Probability space and conditional probability

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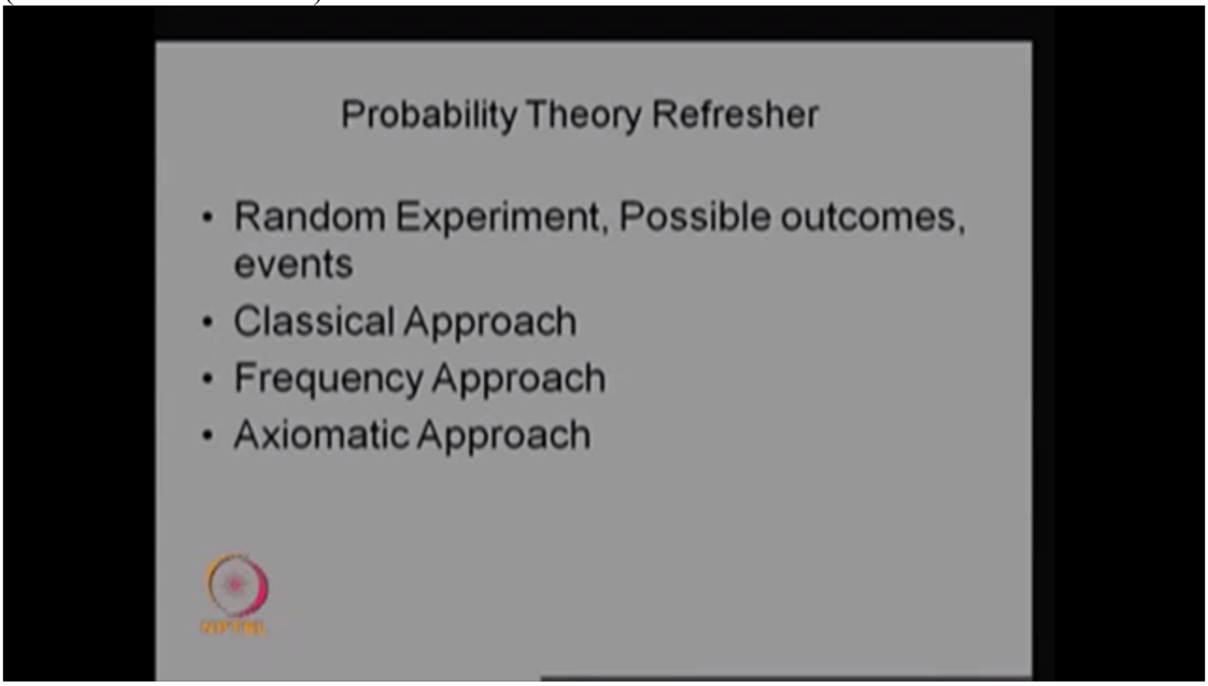

So for that we need the probability theory in detail. So even though we cannot explain the whole probability theory in complete, I am just going to give in -- I'm just making a refresher type of defining what is a probability and what is a random variable and so on. And I will cover up whatever the probability theory knowledge is needed for the stochastic process that I will explain in another -- this lecture as well as the next lecture. And some of the in detailed probability theory concepts which will be used later that I am going to explain whenever the problem comes -- comes into the picture.

So for that, first we need what is random experiment? A random experiment is a experiment in which you can be able to list out what are all the possible outcomes can going to come if that experiment is going to actually takes place. That means before the experiment takes place, you can always able to list out the possible outcomes.

So the possible outcomes that I am going to make it as the collection with the word called Omega. So the Omega is the set of all -- the set of all possible outcomes. The outcomes could be a numerals or non numerals as well as the outcomes, the set, the Omega could be a countably finite. It could be a countably finite or it could be countably infinite or it could be uncountably many also.

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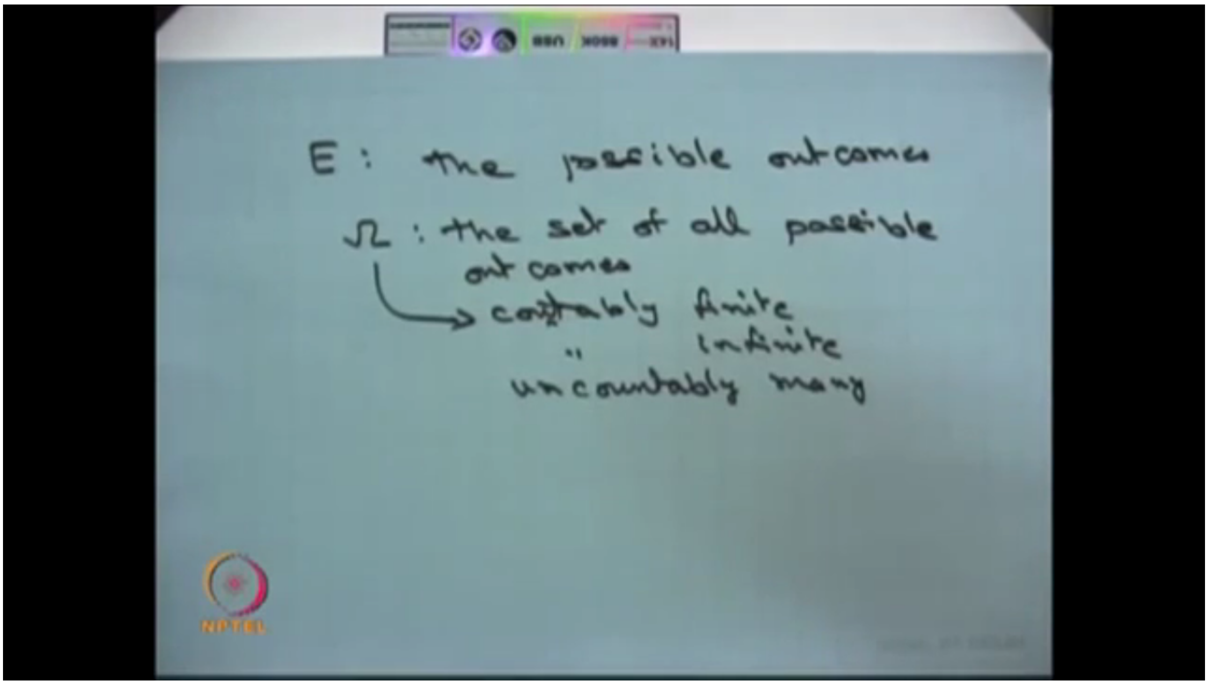

So the way you have chosen the random experiment, when you start collecting the possible outcomes, that collection I am going to use the -- I'm going to put it in the collection called Omega.

Once you have the Omega, then we can go for creating the event. The event is nothing but the subsets of Omega. So the event, the possible events are starting from the empty set as well as these are all the -- these are all the -- just you can get it like that. It's a empty set as well as Omega and you can create all the possible subsets of Omega that is also going to form a event.

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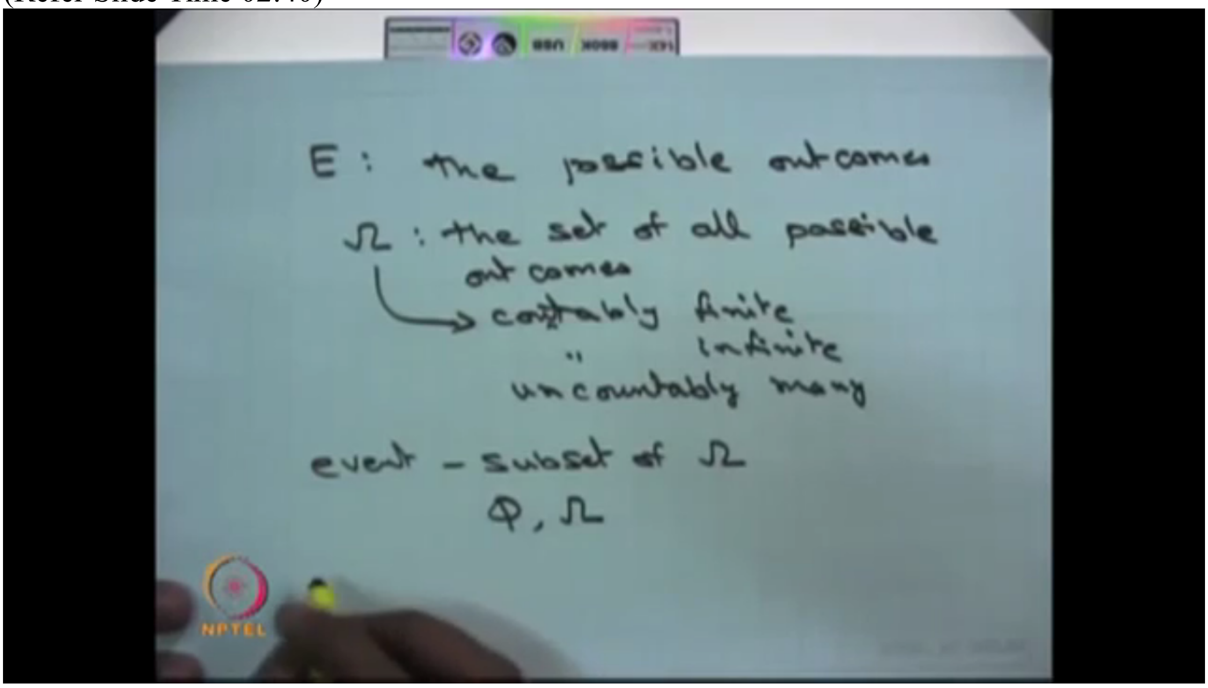

Next we are going to make a probability space. I will just take out. So to define the probability space, you need sigma-algebra. So what is sigma-algebra? What we are going to create a sigma-algebra over Omega. So that I am going to use the word F. F is the sigma-algebra over the Omega that is the collection of all possible subsets of Omega such that the empty set is belonging to F and if I take -- then the union of Ai is also belonging to F.

The third condition if I take one element from the F, then that complement is also belonging to F.

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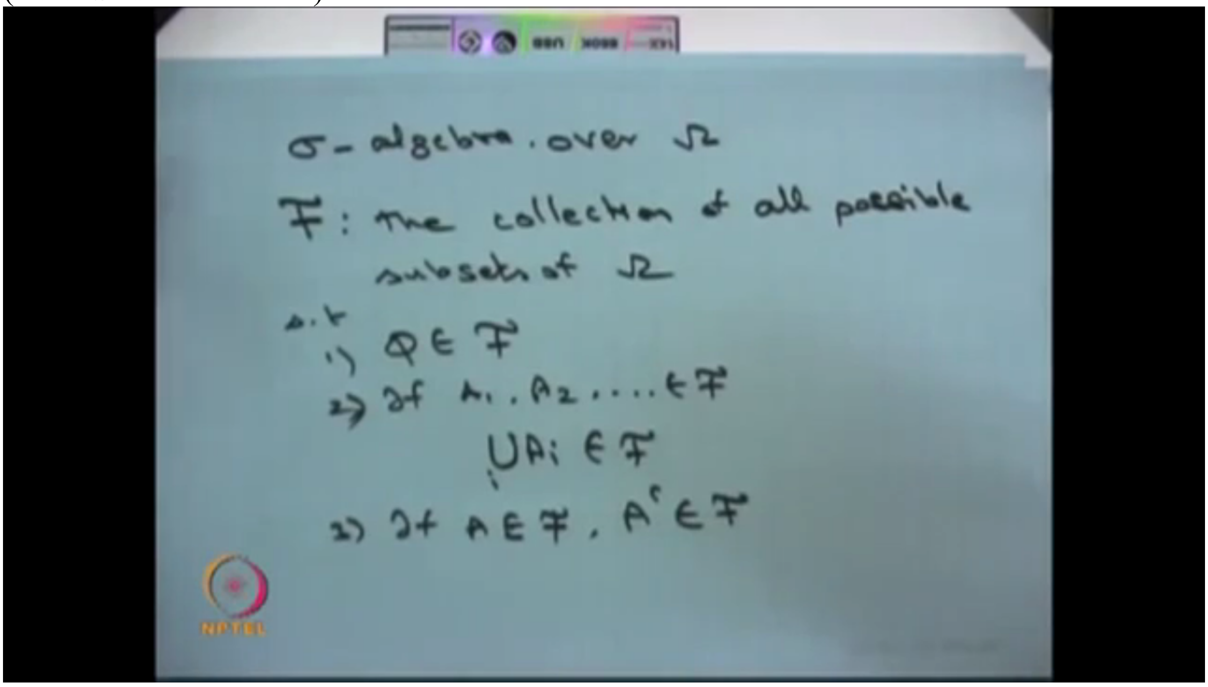

So that means the sigma-algebra over the F -- over the Omega that F contains the collection of all possible subsets of Omega such that these three conditions are satisfied. That means you can go for making the trivial F that is going to be contains only the empty set as well as the whole set. This is also going to be one of the sigma-algebras over the Omega that is at the default one.

Like that I can go for creating many sigma-algebra that by making a few elements of -- few elements of possible outcomes that I make it as the set A. Then I can make it the another sigma-algebra that as a empty set and I can make a one set called U and U consists of a few elements of Omega and you complement. Then I can have a Omega also.

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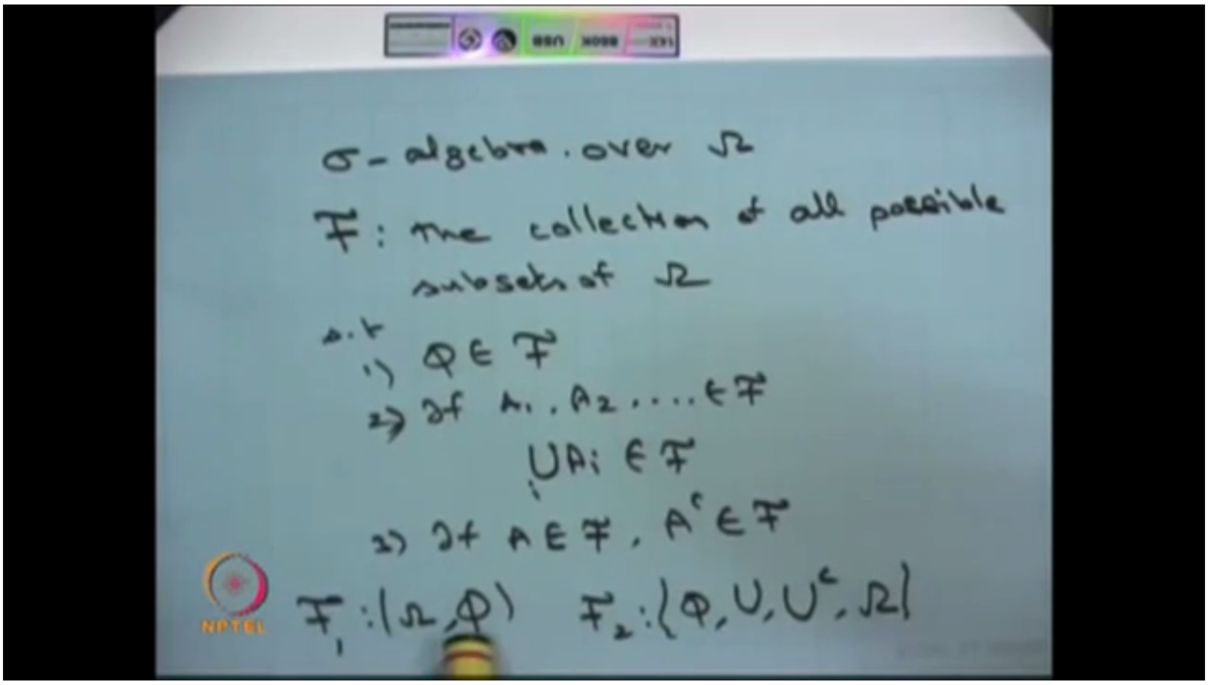

So like that I can keep creating the different sigma-algebra over Omega and the trivial one is the empty set with the Omega set that is going to be the trivial one and now I am going to define the probability space.

What is probability space? The probability space is a triplet in which the Omega is the collection of possible outcomes and F is the sigma-algebra over Omega and P is the set function such that -- P is the set function such that the P of A is always going to be greater than or equal to 0 for any A belonging to F.

The second condition, the P of Omega is going to be 1 always.

The third condition, if I take few A_i's or mutually independent events, then the P of union of A_i's is same as summation of P of A_i's.

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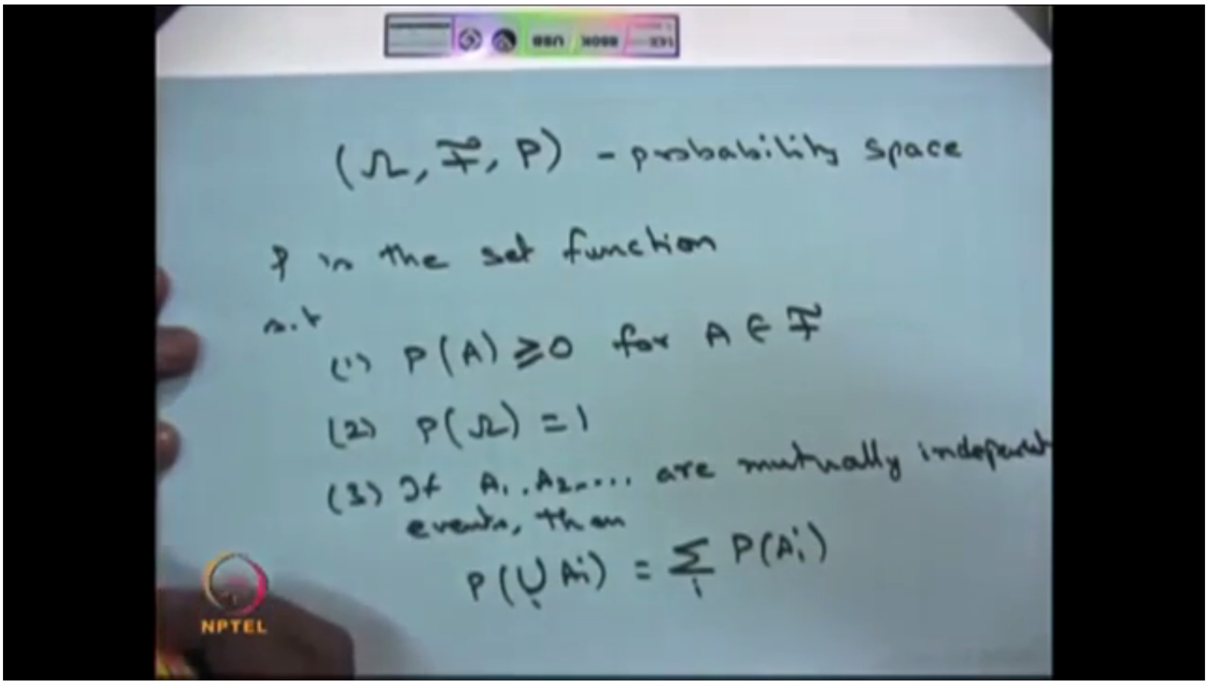

Let me just explain the probability space in a better way. This triplet is going to be call it as a probability space as long as you have the collection of possible outcomes and you have a sigma-algebra. So this sigma-algebra can be anything and you can go for the default one is the largest sigma-algebra which you have created and P is the set function such that whatever the element you are going to take it from F, any elements of F is going to be event. So the P of any event that is going to be always greater than or equal to zero and if you take the event is going to be Omega, therefore the Omega is also one of the element in the F and the P of Omega is equal to 1.

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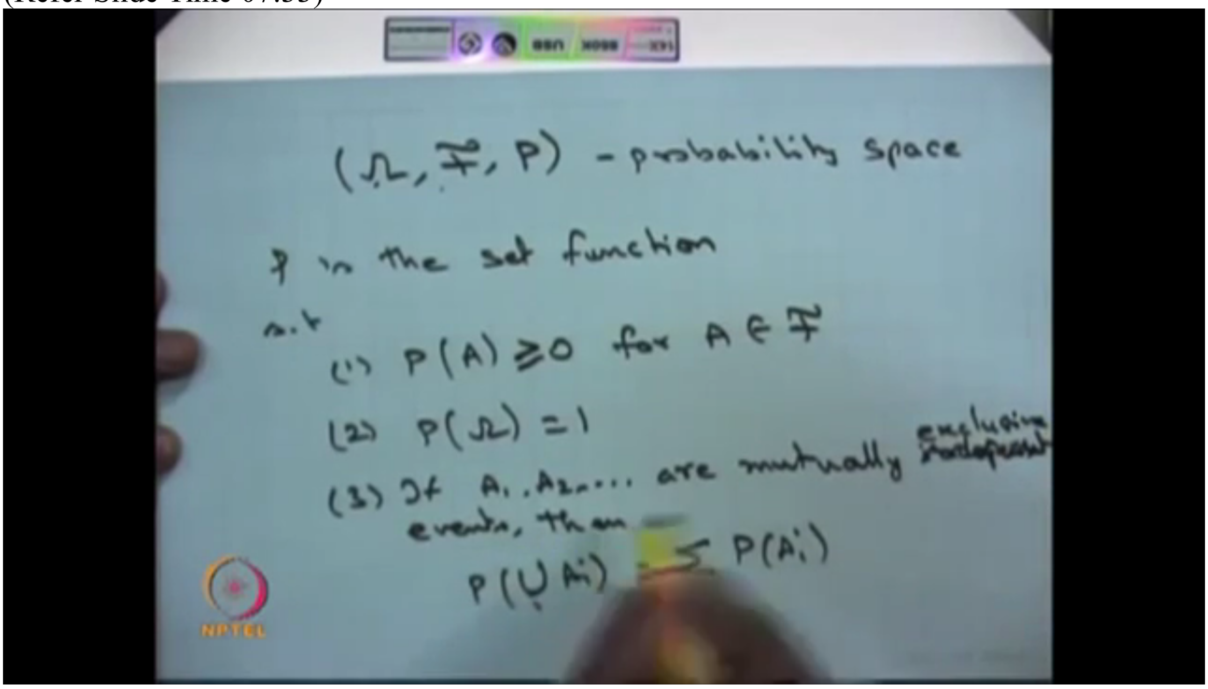

And the third condition, if you take A_i's are mutually exclusive, sorry, if you take A_i's are mutually exclusive events then the probability of a union is going to be the summation of a probabilities, summation of P of A_i's. Then this P is going to be the set function and the P is going to be the probability measure. This P is going to be call it as -- this P is the probability measure and this P is the normed measure also because of the condition P of Omega is equal to 1. Okay.

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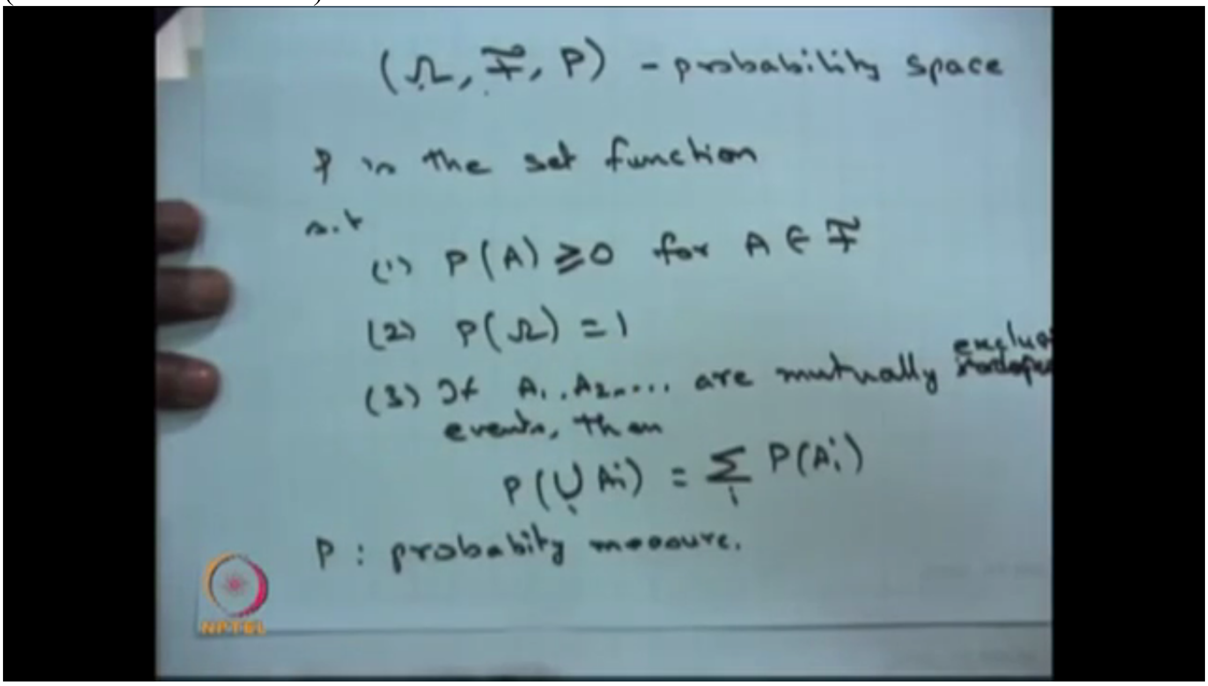

There are many definitions over the probability theory, the classical approach or the frequency approach and what we have given is a axiomatic approach. So the way I have given the definition that is the probability space with the Omega, F and P and this is called the axiomatic approach. And we are going to use axiomatic approach, not the frequency approach or the classical approach.

And you should note that the classical approach is going to be the special case of the axiomatic approach in which you make the collection of possible outcomes are going to be equally likely, then the classical approach is going to be the special case of the axiomatic approach. Therefore, throughout our course, we are going to use the axiomatic approach, not the classical approach.

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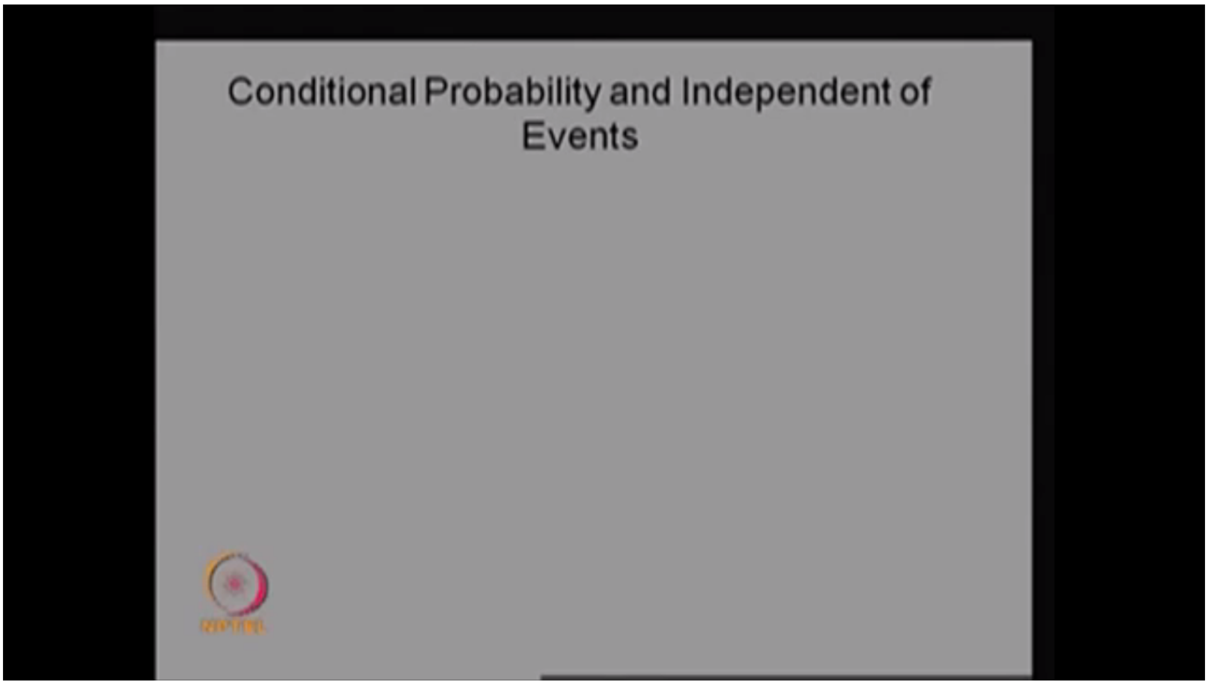

Next I am moving into the concept called Conditional Probability. So suppose you have a probability space be the probability space, if you take -- if P of A is greater than or equal to 0, then let B be the event. You can define the probability of B given A is same as the probability of -- probability of A intersection B divided by probability of A.

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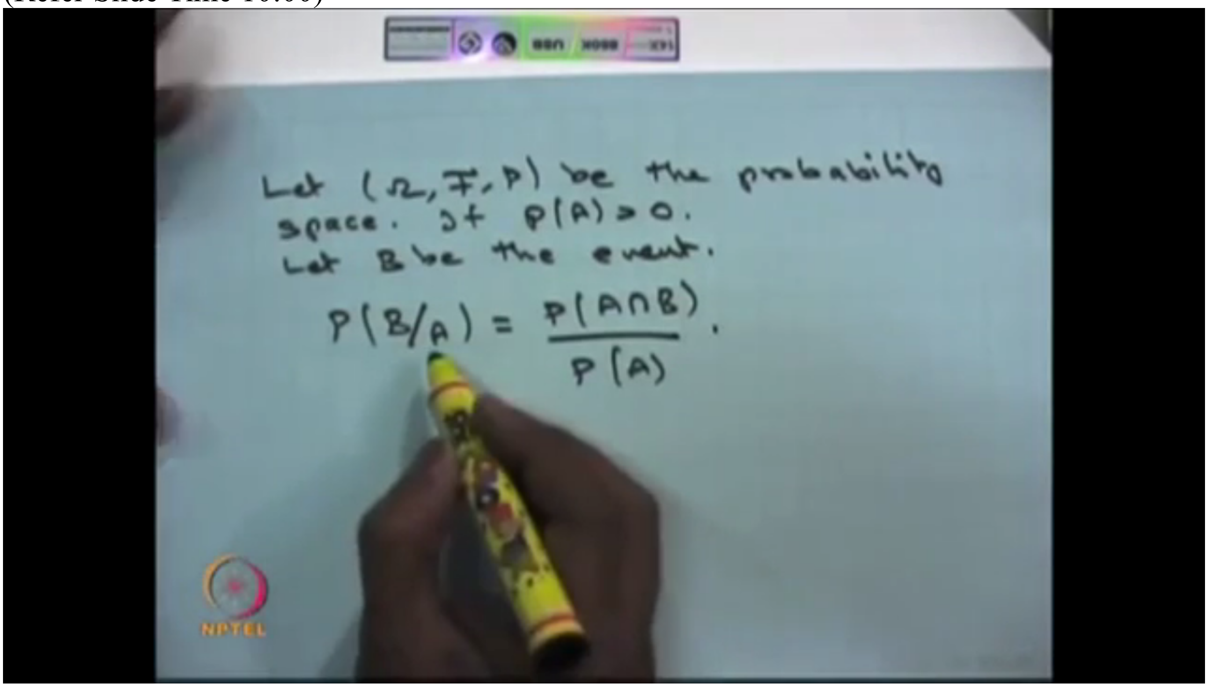

That means if already the event A occurs with the positive probability, then you can find out what is the probability of the event B given that already the event A occurs. That is same as what is the probability that A intersection B divided by probability of A. So this can be -- this is by the definition and this -- this can be visualized from the reduced sample space also.

That means you have a sample space Omega and from the Omega, you take event A. Suppose this is going to be the event A and what we are saying is the event A is already occurred that means with this given condition and suppose you make another event that is a event B and you are asking what is the -- what is the proportion in which already the event A occurs and you are asking what is the probability of event B? That means you find out what is the reduced sample space Omega B and you find out what is the proportion in which or what is the probability of event B occurs in the reduced sample space is same as by using the definition of probability B given A.

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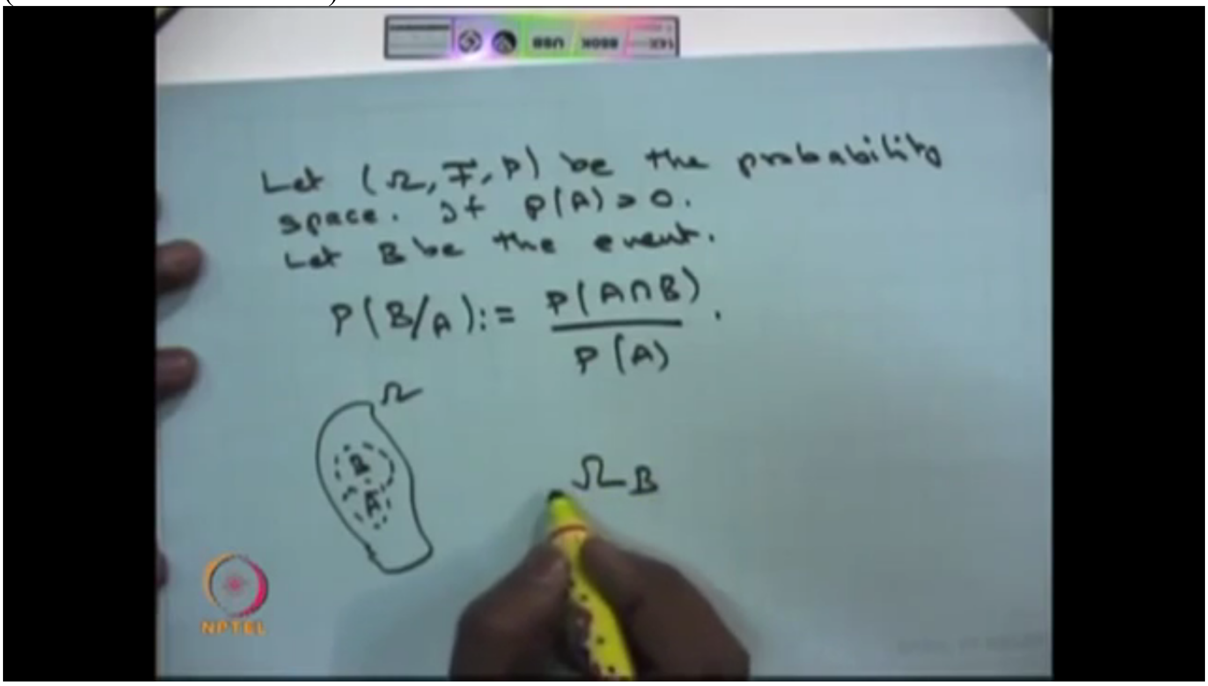

That means you find out what is the intersection A intersection B. That means you find out what is the event which correspond to A intersection B and what is the ratio in which the probability of A intersection B with the probability of A that gives the conditional probability.

If the event A and B are independent, if A and B are independent event, then there is no way of relating the probability of B given A. Then the probability of B given A is same as probability of A. Probability of B, sorry.

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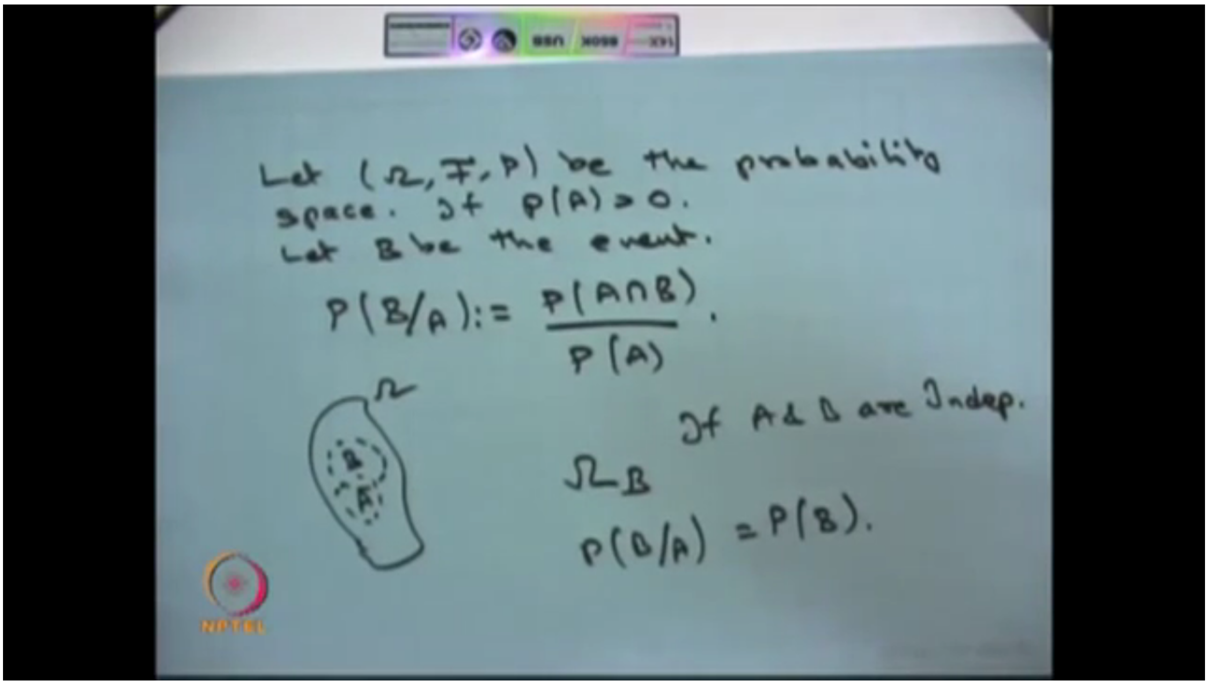

That means there is no dependency over -- there is no dependency over the event B and A. Therefore, it is not going to cause anything with the event B by occurring the event A. Therefore, the probability of B is same as the probability of A intersection, sorry, probability of B intersection A.