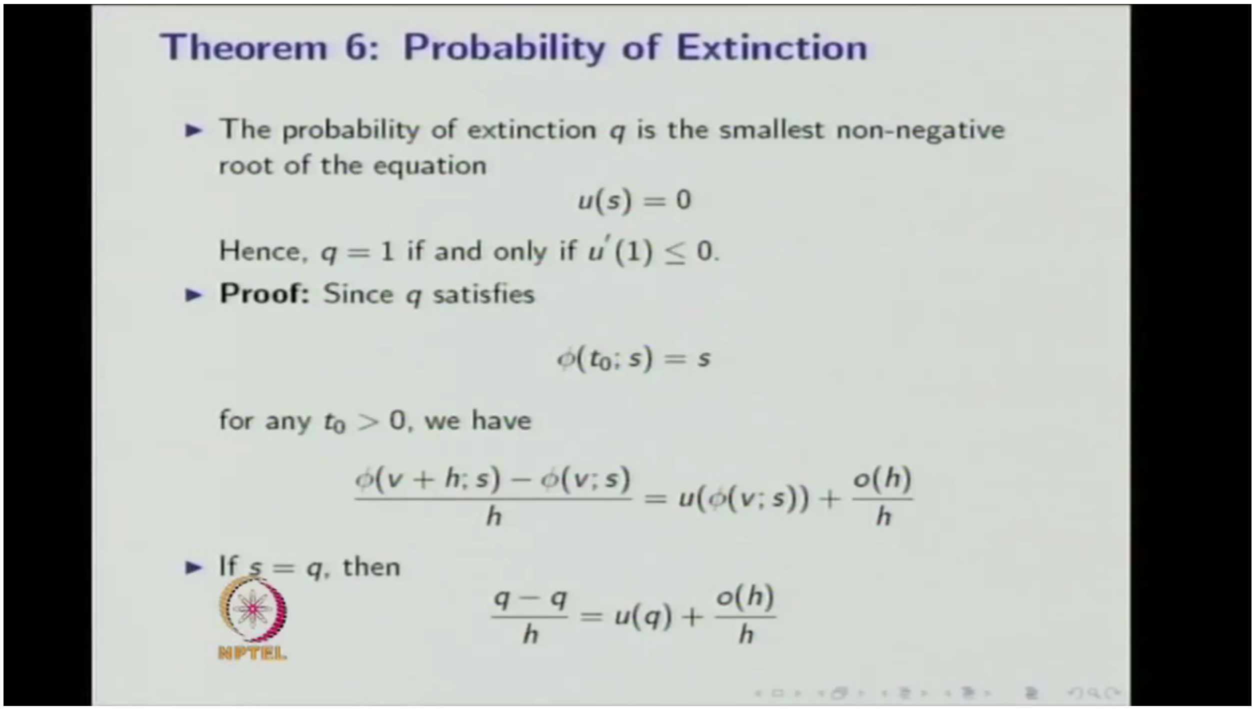
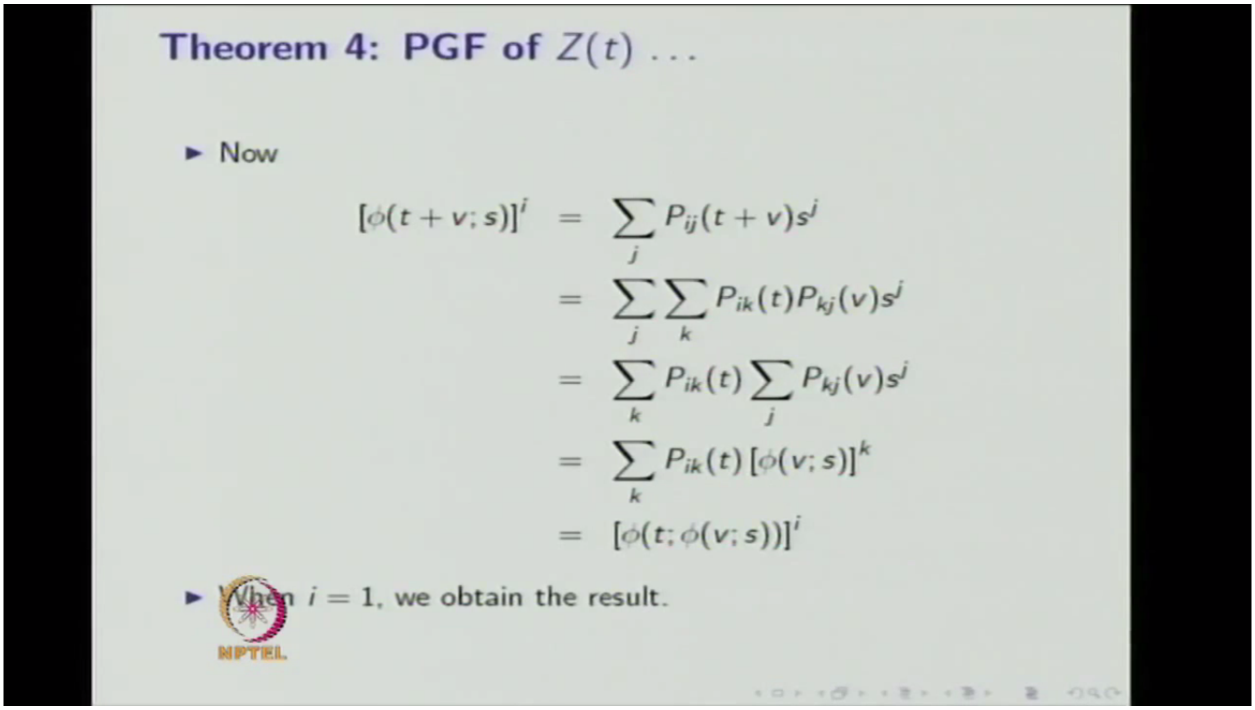
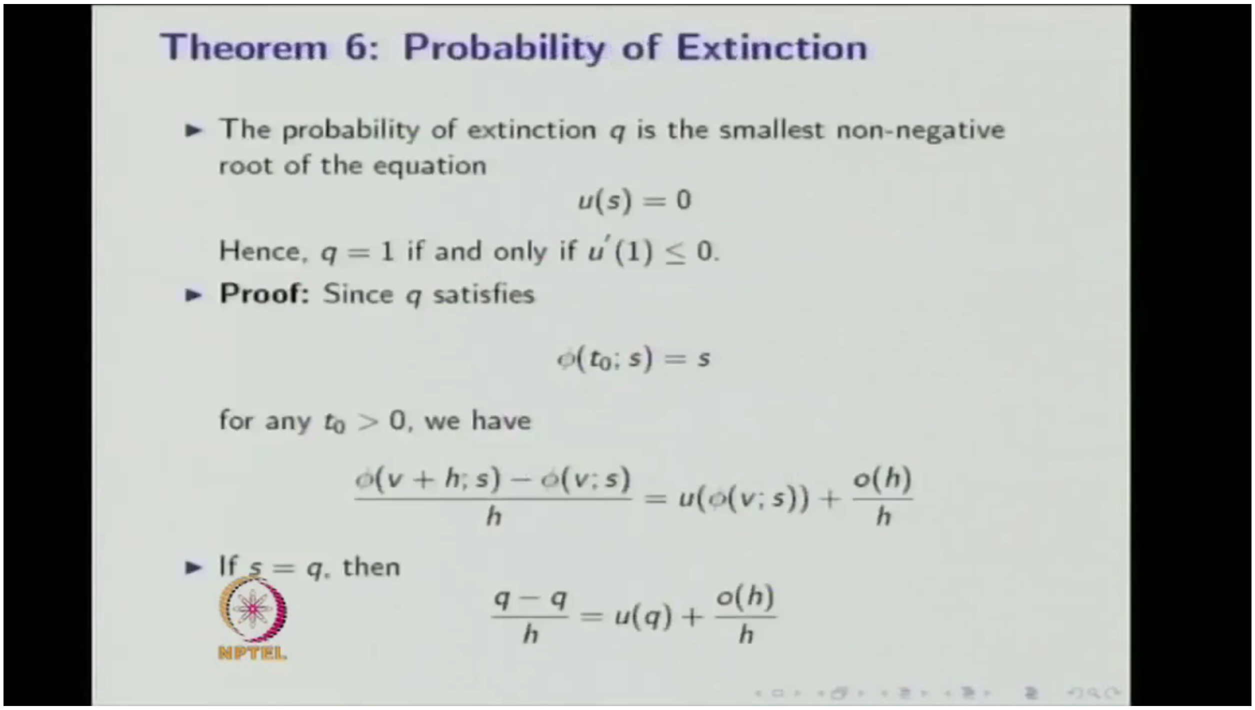


Now we are moving into theorem 6; how to find the probability of extinction. We conclude q is the probability of extinction for the continuous time branching process. So here in this theorem we are giving the probability of extinction q is the smallest non-negative root of the equation u of s equal to 0. Hence q is equal to 1 if and only if q dash is less than or equal to 0. So whenever q dash of 1 is less than or equal to 0 then the probability of extinction will be sure, the probability will be 1. Extinction event will be sure the probability of extinction will be 1.

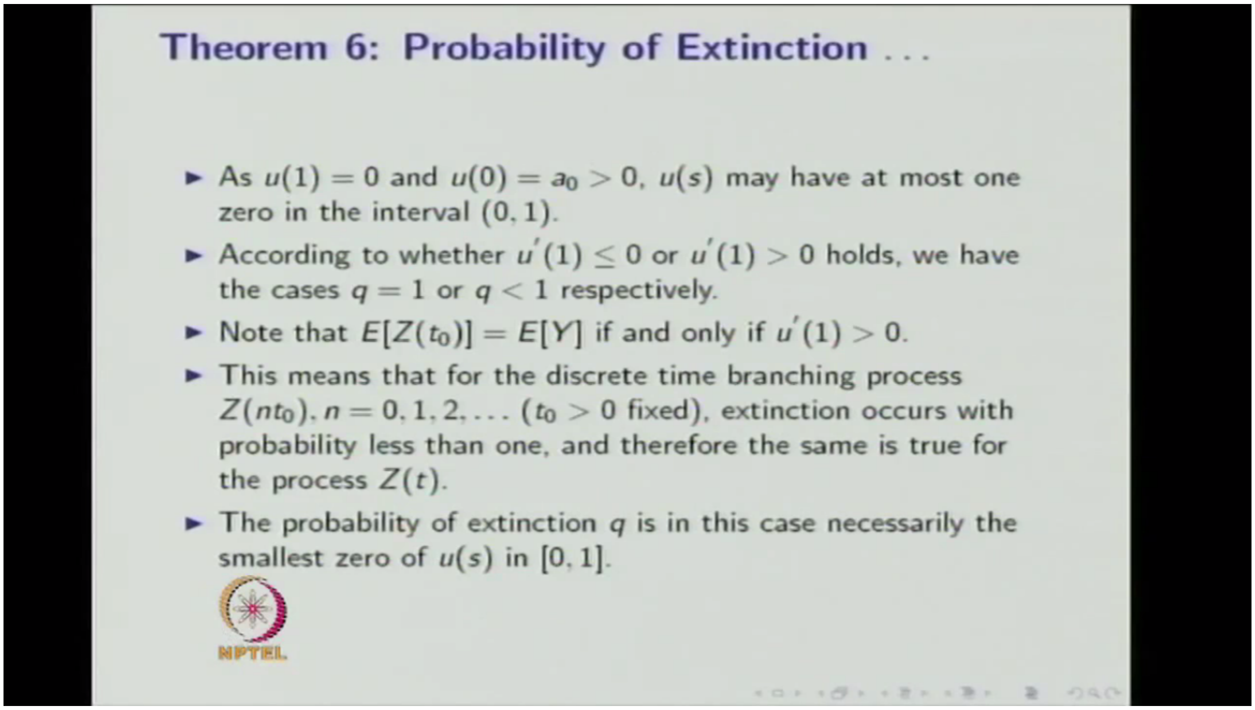


Now we give the roof of a probability of extinction. In the earlier theorem we have concluded q satisfies psi of t naught,s is equal to s. For any t naught greater than 0 we have this relation we have in the theorem for the theorem four. The theorem four, theorem five discuss the partial differential equation and ordinary differential equation satisfied by psi of t,s. So we are using these equations to find the probability of extinction.



So here by using theorem 5 psi of v plus h, s minus psi of v,s divided by h will be u of psi of u,s plus order of h by h. you know that this will be tends to 0 as h tends to 0. if s equal to q because q is the probability of extension is the smallest non-negative root of the equation so psi of t naught,s is equal to s therefore if you substitute s is equal to q here then the above equation becomes the left-hand side becomes 0. When you put s is equal to q here then the psi of v,q will be q therefore this will be u of q plus order of h divided by h by s equal to q in this above equation the left-hand side becomes 0, the right-hand side first term psi of v, q will be q therefore it will be u of q plus order of h divided by h for any h greater than zero it will be 0 is equal to u of q plus order of h divided by h as h tends to 0 plus you will get at u of q equal to 0.

Therefore the earlier theorem we have concluded the probability of extinction will be psi of t naught,s equal to 0 the q will be the probability of extinction is the smallest non-negative root of the equation but here by using this we concluded u of q is equal to 0. hence, the probability of extinction q is the smallest non-negative root of the equation u of s is equal to 0 because we concluded u of q is equal to 0. Suppose you find the double derivative of u that will be great or equal to 0. Hence we conclude u of s is a convex function in the interval 0,1. As u of 1 equal to 0 and the u of 0 is equal to a naught which is greater than 0 us may have at most one 0 in the interval 0,1. The way we defined u of s, u of s is a summation ak s power k therefore u of 1 will be 0 and u naught will be a naught which is greater than 0. With that assumption only the probability of extinction is possible.

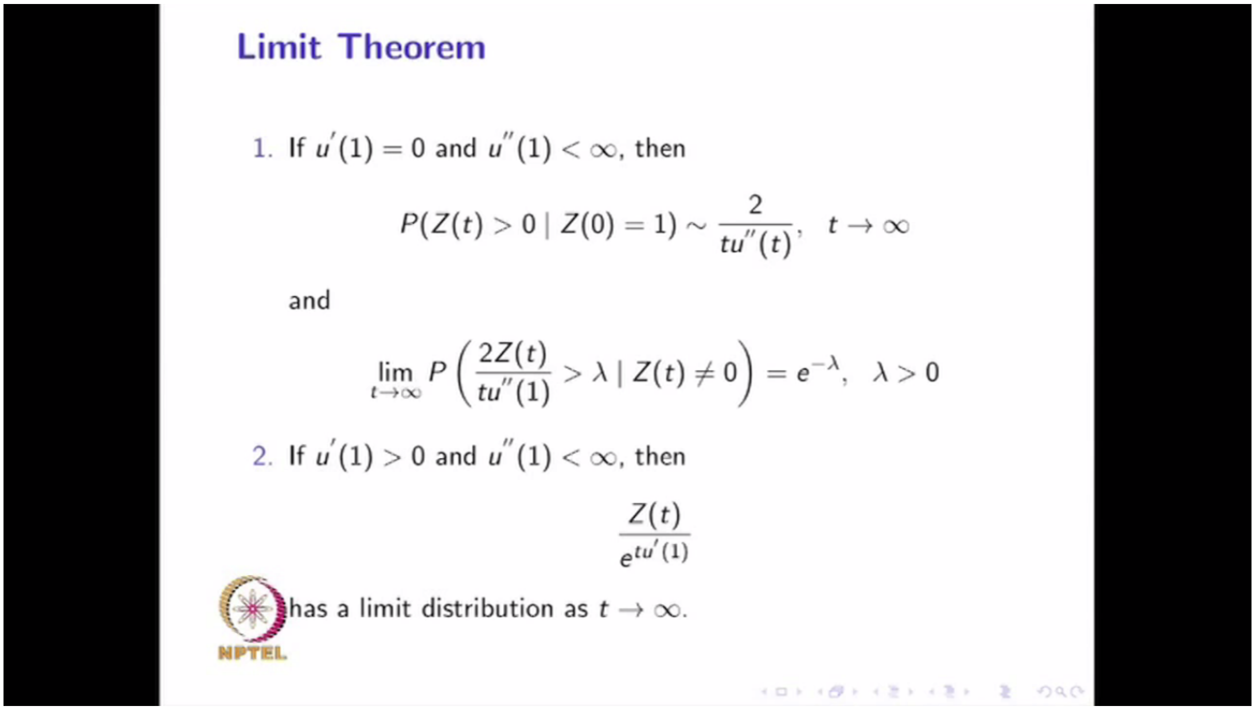


According to u double dash is less than or equal to 0 or greater than 0 we have the case q is equal to 0 or q is less than one respectively. That means when u dash is less than or equal to u dash of 1 is less than or equal to 0 the probability of extinction will be 1. The u dash of 1 is greater than 0 then the probability of extinction will be less than 1. So graphically one can show this is a graph of y is equal to u of s. So here we have two graphs. The graph a later to u dash of 1 is less than or equal to 0. Since u of s is a convex function in the interval 0 to 1 and u naught is a naught u 1 will be 0 so this is the graphical representation of y is equal to u of s in the case u dash of 1 is less than or equal to 0.

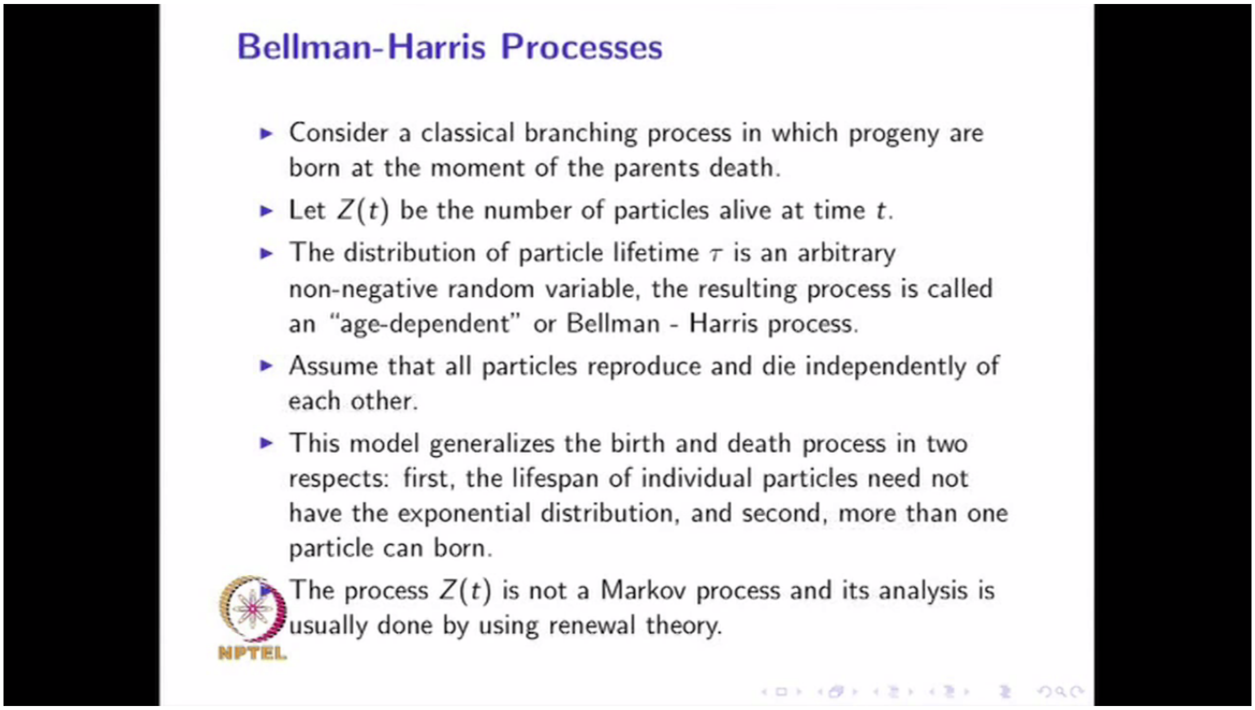
The case two when u dash of 1 is greater than 0 y is equal to u of s will cut the x-axis at some point which is less than 1 because u naught is a naught, u 1 is equal to 0, and u of s is a convex function u dash of 1 is greater than 0, the u of s will cut the x-axis before 1. Hence the probability of extinction when u dash of 1 is less than or equal to 0 that will be 0 and the probability of extinction when u dash of 1 is greater than 0 it will be less than 1.

Note that expectation of Z naught expectation of Z of t naught is equal to expectation of Y if and only u dash of 1 is strictly greater than 0. So whenever u dash of 1 is strictly greater than 0 the probability of extinction is less than 1. This means that for a discreet time branching process Z of n times t naught extension occurs with the probability less than 1 and therefore the same is true for the process above. The probability of extinction q is in the case necessarily the smallest zero of u of s in 0 to 1.

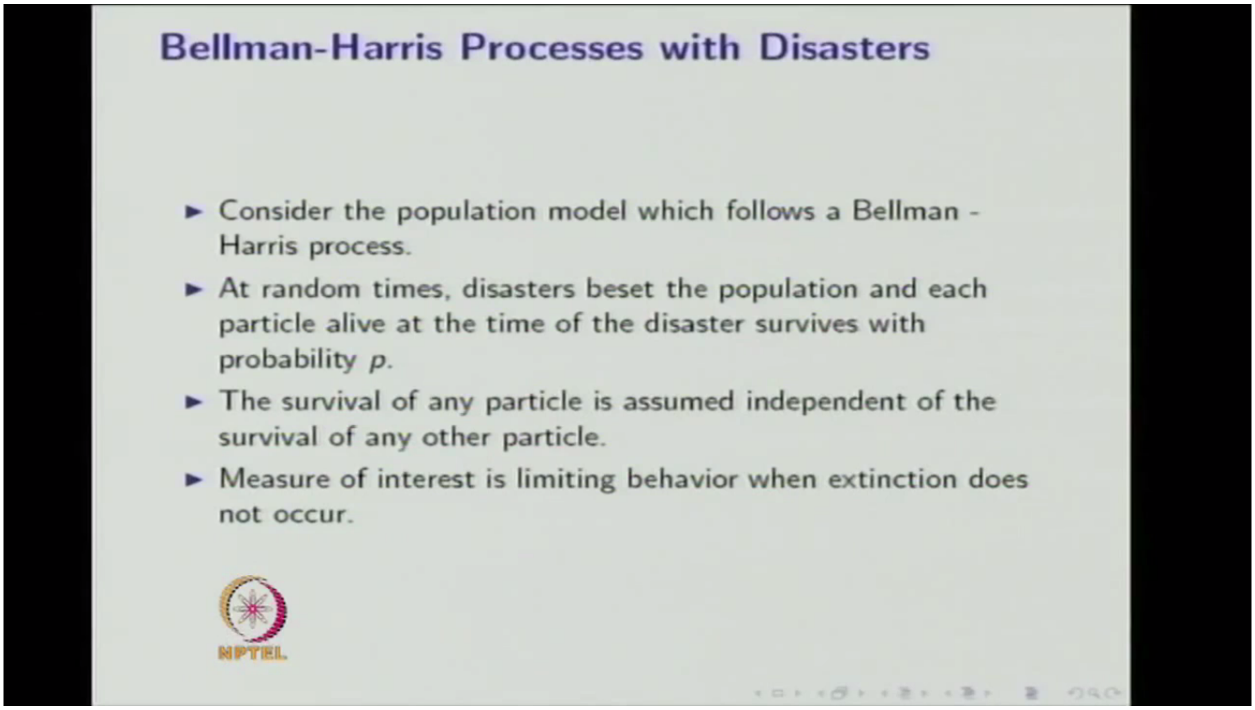
In a similar manner we conclude that if u dash of 1 is less than or equal to 0 q must be equal to 1. In either case q is the smallest non negative root of u of s equal to 0. So hence the probability of extinction q is the smallest non-negative root of the equation u of s equal to 0 when q is equal to -- when u dash of 1 is less than or equal to 0 the probability of extinction will be 1.



Now we will consider the limit theorem. If u dash of 1 is equal to 0 and u double dash of 1 is finite then we can show this conditional probability will be approximately 2 divided by t times u double dash of t as t tends to infinity. And also we can conclude the limit t tends to infinity probability of this event is e power minus lambda where lambda is t greater than 0. When u dash of 1 is strictly greater than 0 and u double dash of 1 is a finite then the Z of t divided by e power t times u dash of 1 has a limit distribution as t tends to infinity. Without proof we are stating this limited theorem.



Till now we have discussed two important branching processes. The first one is Galton-Watson discrete-time branching process. The second one is a Markov branching process which is a continuous type branching process. Now we are going to discuss some more or some other important branching processes. The first one is a Bellman- Harris processes. Consider a classical branching processes in which the progeny are born at the moment of parents death. Let Zt be the number of particles at time t. The distribution of a particle lifetime tau is an arbitrary non-negative random variable. The resulting process is called the age-dependent or Bellman-Harris process. So in the Markov branching process the random variable tau which is a exponential distribution but here it is a arbitrary non-negative random variable then the resulting process is age-dependent or Bellman-Harris process. So wen tau becomes exponential distribution then age-dependent Bellman-Harris process becomes Markov branching process. Assume that all particles reproduce and die independently of each other. The similar assumption we have taken care in the discrete type as well as continuous type branching processes. This model generalizes the birth-death process in two respects; the first the lifespan of individual particles need not have the exponential distribution and second more than one particle can work. Because of these two aspects this model generalized the birth-death process. The process Z of t is not a Markov process and its analysis is usually done by using renewal theory. We have discussed a renewal processes in a model 8.



Now we discuss the Bellman-Harris processes with disaster. Consider the population model which follows Bellman-Harris process at random times disasters reset the population and each particle alive at the time of disaster survives with the probability p. The survival of any particle is assumed independent of survival of any other particle. In this model the measure of interest is limiting behavior when extinction does not occur. We will move into the other important branching process that is Bellman-Harris processes with immigration. In addition to the Bellman-Harris process we allow a certain appearance into the system of newly born particles called immigrants. Immigrants are assumed to arrive in a group of various sizes with the probability of n immigrants in a group immigrating at time t given by Pn of t. Once these particles arrive they reproduce and die according to the Bellman-Harris process. In this model measures of interest or the mean of Z of t, the limiting distribution and asymptotic behavior of Z of t. This process is widely used to describe growth and decay of biological populations. We are not discussing in detail of Bellman-Harris process in this lecture.

In this model we have discussed in detail two important branching processes Galton-Watson process and Markov branching process. We have briefed Bellman-Harris process with a disaster and with immigration. The first two branching processes we have discussed the mean and variance of Z of t limiting distribution probability of extinction in both branching processes. Here are the references for this model 9, branching processes.