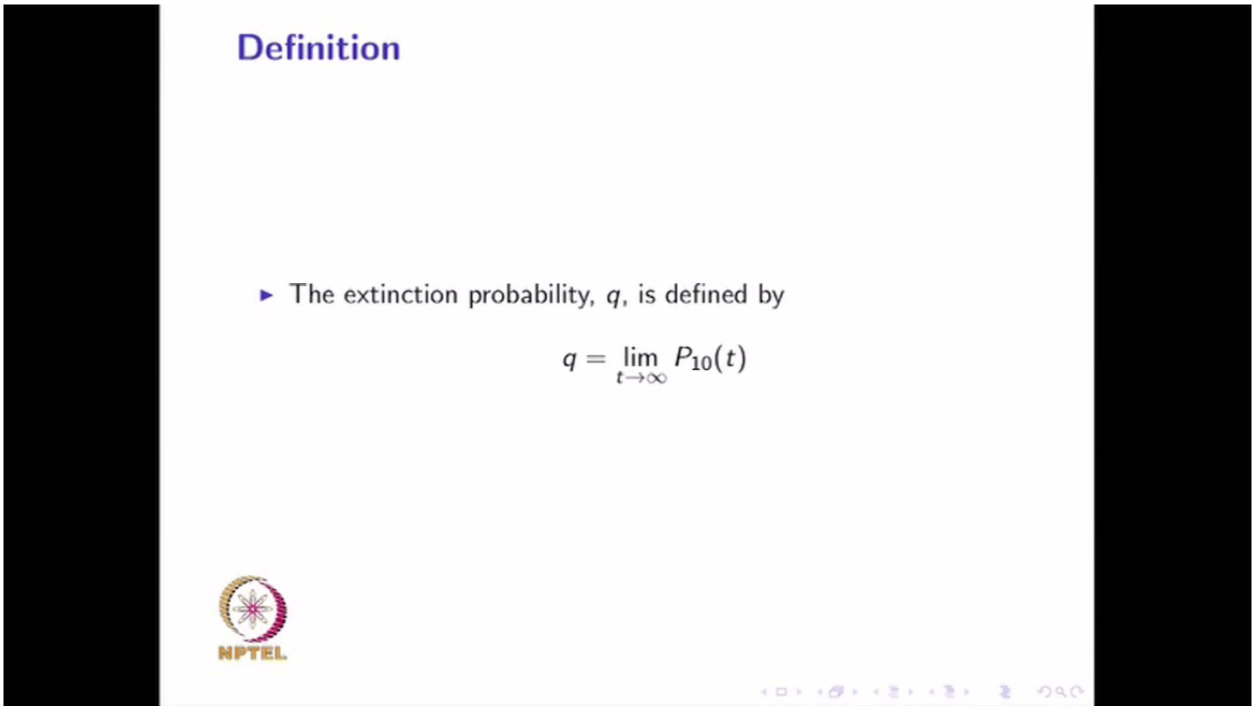
Now we will find out the mean of Z of t. You start with the partial differential equation satisfied by probability generating function. By differentiating with respect to s and the interchanging the order of differentiation on the left-hand side we get the left-hand side is the second order partial derivative of psi with respect to t and with respect to s. The right hand side u of s second order partial derivative of psi with respect to s u dash of s partial derivative of psi with respect to s. If s equal to 1 you know that u of 1 will be 0. Suppose the m of t will be the mean of Z of t that is nothing but the partial derivative of psi with respect to s then substitute s is equal to 1. Therefore this equation becomes partial derivative of m of t with respect to t is equal to u dash of 1 m of t since m is the single variable so this is the ordinary differential equation. So dmt by dt is equal to u dash of 1 times m of t where m of t is a mean of Z of t. But since Z of 0 is equal to 1 m of 0 also 1. Therefore you can solve this ordinary differential equation. The initial condition m of 0 is equal to 1 hence the solution will be m of t is equal to e power t times u dash of 1.



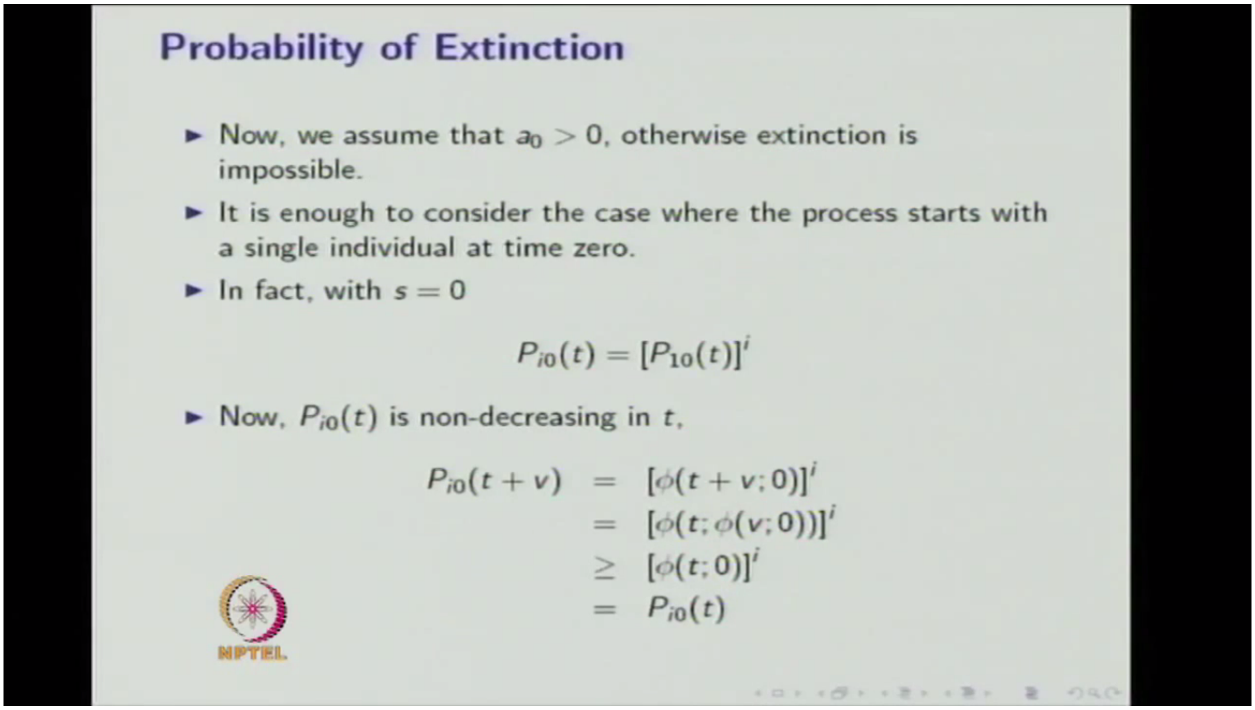
Now we can discuss the mean of Z of t based on the value of u dash of 1. Before that we discuss the probability of extinction. That is defined by q that is nothing but limit t tends to infinity probability of 1,0 of t. This is called a probability of extension that is denoted by the letter quarter.



Now we will try to find out the probability of extinction q. Assume that a naught is strictly greater than 0 otherwise extinction is impossible. It is an enough to consider the case where the process starts with a single individual at time 0. It means Z of 0 is equal to 1. With s equal to 0 you will get a Pi of 0 of t that is nothing but a P10 of t power i in the probability generating function of Pij of t.

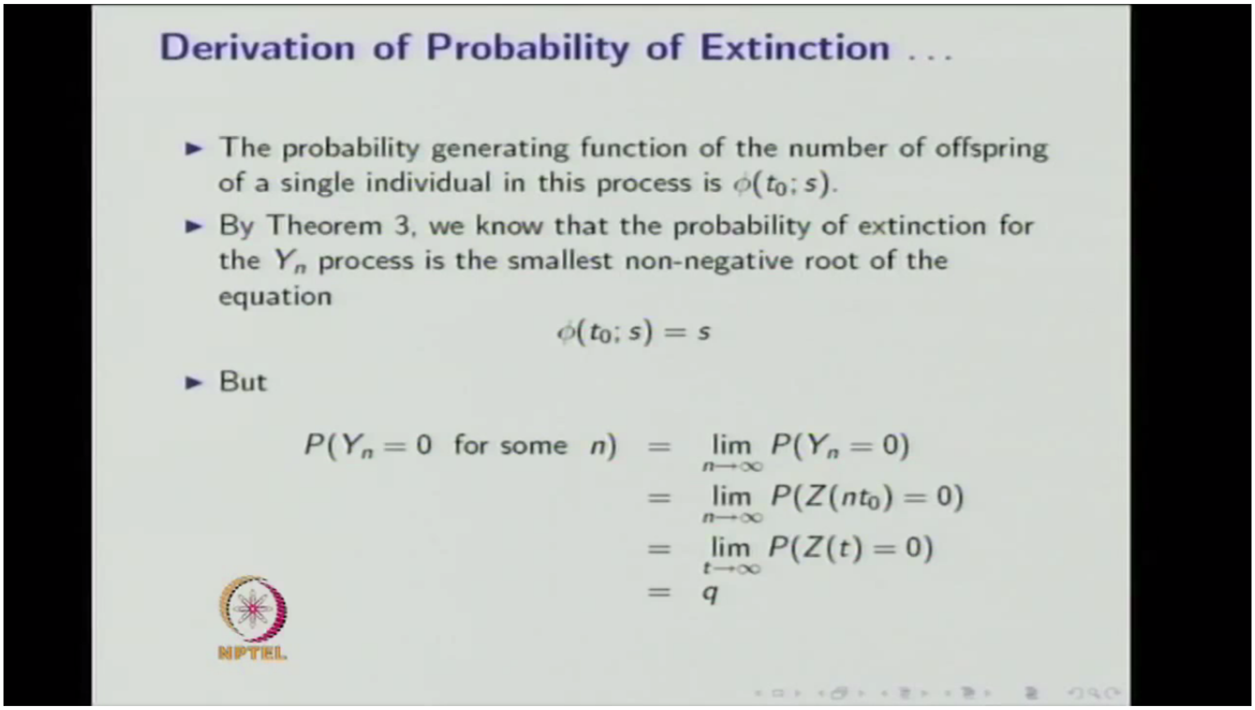


Now we will prove that Pij0 of t is a non decreasing in t. You start with Pi0 of t plus v that is nothing but psi of t plus v,0 of power i that is same as psi of t, psi of v,0 power i that will be greater than or equal to psi of 0,t power i but that is same as Pi0 of t. Hence we proved Pi0 of t is a non decreasing in t.

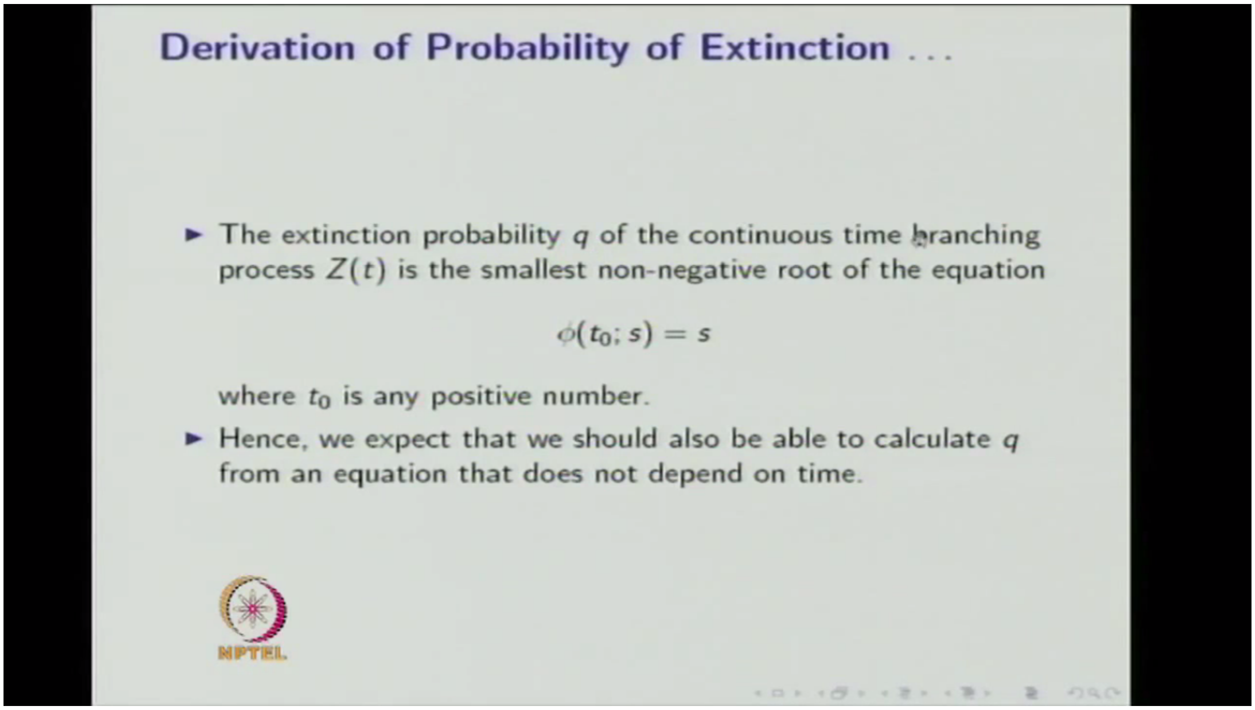


Let t be the fixed positive number consider a discrete time branching process Z of 0, Z of t naught, Z of 2 times Z naught and so on, Z of n times t naught where Z of t is population size at time t. Assume that the population size at time 0 is 1, Z of 0 is equal to 1. Since Z of t is assumed to be Markov process the discrete process Yn Y suffix n that is nothing but Z of n of t naught will be a discrete time Markov chain which is also a discreet time branching process because Z of t is a continuous time branching process therefore Y of n will form a discrete-time branching process which is also a discrete time Markov chain. By the hypothesis of homogeneity of probability function of Z of t and the probability generating function of Pij of t that is nothing but probability generating function of Pij of t power P1 of 1 j of t power i.

This we have proved it in the earlier. We have proved it in earlier therefore using these two we are finding summation over k the conditional probability of Yn plus 1 is equal to k given Yn is equal to i multiplied by s power k that is nothing but expectation of s power Yn plus 1 given Yn is equal to i. That is same as because Yn is nothing but Z of n times t naught therefore Yn plus 1 is nothing but Z of Yn plus 1 times t naught. So it implies Yn by Zn n times t naught and Yn plus 1 by Z of n plus 1 times t naught. This is true for all n therefore that the same as expectation of s power Z of t naught given Z of 0 is equal to i but that is nothing but psi of t naught,s power i but that can be written as expectation of s power Z of t naught given Z naught is equal to 1 whole power i that is same as expectation of s power Y1 given Y naught is equal to 1 the whole power i. This shows that Yn is a branching process. Yn is a discrete-time branching process. So using this we have proved the Yn is a discrete time branching process. The probability generating function for the number of offspring of a single individual in this process is psi of t naught,s. By theorem 3 we know that the probability of extension for Yn that is a discrete time branching process is the smallest non-negative root of the equation psi of t naught,s equal to s.



So by using the theorem 3 we conclude the probability of extinction for the Yn process is the smallest non-negative root of the equation psi of Z naught, s equal to s. But we know that probability of Yn is equal to 0 for some n that is same as limit n tends to infinity of probability of Yn is equal to 0 that is same as a limit n tends to infinity of a probability of n times Z naught is equal to 0 but that is same as limit t tends to infinity of probability Zt is equal to 0. By definition this is nothing but q that is a probability of extinction.



Hence the probability of extinction q of a continuous time branching process Z of t is the smallest non-negative root of the equation psi of t naught,s is equal to s. Here we have concluded by theorem 3 the probability of extinction for the discrete-time branching process Yn is the smallest non-negative root of the equation psi of t naught,s equal to s because of this we conclude the probability of extinction of the continuous-time branching process Z of t is a smallest non-negative root of the equation psi of t naught,s equal to s where t naught is any positive number. Hence we expect that we should be – we should also be able to calculate q from equation that does not depend on time. From this equation can able to calculate q from the equation that does not depends on time.